



Soft α gs-Closed Sets in Soft Čech Closure Space

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Received 10 July, 2016; Accepted 23 July, 2016 © The author(s) 2014. Published with open access at www.questjournals.org

ABSTRACT:- In this paper, we introduce soft α gs-closed sets and soft α gs-open sets in soft Čech closure spaces, which are defined over an initial universe with a fixed set of parameters and studied some of their basic properties.

Keywords:- Soft set, Soft α gs-closed set, Soft α gs-open set.

I. INTRODUCTION

Fuzzy sets [1], theory of rough sets [2], theory of vague sets [3], theory of intuitionistic fuzzy sets [4], and theory of interval mathematics [5,6] are the tools, which are dealing with uncertainties. But all these theories have their own difficulties, namely inadequacy of parameterization. In 1999, D. Molodtsov [6] introduced the notion of soft set to deals with inadequacy of parameterization. Later, he applied this theory to several directions [7,8].

Levine [9] introduced generalized closed sets in topological space in order to extend some important properties of closed sets to a large family of sets. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets.

E. Čech [10] introduced the concept of closure spaces. In Čech's approach the operator satisfies idempotent condition among Kuratowski axioms. This condition need not hold for every set A of X. When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalization of a topological space. In 2010, Chawalit Boonpok [11] introduced generalized closed sets in Čech closure spaces.

R. Gowri and G. Jegadeesan [12,13,14,15,16,17] introduced and studied the concept of lower separation axioms, higher separation axioms, soft generalized closed sets, soft ∂ -closed sets, strongly soft g-closed sets, strongly soft ∂ -closed sets and strongly soft g^{**} -closed sets in soft Čech closure spaces.

In this paper, we introduce soft α gs-closed sets and soft α gs-open sets in soft Čech closure spaces, which are defined over an initial universe with a fixed set of parameters and studied some of their basic properties.

II. PRELIMINARIES

In this section, we recall the basic definitions of soft Čech closure spaces.

Definition 2.1 [12]. Let X be an initial universe set, A be a set of parameters. Then the function $k: P(X_{F_A}) \rightarrow P(X_{F_A})$ defined from a soft power set $P(X_{F_A})$ to itself over X is called Čech closure operator if it satisfies the following axioms:

(C1) $k(\emptyset_A) = \emptyset_A$.

(C2) $U_A \subseteq k(U_A)$.

(C3) $k(U_A \cup V_A) = k(U_A) \cup k(V_A)$.

Then (X, k, A) or (F_A, k) is called a soft Čech closure space.

Definition 2.2 [12]. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft k-closed (soft closed) if $k(U_A) = U_A$.

Definition 2.3 [12]. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft k-open (soft open) if $k(U_A^c) = U_A^c$.

Definition 2.4 [12]. A soft set $Int(U_A)$ with respect to the closure operator k is defined as $Int(U_A) = F_A - k(F_A - U_A) = [k(U_A^c)]^c$. Here $U_A^c = F_A - U_A$.

Definition 2.5 [12]. A soft subset U_A in a soft Čech closure space (F_A, k) is called Soft neighbourhood of e_F if $e_F \in Int(U_A)$.

Definition 2.6 [12]. If (F_A, k) be a soft Čech closure space, then the associate soft topology on F_A is $\tau = \{U_A^c : k(U_A) = U_A\}$.

Definition 2.7 [12]. Let (F_A, k) be a soft Čech closure space. A soft Čech closure space (G_A, k^*) is called a soft subspace of (F_A, k) if $G_A \subseteq F_A$ and $k^*(U_A) = k(U_A) \cap G_A$, for each soft subset $U_A \subseteq G_A$.

Definition 2.8 [16]. Let U_A be a soft subset of a soft Čech closure space (F_A, k) is said to be

1. Soft semi-open set if $U_A \subseteq k[Int(U_A)]$ and a soft semi-closed set if $Int(k[U_A]) \subseteq U_A$.
2. Soft regular-open set if $Int(k[U_A]) = U_A$ and a soft regular-closed set if $U_A = k[Int(U_A)]$.
3. Soft pre-open set if $U_A \subseteq Int(k[U_A])$ and a soft pre-closed set if $k[Int(U_A)] \subseteq U_A$.
4. Soft α -open set if $U_A \subseteq Int(k[Int(U_A)])$ and soft α -closed set if $k[Int(k[U_A])] \subseteq U_A$.
5. Soft semi pre-open (soft β -open) set if $U_A \subseteq k[Int(k[U_A])]$ and soft semi pre-closed set if $Int(k[Int(U_A)]) \subseteq U_A$.

The smallest soft Čech semi-closed set containing U_A is called soft Čech semi-closure of U_A with respect to k and it is denoted by $k_s(U_A)$.

The largest soft Čech semi-open set contained in U_A is called soft Čech semi-interior of U_A with respect to k and it is denoted by $Int_s(U_A)$.

The smallest soft Čech α -closed set containing U_A is called soft Čech α -closure of U_A with respect to k and it is denoted by $k_\alpha(U_A)$.

The largest soft Čech α -open set contained in U_A is called soft Čech α -interior of U_A with respect to k and it is denoted by $Int_\alpha(U_A)$.

Definition 2.9 [14]. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft generalized closed (briefly soft g-closed) set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft open subset of (F_A, k) .

Definition 2.10 [15]. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft ∂ -closed set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft g-open subset of (F_A, k) .

Definition 2.11 [16]. Let (F_A, k) be a soft Čech closure space. A soft subset $U_A \subseteq F_A$ is called a strongly soft generalized closed (briefly strongly soft g-closed) set if $k[Int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft open subset of (F_A, k) .

Definition 2.12 [16]. Let (F_A, k) be a soft Čech closure space. A soft subset $U_A \subseteq F_A$ is called a strongly soft ∂ -closed set if $k[Int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft g-open subset of (F_A, k) .

Definition 2.13 [16]. Let (F_A, k) be a soft Čech closure space. A soft subset $U_A \subseteq F_A$ is called soft regular generalized closed (briefly soft rg-closed) set if $k[U_A] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft regular open subset of (F_A, k) .

Definition 2.14 [17]. Let (F_A, k) be a soft Čech closure space. A soft subset $U_A \subseteq F_A$ is called a strongly soft g^{**} -closed set if $k[Int(U_A)] \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft ∂ -open subset of F_A .

III. SOFT α GS-CLOSED SETS

In this section, we introduce soft α gs-closed sets in soft Čech closure space and investigate some basic properties.

Definition 3.1. A soft subset U_A of a soft Čech closure space (F_A, k) is said to be soft α -generalized semi closed set (briefly soft α gs-closed) if $k_\alpha(U_A) \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft semi open subset of (F_A, k) .

Example 3.2. Let the initial universe set $X = \{u_1, u_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters. Let $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then $P(X_{F_A})$ are

$F_{1A} = \{(x_1, \{u_1\})\}, F_{2A} = \{(x_1, \{u_2\})\}, F_{3A} = \{(x_1, \{u_1, u_2\})\}, F_{4A} = \{(x_2, \{u_1\})\}, F_{5A} = \{(x_2, \{u_2\})\},$
 $F_{6A} = \{(x_2, \{u_1, u_2\})\}, F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\},$
 $F_{9A} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, F_{11A} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\},$
 $F_{12A} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}, F_{13A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\},$
 $F_{14A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, F_{15A} = F_A, F_{16A} = \emptyset_A.$

An operator $k: P(X_{F_A}) \rightarrow P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$k(F_{1A}) = k(F_{5A}) = F_{8A}, k(F_{2A}) = F_{3A}, k(F_{3A}) = k(F_{9A}) = k(F_{13A}) = F_{13A}, k(F_{4A}) = F_{4A}, k(F_{7A}) = F_{7A},$
 $k(F_{6A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{10A}) = F_{14A}, k(F_{12A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A.$

Here, the α gs-closed sets are $\emptyset_A, F_{4A}, F_{6A}, F_{7A}, F_{9A}, F_{11A}, F_{12A}, F_{13A}, F_A.$

Theorem 3.3. In a soft Čech closure space (F_A, k) , every soft closed subset U_A of F_A is soft α gs-closed.

Proof. Let U_A be soft closed subset of a soft Čech closure space (F_A, k) . Let G_A be soft semi open set such that $U_A \subseteq G_A$. Since, U_A be soft closed subset in F_A . Then, $U_A = k[U_A] \subseteq G_A$. This implies, $k[U_A] \subseteq G_A$. Since, $k_\alpha(U_A) \subseteq k[U_A]$. Then, U_A is soft α gs-closed.

Result 3.4. The converse of the above theorem (3.3) is not true as shown in the following example.

Example 3.5. In example 3.2, here $F_{6A} = \{(x_2, \{u_1, u_2\})\}$ is soft α gs-closed set but not soft closed.

Theorem 3.6. Let (F_A, k) be a soft Čech closure space. If U_A and V_A are two non-empty soft α gs-closed sets and so is $U_A \cap V_A$.

Proof. Let U_A and V_A be two non-empty soft α gs-closed sets. Let G_A be soft semi open subset in F_A . Let $(U_A \cap V_A) \subseteq G_A$. Since, $U_A \subseteq G_A$ and $V_A \subseteq G_A$. This implies, $k_\alpha(U_A) \subseteq G_A$ and $k_\alpha(V_A) \subseteq G_A$. Then, $k_\alpha(U_A) \cap k_\alpha(V_A) \subseteq G_A$. Hence, $k_\alpha(U_A \cap V_A) \subseteq G_A$. Thus, $(U_A \cap V_A)$ is soft α gs-closed set.

Theorem 3.7. Let (F_A, k) be a soft Čech closure space and if $U_A \subseteq F_A$ is soft α gs-closed and soft semi open set, then U_A is soft α -closed.

Proof. Since, U_A is soft α gs-closed subset of F_A . Then, $k_\alpha(U_A) \subseteq G_A$, whenever $U_A \subseteq G_A$ and G_A is soft semi open subset in F_A . Since, U_A is soft semi open. Then, $U_A \subseteq U_A$ and $k_\alpha(U_A) \subseteq U_A$. Since, $U_A \subseteq k_\alpha(U_A)$. Then, $U_A = k_\alpha(U_A)$. Hence, U_A is soft α -closed set.

Theorem 3.8. Let (F_A, k) be a soft Čech closure space. If U_A and V_A are two non-empty soft α gs-closed sets and so is $U_A \cup V_A$.

Proof. Let U_A and V_A be two non-empty soft α gs-closed sets. Let G_A be soft semi open subset in F_A . Let $(U_A \cup V_A) \subseteq G_A$. Then, $U_A \subseteq G_A$ and $V_A \subseteq G_A$. This implies, $k_\alpha(U_A) \subseteq G_A$ and $k_\alpha(V_A) \subseteq G_A$. Then, $k_\alpha(U_A) \cup k_\alpha(V_A) \subseteq G_A$. Hence, $k_\alpha(U_A \cup V_A) \subseteq G_A$. Thus, $(U_A \cup V_A)$ is soft α gs-closed set.

Theorem 3.9. Let (F_A, k) be a soft Čech closure space and let $U_A \subseteq F_A$. If U_A be soft α gs-closed set, then $k_\alpha(U_A) - U_A$ contains no non-empty soft semi closed subset.

Proof. Suppose that, U_A is soft α gs-closed set. Let V_A be a soft semi closed subset of $k_\alpha(U_A) - U_A$. Then, $V_A \subseteq k_\alpha(U_A) \cap (F_A - U_A)$ and so, $U_A \subseteq F_A - V_A$. Since, V_A is soft semi closed. Then, $(F_A - V_A)$ is soft semi open. Thus, $k_\alpha(U_A) \subseteq F_A - V_A$. Consequently, $V_A \subseteq F_A - k_\alpha(U_A)$. Since, $V_A \subseteq k_\alpha(U_A)$. Then, $V_A \subseteq k_\alpha(U_A) \cap (F_A - k_\alpha(U_A)) = \emptyset_A$. Thus, $V_A = \emptyset_A$. Therefore, $k_\alpha(U_A) - U_A$ contains no non-empty soft semi closed subset.

Theorem 3.10. Let (F_A, k) be a soft Čech closure space and let $U_A \subseteq F_A$. If U_A be soft α gs-closed subset and soft semi open, then $U_A = k_\alpha(U_A)$.

Proof. Let U_A be soft α gs-closed subset of a soft Čech closure space (F_A, k) . Since, U_A be soft semi open then $k_\alpha(U_A) \subseteq G_A$ whenever $U_A \subseteq G_A$ and G_A is soft semi open subset of (F_A, k) . Since, U_A is soft semi open subset of (F_A, k) . Since, U_A is soft semi open and $U_A \subseteq U_A$. Then, $k_\alpha(U_A) \subseteq U_A$. Since, $U_A \subseteq k_\alpha(U_A)$. Hence, $U_A = k_\alpha(U_A)$.

Theorem 3.11. Let $U_A \subseteq H_A \subseteq F_A$ and if U_A is soft α gs-closed set in F_A , then U_A is soft α gs-closed relative to H_A .

Proof. Let $U_A \subseteq H_A \subseteq F_A$ and suppose that U_A is soft α gs-closed set in F_A . Let $U_A \subseteq H_A \cap G_A$, where G_A is soft semi open in F_A . Since, U_A is soft α gs-closed set in F_A , $U_A \subseteq G_A$ implies $k_\alpha(U_A) \subseteq G_A$. That is, $H_A \cap k_\alpha(U_A) \subseteq H_A \cap G_A$, where $H_A \cap k_\alpha(U_A)$ is soft Čech α -closure of U_A with respect to k in H_A . Thus, U_A is soft α gs-closed relative to H_A .

Theorem 3.12. Let (F_A, k) be a soft Čech closure space. If U_A is soft α gs-closed subset of F_A , then $k_\alpha((x, u)) \cap U_A \neq \emptyset_A$ for all $(x, u) \in k_\alpha(U_A)$.

Proof. Let U_A be a soft α gs-closed subset of F_A . Assume that, $k_\alpha((x, u)) \cap U_A = \emptyset_A$, for some $(x, u) \in k_\alpha(U_A)$. Then, $U_A \subseteq [k_\alpha((x, u))]^C$. Since, $k_\alpha((x, u))$ is soft α -closed subset in F_A . Then, $[k_\alpha((x, u))]^C$ is soft α -open subset in F_A . Therefore, $[k_\alpha((x, u))]^C$ is soft semi open subset in F_A . Since, U_A is soft α gs-closed set. Then, $k_\alpha(U_A) \subseteq [k_\alpha((x, u))]^C$. This implies, $k_\alpha(U_A) \cap k_\alpha((x, u)) = \emptyset_A$. Then, $(x, u) \notin k_\alpha(U_A)$ is a contradiction. Hence, $k_\alpha((x, u)) \cap U_A \neq \emptyset_A$ holds for each $(x, u) \in k_\alpha(U_A)$.

Theorem 3.13. Let (F_A, k) be a soft Čech closure space. For every $(x, u) \in F_A$, $\{(x, u)\}$ is soft semi closed set or $\{(x, u)\}^C$ is soft α gs-closed.

Proof. Let (F_A, k) be a soft Čech closure space. Assume that, $\{(x, u)\}$ is not a soft semi closed subset of F_A . Then, $\{(x, u)\}^C$ is not a soft semi open. Therefore, the only soft semi open set containing $\{(x, u)\}^C$ is F_A . Thus, $\{(x, u)\}^C \subseteq F_A$. Now, $k_\alpha(\{(x, u)\}^C) \subseteq k_\alpha(F_A) = F_A$. Hence, $\{(x, u)\}^C$ is soft α gs-closed subset in F_A .

IV. SOFT α GS-OPEN SETS

In this section, we introduce soft α gs-open sets in soft Čech closure space and investigate some basic properties.

Definition 4.1. A soft subset U_A of a soft Čech closure space (F_A, k) is called soft α gs-open set if U_A^C is soft α gs-closed in F_A .

Theorem 4.2. Let (F_A, k) be a soft Čech closure space. A soft subset U_A of F_A is soft α gs-open set if and only if $G_A \subseteq \text{int}_\alpha(U_A)$, whenever $G_A \subseteq U_A$ and G_A is soft semi closed subset in F_A .

Proof. Suppose that, U_A is soft α gs-open subset in F_A . Let G_A be soft semi closed subset in F_A . Then, $(F_A - G_A)$ is soft semi open subset in F_A and $U_A^C \subseteq G_A^C$. Since, U_A^C is soft α gs-closed set. Then, $k_\alpha(U_A^C) \subseteq G_A^C$. Therefore, $G_A \subseteq [k_\alpha(U_A^C)]^C = \text{int}_\alpha(U_A)$. Conversely, let V_A be any soft semi open set in F_A such that $U_A^C \subseteq V_A$. This implies, $V_A^C \subseteq U_A$ and V_A^C is soft semi closed. Therefore, $V_A^C \subseteq \text{int}_\alpha(U_A)$. Then, $[\text{int}_\alpha(U_A)]^C \subseteq V_A$. That is, $k_\alpha(U_A^C) \subseteq V_A$. Thus, U_A^C is soft α gs-closed set in F_A . Hence, U_A is soft α gs-open subset in F_A .

Corollary 4.3. Let (F_A, k) be a soft Čech closure space. A soft subset U_A of F_A is soft α gs-closed, then $k_\alpha(U_A) - U_A$ is soft α gs-open subset in F_A .

Proof. Let G_A be a soft semi closed subset in F_A such that $G_A \subseteq k_\alpha(U_A) - U_A$. By theorem 3.9, $G_A = \emptyset_A$. Then, $G_A \subseteq \text{int}_\alpha(k_\alpha(U_A) - U_A)$. Therefore, $k_\alpha(U_A) - U_A$ is soft α gs-open set.

Theorem 4.4. Let (F_A, k) be a soft Čech closure space. If U_A and V_A are soft α gs-open sets, then so is $U_A \cap V_A$.

Proof. Let $U_A^C \cup V_A^C \subseteq G_A$, where G_A is soft semi open. This implies $U_A^C \subseteq G_A$ and $V_A^C \subseteq G_A$. Then, $k_\alpha(U_A^C) \subseteq G_A$ and $k_\alpha(V_A^C) \subseteq G_A$. This implies, $k_\alpha(U_A^C) \cup k_\alpha(V_A^C) \subseteq G_A$. Thus, $k_\alpha(U_A^C \cup V_A^C) \subseteq G_A$. Therefore, $U_A \cap V_A$ is soft α gs-open subset in F_A .

Theorem 4.5. Let $U_A \subseteq H_A \subseteq F_A$ and if H_A is soft α -closed in F_A and U_A is soft α gs-open set in F_A , then U_A is soft α gs-open set relative to H_A .

Proof. It is similar to the proof of the theorem 3.11.

V. Conclusion

In the present work, we have introduced soft α gs-closed sets and soft α gs-open sets in soft Čech closure spaces, which are defined over an initial universe with a fixed set of parameters. We studied the behavior relative to union, intersection of soft α gs-closed sets and soft α gs-open sets. Also, we proved that every soft closed set is soft α gs-closed. In future, findings of this paper will contribute to a new types of soft generalized closed sets in soft Čech closure spaces. Also, the findings in this paper will help to carry out a general framework for their applications in practical life.

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