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Research Paper

Analytical Solutions of Nonlinear 2-Dimensional Brusselator Equation Using Natural Decomposition Method.

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ABSTRACT*: In this research paper, the Natural Decomposition Method (NDM) is applied to obtain solution of nonlinear 2-dimensional Brusselator Equation.*
The method which is a combination of Natural Transform Method (NTM) and the Adomian Decomposition Method (ADM) gives an exact solution in the form of a rapid convergent series with easily computable components.
The application further demonstrates the efficiency and accuracy of the method as compared to some other known techniques. The method is effective and applicable for many other NPDEs in mathematical physics.
Keywords*: Adomian Polynomials, Brusselator Equation, Systems of nonlinear Partial differential Equations, NDM.*

I. INTRODUCTION

Partial differential equations (PDEs) can be applied in numerous ways in various fields of science and engineering, notably in physics, thermodynamic, fluid mechanic and heat transfer. Nonlinear partial differential equations (NPDEs) frequently arise in formulating fundamental laws of nature and in mathematical analysis of a wide variety of problems naturally arising from meteorology, solid-state physics, fluid dynamics, plasma physics, ocean and atmospheric waves, mathematical biology, chemistry, material science, etc. The solutions of nonlinear partial differential equations by various methods have attracted interest of many authors in recent years.

There are many integral transform methods [1,2,3] existing in the literature to solve PDEs, ODEs and integral equations. We will discuss a new integral method called the Natural Decomposition Method (NDM) and apply it to find exact solutions to nonlinear 2-dimensional Brusselator equation. Mahmoud Rawashdeh and Shehu Maitama [1] solved coupled system of nonlinear PDE's using the natural decomposition method and M. Akbari [4] applied reduced differential transformation method (RDTM) to solve nonlinear 2-dimensional Brusslator equation. Other methods used recently to solve PDEs and ODEs are; Elzaki transform [5], Sumudu transform [3] and Decomposition method [6]. F.B.M Belgacem, A.A Karaballi and S.L Kalla [2] carried out analytical investigations of the Sumudu transform and applied to Integral production equations, F.B.M Belgacem and R. Silambarasan [7] used the N-Transform to solve the Maxwell's equation, Bessel's differential equation and linear and nonlinear Klein Gordon Equations and A.K. Adio [8] used natural decomposition method to obtain solution of linear and nonlinear Klein Gordon Equations.

The Adomian decomposition method (ADM) [9], proposed by George Adomian, has numerous applications to a wide class of linear and nonlinear PDEs. For nonlinear models, the NDM shows reliable results in supplying exact solutions and analytical approximate solutions that converges rapidly to the exact solutions [1, 8]. Exact solutions of NPDEs play an important role in the proper understanding of qualitative features of many phenomena and processes in the mentioned areas of natural science.
In this paper, we apply the Natural Decomposition Method (NDM) to the nonlinear 2-dimensional Brusselator equation:

$$\begin{aligned} v_t &= v^2 w - 2v + \frac{1}{4}\left(v_{xx} + v_{yy}\right) \\ w_t &= v - v^2 w + \frac{1}{4}\left(w_{xx} + w_{yy}\right) \end{aligned} \qquad (1.1)$$

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Subject to the initial conditions

$$v(x,y,0)=e^{-(x+y)}\;;$$
$$w(x,y,0)=e^{(x+y)} \qquad (1.2)$$

where $v=v(x,y,t)$ is a function of the variables x,y and t and $(x,y,t)\in\Re^2\times[0,2]$.

The rest of the paper is organized as follows: In section 2, the NDM is introduced and in section 3, the definitions and properties of the N-Transform are discussed. In section 4, the NDM is applied to nonlinear 2-dimensional Brusselator equation. Section 5 is the conclusion.

II. BASIC IDEA OF THE NATURAL TRANSFORM METHOD

Here we discuss some preliminaries about the nature of the Natural Transform Method (NTM).
Consider a function $f(t),\ t\in(-\infty,\infty)$, then the general Integral transform is defined as follows [8,9]:

$$\Im[f(t)](s)=\int_{-\infty}^{\infty}k(s,t)f(t)dt \qquad (2.1)$$

where $k(s,t)$ represent the kernel of the transform, s is the real (complex) number which is independent of t.
Note that when $k(s,t)$ is e^{-st}, $tJ_n(st)$ and $t^{s-1}(st)$, then Equation (2.1) gives, respectively, Laplace Transform, Hankel Transform and Mellin Transform.
Now, for $f(t),\ t\in(-\infty,\infty)$ consider the Integral transforms defined by

$$\Im[f(t)](u)=\int_{-\infty}^{\infty}k(t)f(ut)dt \qquad (2.2)$$

and
$$\Im[f(t)](s,u)=\int_{-\infty}^{\infty}k(s,t)f(ut)dt \qquad (2.3)$$

Note that:

(i) when $k(t)=e^{-t}$, Equation (2.2) gives the Integral Sumudu transform, where parameter s is replaced by u. Moreover, for any value of n, the generalized Laplace and Sumudu transform are respectively defined by [1,7]:

$$\ell[f(t)]=F(s)=s^n\int_0^{\infty}e^{-s^{n+1}t}f(s^nt)dt \qquad (2.4)$$

and
$$S[f(t)]=G(u)=u^n\int_0^{\infty}e^{-u^nt}f(tu^{n+1})dt \qquad (2.5)$$

(ii) when $n=0$, Equation (2.4) and Equation (2.5) are the Laplace and Sumudu transform respectively.

III. DEFINITIONS AND PROPERTIES OF THE N-TRANSFORM

The natural transform of the function $f(t),\ t\in(-\infty,\infty)$ is defined by [1,7]:

$$\mathrm{N}[f(t)]=R(s,u)=\int_{-\infty}^{\infty}e^{-st}f(ut)dt\;;\quad s,u\in(-\infty,\infty) \qquad (3.1)$$

where $\mathrm{N}[f(t)]$ is the natural transformation of the time function $f(t)$ and the variable s and u are the natural transform variables.

Note:

(i) Equation (3.1) can be written in the form [1,5]:

$$\mathrm{N}[f(t)]=\int_{-\infty}^{\infty}e^{-st}f(ut)dt\;;\quad s,u\in(-\infty,\infty)$$

$$=\left[\int_{-\infty}^{0} e^{-st} f(ut)dt \ ; s,u \in (-\infty,0)\right] + \left[\int_{0}^{\infty} e^{-st} f(ut)dt \ ; s,u \in (0,\infty)\right]$$

$$= N^{-}[f(t)] + N^{+}[f(t)] = N[f(t)H(-t)] + N[f(t)H(t)] = R^{-}(s,u) + R^{+}(s,u)$$

where $H(\cdot)$ is the Heaviside function.

(ii) If the function $f(t)H(t)$ is defined on the positive real axis, with $t \in (0,\infty)$ and in the set $A = \left\{ f(t) : \exists\, M, \tau_1, \tau_2 > 0, s.t. |f(t)| < Me^{\frac{|t|}{\tau_j}}, \text{where } t \in (-1)^j \times [0,\infty),\ j = 1,2 \right\}$, then we define the Natural transform (N-Transform) as [1,7]:

$$N[f(t)H(t)] = N^{+}[f(t)] = R^{+}(s,u) = \int_{0}^{\infty} e^{-st} f(ut)dt \ ; \quad s,u \in (0,\infty) \qquad (3.2)$$

(iii) If $u = 1$, Equation (3.2) can be reduced to the Laplace transform and if $s = 1$, then Equation (3.2) can be reduced to the Sumudu transform. Now, we give some of the N-Transforms and the conversion to Sumudu and Laplace [1,10].

Table 1 : Special N-Transforms and the conversion to Sumudu and Laplace.

$f(t)$	$N[f(t)]$	$S[f(t)]$	$\ell[f(t)]$
1	$\frac{1}{s}$	1	$\frac{1}{s}$
t	$\frac{u}{s^2}$	u	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-au}$	$\frac{1}{1-au}$	$\frac{1}{s-a}$
$\frac{t^{n-1}}{(n-1)!}, n=1,2,\ldots$	$\frac{u^{n-1}}{s^n}$	u^{n-1}	$\frac{1}{s^n}$
$\cos t$	$\frac{s}{s^2+u^2}$	$\frac{1}{1+u^2}$	$\frac{s}{1+s^2}$
$\sin t$	$\frac{u}{s^2+u^2}$	$\frac{u}{1+u^2}$	$\frac{1}{1+s^2}$

Some basic properties of the N-Transform are given as follows [1,10]:

(i) If $R(s,u)$ is the natural transform and $F(s)$ is the Laplace transform of the function $f(t)$, then

$$N^{+}[f(t)] = R(s,u) = \frac{1}{u}\int_{0}^{\infty} e^{-\frac{st}{u}} f(t)dt = \frac{1}{u}F\left(\frac{s}{u}\right).$$

(ii) If $R(s,u)$ is the natural transform and $G(u)$ is the Sumudu transform of the function $f(t)$, then

$$N^{+}[f(t)] = R(s,u) = \frac{1}{s}\int_{0}^{\infty} e^{-t} f\left(\frac{ut}{s}\right)dt = \frac{1}{s}G\left(\frac{u}{s}\right).$$

(iii) If $N^{+}[f(t)] = R(s,u)$, then $N^{+}[f(at)] = \frac{1}{a}R(s,u)$.

(iv) If $N^{+}[f(t)] = R(s,u)$, then $N^{+}[f'(t)] = \frac{s}{u}R(s,u) - \frac{f(0)}{u}$.

(v) If $N^{+}[f(t)] = R(s,u)$, then $N^{+}[f''(t)] = \frac{s^2}{u^2}R(s,u) - \frac{s}{u^2}f(0) - \frac{f'(0)}{u}$.

(vi) Linearity property [10]: If a and b are non-zero constants, and $f(t)$ and $g(t)$ are functions, then $\mathrm{N}^{+}[af(t)\pm bg(t)]=a\,\mathrm{N}^{+}[f(t)]\pm b\,\mathrm{N}^{+}[g(t)]=a\,\mathrm{F}^{+}(s,u)\pm b\,\mathrm{G}^{+}(s,u)$.Moreover, $\mathrm{F}^{+}(s,u)$ and $\mathrm{G}^{+}(s,u)$ are the N-transforms of $f(t)$ and $g(t)$ respectively.

IV. APPLICATIONS

Here, we consider the following nonlinear 2-dimensional Brusselator equation:

$$\begin{aligned} v_t &= v^2 w - 2v + \frac{1}{4}\left(v_{xx}+v_{yy}\right) \\ w_t &= v - v^2 w + \frac{1}{4}\left(w_{xx}+w_{yy}\right) \end{aligned} \tag{4.1}$$

Subject to the initial conditions

$$\begin{aligned} v(x,y,0) &= e^{-(x+y)}; \\ w(x,y,0) &= e^{(x+y)} \end{aligned} \tag{4.2}$$

We first take the N-Transform of Equation (4.1), to obtain

$$\begin{aligned} \mathrm{N}^{+}[v_t] &= \mathrm{N}^{+}[v^2 w] - 2\,\mathrm{N}^{+}[v] + \frac{1}{4}\mathrm{N}^{+}[v_{xx}+v_{yy}] \\ \mathrm{N}^{+}[w_t] &= \mathrm{N}^{+}[v] - \mathrm{N}^{+}[v^2 w] + \frac{1}{4}\mathrm{N}^{+}[w_{xx}+w_{yy}] \end{aligned}$$

Using the properties in Table 1 and properties of the N-Transform, we have

$$\begin{aligned} \frac{s}{u}v(x,s,u) - \frac{v(x,y,0)}{u} &= \mathrm{N}^{+}[v^2 w] - 2\,\mathrm{N}^{+}[v] + \frac{1}{4}\mathrm{N}^{+}[v_{xx}+v_{yy}] \\ \frac{s}{u}w(x,s,u) - \frac{w(x,y,0)}{u} &= \mathrm{N}^{+}[v] - \mathrm{N}^{+}[v^2 w] + \frac{1}{4}\mathrm{N}^{+}[w_{xx}+w_{yy}] \end{aligned} \tag{4.3}$$

Substituting Equation (4.2) into Equation (4.3), we have:

$$\begin{aligned} v(x,s,u) &= \frac{1}{s}e^{-(x+y)} + \frac{u}{s}\mathrm{N}^{+}\left[v^2 w - 2v + \frac{1}{4}\left(v_{xx}+v_{yy}\right)\right] \\ w(x,s,u) &= \frac{1}{s}e^{(x+y)} + \frac{u}{s}\mathrm{N}^{+}\left[v - v^2 w + \frac{1}{4}\left(w_{xx}+w_{yy}\right)\right] \end{aligned} \tag{4.4}$$

Now , taking the inverse N-Transform of Equation (4.4), we have

$$\begin{aligned} v(x,y,t) &= e^{-(x+y)} + \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[v^2 w - 2v + \frac{1}{4}\left(v_{xx}+v_{yy}\right)\right]\right] \\ w(x,y,t) &= e^{(x+y)} + \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[v - v^2 w + \frac{1}{4}\left(w_{xx}+w_{yy}\right)\right]\right] \end{aligned} \tag{4.5}$$

we now assume a series solutions for the unknown functions v(x,y,t) and w(x,y,t) of the form:

$$\begin{aligned} v(x,y,t) &= \sum_{n=0}^{\infty} v_n(x,y,t) \\ w(x,y,t) &= \sum_{n=0}^{\infty} w_n(x,y,t) \end{aligned} \tag{4.6}$$

ThenEquation (4.5)

becomes

$$
\begin{aligned}
v(x,y,t) &= e^{-(x+y)} + \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\sum_{n=0}^{\infty}\left(v_{nxx}+v_{nyy}\right)-2\sum_{n=0}^{\infty}v_n+\sum_{n=0}^{\infty}A_n(v,w)\right]\right] \\
w(x,y,t) &= e^{(x+y)} + \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\sum_{n=0}^{\infty}\left(w_{nxx}+w_{nyy}\right)+\sum_{n=0}^{\infty}v_n-\sum_{n=0}^{\infty}A_n(v,w)\right]\right]
\end{aligned}
\qquad (4.7)
$$

where A_n are the Adomian polynomials representing the nonlinear terms v^2w.

Then by applying the natural decomposition techniques, we can generate the recursive relation as follows:

$$v_0(x,y,t) = e^{-(x+y)} \qquad (4.8)$$

$$v_1(x,y,t) = \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\left(v_{0xx}+v_{0yy}\right)-2v_0+A_0(v,w)\right]\right]$$

$$v_2(x,y,t) = \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\left(v_{1xx}+v_{1yy}\right)-2v_1+A_1(v,w)\right]\right]$$

Thus,

$$v_{n+1}(x,y,t) = \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\left(v_{nxx}+v_{nyy}\right)-2v_n+A_n(v,w)\right]\right] \quad n\geq 0 \qquad (4.9)$$

Similarly,

$$w_0(x,y,t) = e^{(x+y)} \qquad (4.10)$$

$$w_1(x,y,t) = \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\left(w_{0xx}+w_{0yy}\right)+v_0-A_0(v,w)\right]\right]$$

$$w_2(x,y,t) = \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\left(w_{1xx}+w_{1yy}\right)+v_1-A_1(v,w)\right]\right]$$

Eventually ,

$$w_{n+1}(x,y,t) = \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\left(w_{nxx}+w_{nyy}\right)+v_n-A_n(v,w)\right]\right] \quad n\geq 0 \qquad (4.11)$$

Therefore from the recursive relation derived in Equation (4.9) and Equation (4.11) we can compute the remaining components of the solution as follows:

$$
\begin{aligned}
v_1(x,y,t) &= \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\left(v_{0xx}+v_{0yy}\right)-2v_0+A_0(v,w)\right]\right] \qquad (4.12)\\
&= \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{4}\left(v_{0xx}+v_{0yy}\right)-2v_0+v_0v_0w_0\right]\right] \\
&= \mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}\left[\frac{1}{2}e^{-(x+y)}-2e^{-(x+y)}+e^{-(x+y)}\right]\right] \\
&= -\frac{1}{2}e^{-(x+y)}\,\mathrm{N}^{-1}\left[\frac{u}{s}\mathrm{N}^{+}[1]\right] = -\frac{1}{2}te^{-(x+y)}.
\end{aligned}
$$

Similarly,

$$w_1(x,y,t) = N^{-1}\left[\frac{u}{s}N^{+}\left[\frac{1}{4}\left(w_{0xx}+w_{0yy}\right)+v_0-A_0(v,w)\right]\right] \qquad (4.13)$$

$$= N^{-1}\left[\frac{u}{s}N^{+}\left[\frac{1}{4}\left(w_{0xx}+w_{0yy}\right)+v_0-v_0v_0w_0\right]\right]$$

$$= N^{-1}\left[\frac{u}{s}N^{+}\left[\frac{1}{2}e^{(x+y)}+e^{-(x+y)}-e^{-(x+y)}\right]\right]$$

$$= \frac{1}{2}e^{(x+y)}N^{-1}\left[\frac{u}{s}N^{+}[1]\right] = \frac{1}{2}te^{(x+y)}.$$

and

$$v_2(x,y,t) = N^{-1}\left[\frac{u}{s}N^{+}\left[\frac{1}{4}\left(v_{1xx}+v_{1yy}\right)-2v_1+A_1(v,w)\right]\right] \qquad (4.14)$$

$$= N^{-1}\left[\frac{u}{s}N^{+}\left[\frac{1}{4}\left(v_{1xx}+v_{1yy}\right)-2v_1+2v_1v_0w_0+v_0v_0w_1\right]\right]$$

$$= N^{-1}\left[\frac{u}{s}N^{+}\left[-\frac{1}{4}te^{-(x+y)}+te^{-(x+y)}-te^{-(x+y)}+\frac{1}{2}te^{-(x+y)}\right]\right]$$

$$= \frac{1}{4}e^{-(x+y)}N^{-1}\left[\frac{u}{s}N^{+}[t]\right] = \frac{1}{4}e^{-(x+y)}N^{-1}\left[\frac{u^2}{s^3}\right]$$

$$= \frac{1}{4}e^{-(x+y)}\frac{t^2}{2!}.$$

$$w_2(x,y,t) = N^{-1}\left[\frac{u}{s}N^{+}\left[\frac{1}{4}\left(w_{1xx}+w_{1yy}\right)+v_1-A_1(v,w)\right]\right] \qquad (4.15)$$

$$= N^{-1}\left[\frac{u}{s}N^{+}\left[\frac{1}{4}\left(v_{1xx}+v_{1yy}\right)+v_1-2v_1v_0w_0-v_0v_0w_1\right]\right]$$

$$= N^{-1}\left[\frac{u}{s}N^{+}\left[\frac{1}{4}te^{(x+y)}-\frac{1}{2}te^{-(x+y)}+te^{-(x+y)}-\frac{1}{2}te^{-(x+y)}\right]\right]$$

$$= \frac{1}{4}e^{(x+y)}N^{-1}\left[\frac{u}{s}N^{+}[t]\right] = \frac{1}{4}e^{(x+y)}N^{-1}\left[\frac{u^2}{s^3}\right]$$

$$= \frac{1}{4}e^{(x+y)}\frac{t^2}{2!}.$$

Eventually, the approximate solution of the unknown functions v(x,y,t) and w(x,y,t) are given by:

$$v(x,y,t)=\sum_{n=0}^{\infty}v_n(x,y,t) \qquad (4.16)$$

$$=v_0(x,y,t)+v_1(x,y,t)+v_2(x,y,t)+\dots$$

$$=e^{-(x+y)}-\frac{1}{2}te^{-(x+y)}+\frac{1}{4}.\frac{t^2}{2!}e^{-(x+y)}+\dots$$

$$=e^{-(x+y)}\left(1-\frac{t}{2}+\frac{1}{4}.\frac{t^2}{2!}+\dots\right)$$

$$=e^{-\left(x+y+\frac{t}{2}\right)}.$$

And

$$w(x,y,t)=\sum_{n=0}^{\infty}w_n(x,y,t) \qquad (4.17)$$

$$=w_0(x,y,t)+w_1(x,y,t)+w_2(x,y,t)+\dots$$

$$=e^{(x+y)}+\frac{1}{2}te^{(x+y)}+\frac{1}{4}.\frac{t^2}{2!}e^{(x+y)}+\dots$$

$$=e^{(x+y)}\left(1+\frac{t}{2}+\frac{1}{4}.\frac{t^2}{2!}+\dots\right)$$

$$=e^{\left(x+y+\frac{t}{2}\right)}.$$

Hence, the exact solutions of the given nonlinear 2-dimensional Brusselator equation are given by:

$$v(x,y,t)=e^{-\left(x+y+\frac{t}{2}\right)}$$
$$w(x,y,t)=e^{\left(x+y+\frac{t}{2}\right)} \qquad (4.18)$$

Which is in agreement with the result obtained by RDTM [4].

V. CONCLUSION

In this article, the Natural Decomposition Method (NDM) was proposed for solving nonlinear 2-dimensional Brusselator Equation with initial conditions.

We successfully found an exact solutions and compare the result with RDTM [4]. This clearly shows that the Natural Decomposition Method NDM introduces a significant improvement in the field over existing technique.

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