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Research Paper

Stability Analysis of Three Different fish Harvesting Models

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ABSTRACT: *In this paper we have discussed three different fish harvesting models where we have assumed that the particular species of fish we considered follows the well known logistic law. In our discussion we have assumed the controlled fishing. Analyzing the stability of critical points we have predicted how many fish can be harvested or maximum how many licenses can be issued in a year for a sustainable population of the fish species.*

I. INTRODUCTION

Suppose in a lake we model the population of a particular species of with the logistic law $\frac{dx}{dt} = \alpha x(K - x)$, where $x(t)$ is the number of fish in tens of thousands at time t, α is the intrinsic growth rate parameter and K is the carrying capacity of the lake. Then our main problem is that how many fish can be harvested in a year so that the species does not get extinct from the lake over the period of time.

The fish harvesting problem is then can be modeled as $\frac{dx}{dt} = \alpha x(K - x) - g(x)$, where $g(x)$ is the number of fish harvested at time t.

In this paper we have considered three different harvesting models by taking three different forms of the harvesting function $g(x)$ viz.

1. $g(x) = h$, i.e fishing is done at a constant rate.
2. $g(x) = hx$, i.e every time fishing depends on the population size at that time.
3. $g(x) = {hx}/{(\alpha + x)}$, i.e fishing can be from zero to a maximum value h, no matter how large the population size becomes.

$h \le 0$ is discounted because of negative fishing and no fishing case.

II. MAIN DISCUSSIONS

Model (1):

In this model we have considered the harvesting at a constant rate i.e fishing is independent of the present population size.

The explicit equation of the model is given by, $\frac{dx}{dt} = \alpha x(K - x) - h$ This model has no critical point for $h > \frac{K^2\alpha}{4}$, i.e the population does not stabilize to any size for this range of the harvesting parameter and so this range is discarded.

It has only one critical point viz. $x_0 = \frac{K}{2}$ when $h = \frac{K^2\alpha}{4}$. But this critical point is found to be neither stable nor unstable i.e this is a point of inflexion. So in this case also population does not stabilize to any size and hence this case is also discounted.

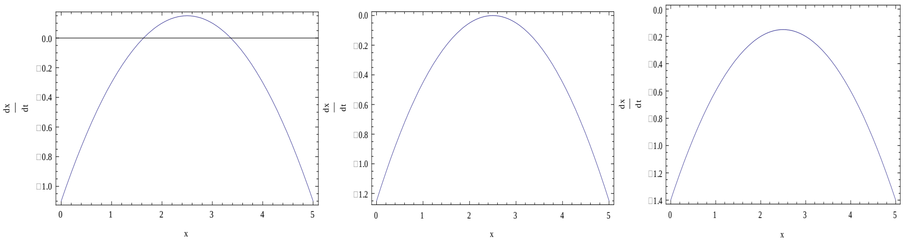
For $h < \frac{K^2\alpha}{4}$, the model has two critical points viz. $x_1 = \frac{K\alpha+\sqrt{K^2\alpha^2-4\alpha h}}{2\alpha}$ and $x_2 = \frac{K\alpha-\sqrt{K^2\alpha^2-4\alpha h}}{2\alpha}$.

The critical point x_1 is stable whereas x_2 is unstable. Thus when $h < \frac{K^2\alpha}{4}$, the population stabilizes to $x_1 \times 10^4$.

Thus discussing the first model we have found that the harvesting is sustainable for $0 < h < \frac{K^2\alpha}{4}$, where the fish persist, and it is unsustainable if $h \ge \frac{K^2\alpha}{4}$, where the fish becomes extinct from the lake.

To get an idea of the above discussion we suppose $\alpha = 0.2$ and $K = 5$ $(i.e\ 50000)$. If for fishing, license is required and each license holder can catch 1000 fish in a year, then the natural question is that how many licenses can be issued for sustainable population. Let "L" be the number of licenses that can be issued. Then, $h = (0.1)L$. For sustainable population we must have $h < \frac{K^2\alpha}{4} \Rightarrow (o.1)L < \frac{5^2\times0.2}{4} \Rightarrow L < 12.5$ i.e maximum 12 licenses can be issued for the species to persist.

Following are the plots of $\frac{dx}{dt}$ $Vs\ x$ for the parameter values $h = 1.1, 1.25\ and\ 1.4$ respectively. (For all the plots we assumed $\alpha = 0.2$ and $K = 5$)



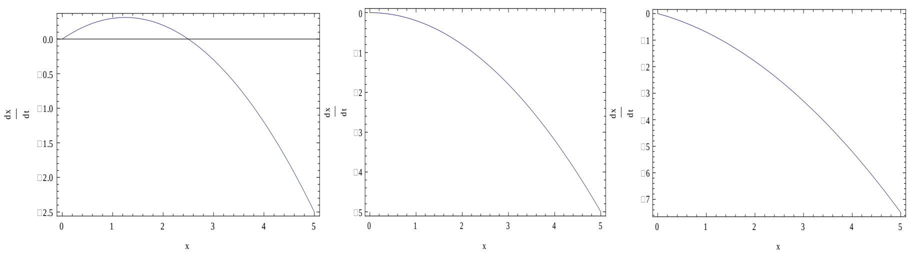
It is seen from the above figures that the fish population persist for $h = 1.1$ and dies out for $h = 1.25$ and for $h = 1.4$.

Model (2):

In this model we have considered the harvesting which is dependent of the present size of the population. The explicit equation of the model is given by, $\frac{dx}{dt} = \alpha x(K - x) - hx$ This model has only one critical point $x_0 = 0$ for $h \geq K\alpha$ and has two critical points $x_0 = 0$ and $x_1 = \frac{K\alpha - h}{\alpha}$ for $h < K\alpha$. We are not interested on the critical point $x_0 = 0$ as it indicates the zero population. So the cases $h \geq K\alpha$ are discarded.
When $h < K\alpha$, the critical point $x_1 = \frac{K\alpha - h}{\alpha}$ is stable and the population stabilizes to $x_1 \times 10^4$.

Thus we have found that for the second model the harvesting is sustainable for $0 < h < K\alpha$, where the fish persist, and it is unsustainable if $h \geq K\alpha$, where the fish becomes extinct from the lake.
We assume for example, $\alpha = 0.2$ and $K = 5\ (i.e\ 50000)$. We suppose that only 10% of the present population of fish are allowed to harvest by each license holder. Then, $h = 0.01 \times L < K\alpha = 5 \times 0.2 => L < 100$
i.e maximum 99 number of licenses can be issued for sustainable population.

Following are the plots of $\frac{dx}{dt}$ $Vs\ x$ for the parameter values $h = 0.5, 1\ and\ 1.5$ respectively. (For all the plots we assumed $\alpha = 0.2$ and $K = 5$)



It is seen from the above figures that the fish population persist for $h = 0.5$ and dies out for $h = 1$ and for $h = 1.5$.

Model (3):

In this model harvesting can be from zero to a maximum limit h, no matter how big is the population size.
The explicit equation of the model is given by, $\frac{dx}{dt} = \alpha x(K - x) - h\frac{x}{\alpha + x}$ This model has only one critical point $x_0 = 0$ for $h > \frac{\alpha(K+\alpha)^2}{4}$, two critical points viz. $x_0 = 0$ and $x_1 = \frac{K-\alpha}{2}$ for $h = \frac{\alpha(K+\alpha)^2}{4}$, three critical points viz. $x_0 = 0$, $x_1 = \frac{\alpha(K-\alpha)+\sqrt{\alpha^2(K-\alpha)^2-4\alpha(h-K\alpha^2)}}{2\alpha}$ and $x_1 = \frac{\alpha(K-\alpha)-\sqrt{\alpha^2(K-\alpha)^2-4\alpha(h-K\alpha^2)}}{2\alpha}$ for $h < \frac{\alpha(K+\alpha)^2}{4}$. As stated before the critical point $x_0 = 0$ is not of our interest. So the case $h > \frac{\alpha(K+\alpha)^2}{4}$ is discounted because in this case the only critical point is $x_0 = 0$ i.e no population persist.
For $h = \frac{\alpha(K+\alpha)^2}{4}$, the critical point $x_1 = \frac{K-\alpha}{2}$ is the point of inflexion and so this parameter range is also not acceptable for a sustainable population.
For $h < \frac{\alpha(K+\alpha)^2}{4}$, the critical point $x_1 = \frac{\alpha(K-\alpha)+\sqrt{\alpha^2(K-\alpha)^2-4\alpha(h-K\alpha^2)}}{2\alpha}$ is stable whereas the critical point $x_1 = \frac{\alpha(K-\alpha)-\sqrt{\alpha^2(K-\alpha)^2-4\alpha(h-K\alpha^2)}}{2\alpha}$ is unstable. Thus for this parameter range the population stabilizes to the value $x_1 \times 10^4$.

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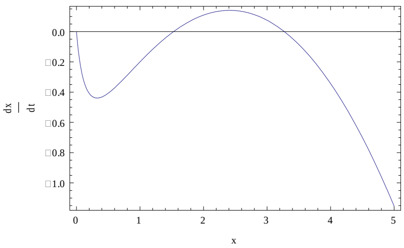
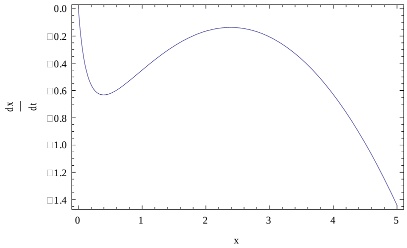
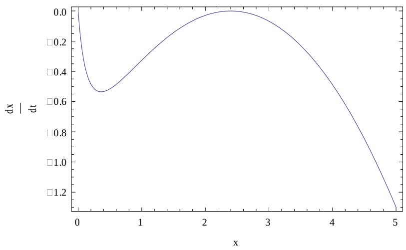
Thus discussing the third model we have found that the harvesting is sustainable for $0 < h < \frac{\alpha(K+\alpha)^2}{4}$, where the fish persist, and it is unsustainable if $h \geq \frac{\alpha(K+\alpha)^2}{4}$, where the fish species becomes extinct from the lake.
As before we assume, $\alpha = 0.2$ and $K = 5$ $(i.e\ 50000)$. Then $h < \frac{0.2(5+0.2)^2}{4} = 1.352$ i.e maximum 13520 fish of the particular species can be harvested in a year. Suppose P number of licenses are issued which are allowed to catch maximum of 500 fish in a year and Q number of licenses are issued each of which can catch a maximum of 1000 fish in a year.
So we must have, $P \times 500 + Q \times 1000 < 13520 \Rightarrow P + 2Q < 27.04$. Thus for maximum harvesting we get $P + 2Q = 27$. The incongruent solutions of this linear Diophantine equation are the possible number of licenses that can be issued, one such solution is P=7, Q=10.
Following are the plots of $\frac{dx}{dt}$ $Vs\ x$ for the parameter values $h = 1.2, 1.352$ and 1.5 respectively. (For all the plots we assumed $\alpha = 0.2$ and $K = 5$)







It is seen from the above figures that the fish population persist for $h = 1.2$ and dies out for $h = 1.352$ and for $h = 1.5$.

III. CONCLUSION

From the stability analysis of critical points it is seen that all the three harvesting models possess a particular value of the harvesting parameter below which the population is sustainable. On crossing the particular value the species die out, thus showing a behavioral change in the dynamics of the system at that particular value of the harvesting parameter and thus this value is the bifurcation value. We have analyzed three harvesting models out of which the third model is the more realistic one because in the first model fishing is done at a constant rate and in the second model fishing is proportional to the present population size but in the third model harvesting is independent of the population size and can vary from zero harvesting to a maximum value.

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