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Research Paper

Common Fixed Point Theorems for $\mathcal{R}$-weakly commuting Mappings in Fuzzy Metric Space

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ABSTRACT: *In this paper, we prove some common fixed point theorems for $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$, type $(\mathcal{A}_f)$ and type (P) using control function in Fuzzy metric space. At the last we provide an application in support of our theorems.*
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I. INTRODUCTION

In 1965, when Zadeh [27] presented the concept of fuzzy set, it marked a new era in the development of fuzzy mathematics. Many applications of fuzzy set theory can be found in neural network theory, applied science, stability theory, mathematical programming, modelling theory, engineering sciences etc. There are many view points of the notion of the metric space in fuzzy topology, see, e.g., Erceg [3], Deng [2], Kaleva and Seikkala[12], Kramosil and Michalek [11], George and Veermani [4]. In this paper, we are considering the Fuzzy metric space in the sense of Kramosil and Michalek [11].

Definition 1.1 A binary operation $*$ on $[0,1]$ is a t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,
(ii) $a * 1 = a$ for every $a \in [0,1]$,
(iii) $a * b \le c * d$ whenever $a \le c$ and $b \le d$.

Basics examples of t-norm are: $\Delta_L(a,b) = \max(a+b-1, 0)$,
t- norm Δ_P, $\Delta_P(a,b) = ab$ and t- norm Δ_M, $\Delta_M(a,b) = min\{a,b\}$.

Definition 1.2[11] The 3- tuple $(\mathfrak{D}, \mathcal{M}, *)$ is called a KM- fuzzy metric space if $\mathfrak{D}$ is an arbitrary set, $*$ is a continuous t- norm and $\mathcal{M}$ is a fuzzy set on $\mathfrak{D}^2 \times [0,\infty)$ satisfying the following conditions for all $x, y, z \in \mathfrak{D}$ and $s, t > 0$;

(KMF-1) $\mathcal{M}(x,y,0) = 0, \mathcal{M}(x,y,t) > 0$;
(KMF-2) $\mathcal{M}(x,y,t) = 1$, for all $t > 0$ if and only $if\ x = y$;
(KMF-3) $\mathcal{M}(x,y,t) = \mathcal{M}(y,x,t)$;
(KMF-4) $\mathcal{M}(x,z, t+s) \ge (\mathcal{M}(x,y,t) * \mathcal{M}(y,z,s))$;
(KMF-5) $\mathcal{M}(x,y,.): [0,\infty) \to [0,1]$ is left continuous.

Note that $\mathcal{M}(x,y,t)$ can be thought of as the degree of nearness between x and y with respect to t.

Definition 1.3[11] A sequence $\{x_n\}$ in $(\mathfrak{D}, \mathcal{M}, *)$ is said to be:

(i) Convergent with limit x if $lim_{n\to\infty} \mathcal{M}(x_n, x, t) = 1$ for all $t > 0$.
(ii) Cauchy sequence in $\mathfrak{D}$ if given $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer $N_{\epsilon,\lambda}$ such that $\mathcal{M}(x_n, x_m, \epsilon) > 1 - \lambda$ for all $n, m \ge N_{\epsilon,\lambda}$.
(iii) Complete if every Cauchy sequence in $\mathfrak{D}$ is convergent in $\mathfrak{D}$.

Fixed point theory in fuzzy metric spaces has been developing since the paper of Grabiec [5]. Subramanian [24] gave a generalization of Jungck 's [7] theorem for commuting mapping in the setting of fuzzy metric spaces.

In 1996, Jungck[7] introduced the notion of weakly compatible as follows:

Definition 1.4[7] Two maps $\mathcal{A}$ and $\mathcal{S}$ are said to be weakly compatible if they commute at their coincidence points.

In 1999, Vasuki [25] introduced the notion of weakly commuting as follows:

Definition 1.5[25] Two self-mapping $\mathcal{A}$ and $\mathcal{S}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ are said to be weakly commuting if $\mathcal{M}(\mathcal{A}\mathcal{S}u, \mathcal{S}\mathcal{A}u, t) \geq \mathcal{M}(\mathcal{A}u, \mathcal{S}u, t)$, for each $u \in \mathfrak{D}$ and for each $t > 0$.

In 1994, Mishra [14] gives the notion of compatible mappings in fuzzy metric space as follows:

Definition 1.6[14] A pair of self-mappings $\{\mathcal{A},\mathcal{B}\}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ is said to be compatible if $lim_{n\to\infty} \mathcal{M}(\mathcal{A}\mathcal{B}x_n,\mathcal{B}\mathcal{A}x_n,t) = 1$, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty} \mathcal{A}x_n = lim_{n\to\infty} \mathcal{B}x_n = u$, for some $u \in \mathfrak{D}$ and for all $t > 0$.

In 1994, Pant [15] introduced the concept of R-weakly commuting maps in metric spaces. Later on, Vasuki [25] initiated the concept of non-compatible mapping in fuzzy metric space.

Definition 1.7[25] A pair of self-mappings $\{\mathcal{A},\mathcal{B}\}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ is said to be non -compatible if $lim_{n\to\infty} \mathcal{M}(\mathcal{A}\mathcal{B}x_n,\mathcal{B}\mathcal{A}x_n,t) \neq 1$ or nonexistent, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty} \mathcal{A}x_n = lim_{n\to\infty} \mathcal{B}x_n = u$, for some $u \in \mathfrak{D}$ and for all $t > 0$.

Definition 1.8[15,17] A pair of self mappings $\{f, g\}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ is said to be $\mathcal{R}$-weakly commuting at a point u in $\mathfrak{D}$ if $\mathcal{M}(fgx_n, gfx_n, t) \geq \mathcal{M}(fx_n, gx_n, \frac{t}{\mathcal{R}})$, for some $\mathcal{R} > 0$.

Definition 1.10[17] A pair of self mappings $\{f, g\}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ are called (i) $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$ if there exists some $\mathcal{R} > 0$ such that

$$\mathcal{M}(ffu, gfu, t) \geq \mathcal{M}(fu, gu, \tfrac{t}{\mathcal{R}}), \text{ for all } u \in \mathfrak{D}.$$

(ii) $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_f)$ if there exists some $\mathcal{R} > 0$ such that

$$\mathcal{M}(fgu, ggu, t) \geq \mathcal{M}(fu, gu, \tfrac{t}{\mathcal{R}}), \text{ for all } u \in \mathfrak{D}.$$

(iii) $\mathcal{R}$-weakly commuting of type (P) if there exists some $\mathcal{R} > 0$ such that

$$\mathcal{M}(ffu, ggu, t) \geq \mathcal{M}(fu, gu, \tfrac{t}{\mathcal{R}}), \text{ for all } u \in \mathfrak{D}.$$

Both compatible and non compatible mappings can be $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$or $(\mathcal{A}_f)$ but converse need not be true.

In 1999, Pant [16] introduced a new continuity condition, known as reciprocal continuity as follows:

Definition 1.12[16] Two self-maps $\mathcal{A}$ and $\mathcal{B}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ are called reciprocally continuous if $lim_{n\to\infty} \mathcal{A}\mathcal{B}x_n = \mathcal{A}z$ *and* $lim_{n\to\infty} \mathcal{B}\mathcal{A}x_n = \mathcal{B}z$, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty} \mathcal{A}x_n = lim_{n\to\infty} \mathcal{B}x_n = u$, for some $u \in \mathfrak{D}$ and for all $t > 0$.

If $\mathcal{A}$ and $\mathcal{B}$ are both continuous, then they are obviously reciprocally continuous, but the converse is need not be true.

In 2011, Pant et al.[17] generalized the notion of reciprocal continuity to weak reciprocal continuity as follows:

Definition 1.13[17] Two self-maps $\mathcal{A}$ and $\mathcal{B}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ are called weakly reciprocally continuous if $lim_{n\to\infty} \mathcal{A}\mathcal{B}x_n = \mathcal{A}z$ *or* $lim_{n\to\infty} \mathcal{B}\mathcal{A}x_n = \mathcal{B}z$, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty} \mathcal{A}x_n = lim_{n\to\infty} \mathcal{B}x_n = u$, for some $u \in \mathfrak{D}$ and for all $t > 0$

Reciprocally continuous implies weak reciprocally continuous, but the converse is not true.

Lemma 1.1[14] Let $\{x_n\}$ be a sequence in a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ with continuous t-norm $*$ and $*(t,t) \geq t$. If there exists a constant $k \in (0,1)$ such that

$$\mathcal{M}(x_n, x_{n+1}, kt) \geq \mathcal{M}(x_{n-1}, x_n, t)$$

for all $t > 0$ and $n = 1,2,3$ ..., then the sequence $\{x_n\}$ is a Cauchy's sequence.

Lemma 1.2[14] Let $(\mathfrak{D},\mathcal{M},*)$ be a fuzzy metric space. If there exists a constant $k \in (0,1)$ such that $\mathcal{M}(x, y, kt) \geq \mathcal{M}(x, y, t)$, for all $t > 0$ and $x, y \in \mathfrak{D}$. Then $x = y$.

II. WEAK RECIPROCAL CONTINUITY AND FIXED POINT THEOREM

Let $\boldsymbol{\Phi}$ be class of all the mappings $\phi: [0,1] \to [0,1]$ satisfying the following properties:

$(\boldsymbol{\phi_1})$ ϕ is continuous and non decreasing on [0,1],

$(\boldsymbol{\phi_2})$ $\phi(m) > m$ for all m in [0,1].

We note that if $\phi \in \boldsymbol{\Phi}$, then $\phi(1) = 1$ and $\phi(m) \geq m$ for all m in [0,1].

In 2014 Kang et al,.[13] prove common fixed point theorems for weakly reciprocally continuous mappings in fuzzy metric space. We generalize the same with a control function in fuzzy metric space.

Theorem 2.1 Let f and g be weakly reciprocally continuous self mappings of a complete fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ with $*(t,t) \geq t$ and satisfying the following conditions:

(C_1) $f(\mathfrak{D}) \subset g(\mathfrak{D})$,

(C_2) $\quad \mathcal{M}(fx, fy, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gx, gy, t), \mathcal{M}(gx, fy, (2-\lambda)t)\\ \mathcal{M}(fx, gx, t), \mathcal{M}(fx, gy, \lambda t)\\ \mathcal{M}(fy, gy, t)\end{Bmatrix}\right),$

for all $x, y \in \mathfrak{D}$, where $k \in (0,1)$, $\lambda \in (0,2)$, $\phi \in \mathbf{\Phi}$ $t > 0$,

(C_3) If f and g are either compatible or $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$ or $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$ or $\mathcal{R}$-weakly commuting of the type (P). Then f and g have a unique common fixed point.

Proof Let $x_0 \in \mathfrak{D}$ be an arbitrary point. From (C_1), we can find a point x_1 such that $f(x_0) = g(x_1) = y_0$.Continuing in this way, one can construct a sequence $\{x_n\}$ such that

$y_n = f(x_n) = g(x_{n+1})$. $\qquad (C_4)$

Now we prove that $\{y_n\}$ is Cauchy sequence in $\mathfrak{D}$.

Putting $x = x_n$, y = x_{n+1} , $\lambda = 1 - \xi$ with $\xi \in (0,1)$ in (C2), we have

$$\mathcal{M}(fx_n, fx_{n+1}, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gx_n, gx_{n+1}, t), \mathcal{M}(gx_n, fx_{n+1}, (2-\lambda)t)\\ \mathcal{M}(fx_n, gx_n, t), \mathcal{M}(fx_n, gx_{n+1}, \lambda t)\\ \mathcal{M}(fx_{n+1}, g\, x_{n+1}, t)\end{Bmatrix}\right)$$

or $$\mathcal{M}(y_n, y_{n+1}, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(y_{n-1}, y_n, t), \mathcal{M}(y_{n-1}, y_{n+1}, (1+\xi)t)\\ \mathcal{M}(y_n, y_{n-1}, t), \mathcal{M}(y_n, y_n, (1-\xi)t)\\ \mathcal{M}(y_{n+1}, y_n, t)\end{Bmatrix}\right)$$

$$\geq \phi\left(min\begin{Bmatrix}\mathcal{M}(y_{n-1}, y_n, t), \mathcal{M}(y_{n-1}, y_n, t),\\ \mathcal{M}(y_n, y_{n-1}, \xi t)\\ \mathcal{M}(y_n, y_{n-1}, t), 1\\ \mathcal{M}(y_{n+1}, y_n, t)\end{Bmatrix}\right)$$

As $*$ is continuous, letting $\xi \to 1$ we get

$$\mathcal{M}(y_n, y_{n+1}, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(y_{n-1}, y_n, t), \mathcal{M}(y_n, y_{n+1}, t),\\ \mathcal{M}(y_n, y_{n+1}, t)\end{Bmatrix}\right)$$

$$= \phi(min\{\mathcal{M}(y_{n-1}, y_n, t), \mathcal{M}(y_n, y_{n+1}, t)\})$$

Hence $\quad \mathcal{M}(y_n, y_{n+1}, kt) \geq \phi(min\{\mathcal{M}(y_{n-1}, y_n, t), \mathcal{M}(y_n, y_{n+1}, t)\})$.

$\mathcal{M}(y_n, y_{n+1}, kt) \geq \phi(\mathcal{M}(y_{n-1}, y_n, t))$, then by property of ϕ, we have

$\mathcal{M}(y_n, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, t)$.

Therefore, by Lemma 1.1, $\{y_n\}$ is a Cauchy sequence in $\mathfrak{D}$. Since $\mathfrak{D}$ is complete, therefore, there exists a point m in $\mathfrak{D}$ such that $lim_{n\to\infty}\, y_n = m$. Therefore, by (C_4), we have

$lim_{n\to\infty}\, y_n = lim_{n\to\infty}\, f(x_n) = lim_{n\to\infty}\, gx_{n+1} = m$, for all $n \in N$. $\qquad (C_5)$

Suppose that f and g are compatible mappings and by weak reciprocal continuity of f and g, we have $lim_{n\to\infty}\, fgx_n = fm$ or $lim_{n\to\infty}\, gfx_n = gm$.

Let $lim_{n\to\infty}\, gfx_n = gm$, then compatibility of f and g gives,

$lim_{n\to\infty}\, \mathcal{M}(fgx_n, gfx_n, t) = 1$ implies that $lim_{n\to\infty}\, \mathcal{M}(fgx_n, gm, t) = 1$.

Hence $lim_{n\to\infty}\, fgx_n = gm$.

By (C_5), we get $lim_{n\to\infty}\, fgx_{n+1} = lim_{n\to\infty}\, ffx_n = gm$.

We claim that $fm = gm$. Putting $x = m$ and $y = fx_n$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fm, ffx_n, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gm, gfx_n, t), \mathcal{M}(gm, ffx_n, t)\\ \mathcal{M}(fm, gm, t), \mathcal{M}(fm, gfx_n, t)\\ \mathcal{M}(ffx_n, gfx_n, t)\end{Bmatrix}\right)$$

Letting limit as $n \to \infty$, we get

$$\mathcal{M}(fm, gm, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gm, gm, t), \mathcal{M}(gm, gm, t)\\ \mathcal{M}(fm, gm, t), \mathcal{M}(fm, gm, t)\\ \mathcal{M}(gm, gm, t)\end{Bmatrix}\right)$$

$\mathcal{M}(fm, gm, kt) \geq \phi(\mathcal{M}(fm, gm, t))$, then by property of ϕ, we have

$\mathcal{M}(fm, gm, kt) \geq \mathcal{M}(fm, gm, t)$, using Lemma 1.2, we have $fm = gm$.

Now f and g are compatible mappings so the commute at their coincidence point, then $gfm = ffm = ggm = fgm$

Next we claim that $fm = ffm$. Putting $x = m$ and $y = fm$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fm, ffm, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gm, gfm, t), \mathcal{M}(gm, ffm, t)\\ \mathcal{M}(fm, gm, t), \mathcal{M}(fm, gfm, t)\\ \mathcal{M}(ffm, gfm, t)\end{Bmatrix}\right)$$

$$\mathcal{M}(fm, ffm, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(fm, ffm, t), \mathcal{M}(fm, ffm, t)\\ \mathcal{M}(fm, gm, t), \mathcal{M}(fm, ffm, t)\\ \mathcal{M}(ffm, ffm, t)\end{Bmatrix}\right)$$
$$= \phi(min\{\mathcal{M}(fm, ffm, t), \mathcal{M}(fm, ffm, t), 1,1\})$$
$$= \phi(\mathcal{M}(fm, ffm, t))$$

Then $\mathcal{M}(fm, ffm, kt) \geq \phi(\mathcal{M}(fm, ffm, t))$, then by property of ϕ, we have
$\mathcal{M}(fm, ffm, kt) \geq \mathcal{M}(fm, ffm, t)$, using Lemma 1.2, we have $fm = ffm$.
Hence $fm = ffm = gfm$. Thus fm is the common fixed point of the mappings f and g.
Uniqueness If possible let u_1 and v_1 be two fixed point of the mappings f and g.
Finally, we claim that $u_1 = v_1$. Putting $x = u_1$ and $y = v_1$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fu_1, fv_1, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gu_1, gv_1, t), \mathcal{M}(gu_1, fv_1, t)\\ \mathcal{M}(fu_1, gu_1, t), \mathcal{M}(fu_1, gv_1, t)\\ \mathcal{M}(fv_1, gv_1, t)\end{Bmatrix}\right),$$

or $\mathcal{M}(u_1, v_1, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(u_1, v_1, t), \mathcal{M}(u_1, v_1, t)\\ \mathcal{M}(u_1, u_1, t), \mathcal{M}(u_1, v_1, t)\\ \mathcal{M}(v_1, v_1, t)\end{Bmatrix}\right)$,

$\mathcal{M}(u_1, v_1, kt) \geq \phi(\mathcal{M}(u_1, v_1, t))$, then by property of ϕ, we have
$\mathcal{M}(u_1, v_1, kt) \geq \mathcal{M}(u_1, v_1, t)$, using Lemma 1.2, we have $u_1 = v_1$.
Thus $u_1 = v_1 = fm$ is the common fixed point of the mappings f and g.
Next suppose that $lim_{n\to\infty} fgx_n = fm$. Then (C_1) implies that $fm = gn$, for some $n \in \mathfrak{D}$ so $lim_{n\to\infty} fgx_n = gn$ and from compatibility of f and g, we have $lim_{n\to\infty} gfx_n = gn$. Then by (C_5), we have $lim_{n\to\infty} fgx_{n+1} = lim_{n\to\infty} ffx_n = gn$.
Now we claim that $fn = gn$. Putting $x = n$ and $y = fx_n$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fn, ffx_n, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gn, gfx_n, t), \mathcal{M}(gn, ffx_n, t)\\ \mathcal{M}(fn, gn, t), \mathcal{M}(fn, gfx_n, t)\\ \mathcal{M}(ffx_n, gfx_n, t)\end{Bmatrix}\right)$$

Letting limit as $n \to \infty$, we get

$$\mathcal{M}(fn, gn, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gn, gn, t), \mathcal{M}(gn, gn, t)\\ \mathcal{M}(fn, gn, t), \mathcal{M}(fn, gn, t)\\ \mathcal{M}(gn, gn, t)\end{Bmatrix}\right)$$

$\mathcal{M}(fn, gn, kt) \geq \phi(\mathcal{M}(fn, gn, t))$, then by property of ϕ, we have
$\mathcal{M}(fn, gn, kt) \geq \mathcal{M}(fn, gn, t)$, using Lemma 1.2, we have $fn = gn$.
Now f and g are compatible mappings so the commute at their coincidence point, then $gfn = ffn = ggn = fgn$.
Next we claim that $fn = ffn$. Putting $x = n$ and $y = fn$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fn, ffn, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gn, gfn, t), \mathcal{M}(gn, ffn, t)\\ \mathcal{M}(fn, gn, t), \mathcal{M}(fn, gfn, t)\\ \mathcal{M}(ffn, gfn, t)\end{Bmatrix}\right)$$
$$\mathcal{M}(fn, ffn, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(fn, ffn, t), \mathcal{M}(fn, ffn, t)\\ \mathcal{M}(fn, gn, t), \mathcal{M}(fn, ffn, t)\\ \mathcal{M}(ffn, ffn, t)\end{Bmatrix}\right)$$
$$= \phi(min\{\mathcal{M}(fn, ffn, t), \mathcal{M}(fn, ffn, t), 1,1\})$$
$$= \phi(\mathcal{M}(fn, ffn, t))$$

Then $\mathcal{M}(fn, ffn, kt) \geq \phi(\mathcal{M}(fn, ffn, t))$, then by property of ϕ, we have
$\mathcal{M}(fn, ffn, kt) \geq \mathcal{M}(fn, ffn, t)$, using Lemma 1.2, we have $fn = ffn$.
Uniqueness easily follows from first part.
Hence $fn = ffn = gfn$. Thus fn is the common fixed point of the mappings f and g.
Now suppose that f and g are $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$ and by weak reciprocal continuity of f and g implies that $lim_{n\to\infty} fgx_n = fu$ or $lim_{n\to\infty} gfx_n = gu$.
Let us first assume that $lim_{n\to\infty} gfx_n = gu$. Then by $\mathcal{R}$-weakly commuting of the type $(\mathcal{A}_g)$ implies that $\mathcal{M}(ffx_n, gfx_n, t) \geq \mathcal{M}\left(fx_n, gx_n, \frac{t}{\mathcal{R}}\right)$,
Taking limit as $n \to \infty$, we get
$\mathcal{M}(ffx_n, gu, t) \geq \mathcal{M}\left(fx_n, gx_n, \frac{t}{\mathcal{R}}\right) = \mathcal{M}\left(u, u, \frac{t}{\mathcal{R}}\right) = 1$, then $lim_{n\to\infty} ffx_n = gu$.
Next we claim that $fu = gu$. Putting $x = u$ and $y = fx_n$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fu, ffx_n, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gu, gfx_n, t), \mathcal{M}(gu, ffx_n, t)\\ \mathcal{M}(fu, gu, t), \mathcal{M}(fu, gfx_n, t)\\ \mathcal{M}(ffx_n, gfx_n, t)\end{Bmatrix}\right)$$

Letting limit as $n \to \infty$, we get

$$\mathcal{M}(fu, gu, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gu, gu, t), \mathcal{M}(gu, gu, t)\\ \mathcal{M}(fu, gu, t), \mathcal{M}(fu, gu, t)\\ \mathcal{M}(gu, gu, t)\end{Bmatrix}\right)$$

$\mathcal{M}(fu, gu, kt) \geq \phi(\mathcal{M}(fu, gu, t))$, then by property of ϕ, we have
$\mathcal{M}(fu, gu, kt) \geq \mathcal{M}(fu, gu, t)$, using Lemma 1.2, we have $fu = gu$.
By $\mathcal{R}$-weakly commuting of the type $(\mathcal{A}_g)$ implies that
$\mathcal{M}(ffu, gfu, t) \geq \mathcal{M}\left(fu, gu, \frac{t}{\mathcal{R}}\right)$ =1, gives $ffu = gfu$.Then $ffu = gfu = fgu = ggu$.
Next we claim that $fu = ffu$. Putting $x = u$ and $y = fu$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fu, ffu, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gu, gfu, t), \mathcal{M}(gu, ffu, t)\\ \mathcal{M}(fu, gu, t), \mathcal{M}(fu, gfu, t)\\ \mathcal{M}(ffu, gfu, t)\end{Bmatrix}\right),$$
$$= \phi\left(min\begin{Bmatrix}\mathcal{M}(fu, ffu, t), \mathcal{M}(fu, ffu, t)\\ \mathcal{M}(fu, ffu, t), 1\\ 1\end{Bmatrix}\right),$$
$$= \phi(\mathcal{M}(fu, ffu, t))$$

$\mathcal{M}(fu, ffu, kt) \geq \phi(\mathcal{M}(fu, ffu, t))$, then by property of ϕ, we have
$\mathcal{M}(fu, ffu, kt) \geq \mathcal{M}(fu, ffu, t)$, using Lemma 1.2, we have $fu = ffu$.
Hence $fu = ffu = gfu$ and fu is a common fixed point of f and g. Similarly, we prove if $lim_{n\to\infty} fgx_n = fu$.
Suppose f and g are $\mathcal{R}$-weakly commuting of the type $(\mathcal{A}_g)$. Again, as done above, we can easily prove that fu is a common fixed point of f and g.
Now suppose that f and g are $\mathcal{R}$-weakly commuting of type (P) by weak reciprocal continuity of f and g implies that $lim_{n\to\infty} fgx_n = fv$ or $lim_{n\to\infty} gfx_n = gv$.
Let us first assume that $lim_{n\to\infty} gfx_n = gv$. Then $\mathcal{R}$-weakly commuting of type (P) gives
$\mathcal{M}(ffx_n, ggx_n, t) \geq \mathcal{M}\left(fx_n, gx_n, \frac{t}{\mathcal{R}}\right) = \mathcal{M}\left(v, v, \frac{t}{\mathcal{R}}\right)$=1,
gives $lim_{n\to\infty} ffx_n = lim_{n\to\infty} ggx_n = lim_{n\to\infty} gfx_n = gv$.
Next we claim that $fv = gv$. Putting $x = v$ and $y = fx_n$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fv, ffx_n, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gv, gfx_n, t), \mathcal{M}(gv, ffx_n, t)\\ \mathcal{M}(fv, gv, t), \mathcal{M}(fv, gfx_n, t)\\ \mathcal{M}(ffx_n, gfx_n, t)\end{Bmatrix}\right),$$

Letting limit as $n \to \infty$, we get

$$\mathcal{M}(fv, gv, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gv, gv, t), \mathcal{M}(gv, gv, t)\\ \mathcal{M}(fv, gv, t), \mathcal{M}(fv, gv, t)\\ \mathcal{M}(gv, gv, t)\end{Bmatrix}\right),$$

$\mathcal{M}(fv, gv, kt) \geq \phi(\mathcal{M}(fv, gv, t))$, then by property of ϕ, we have
$\mathcal{M}(fv, gv, kt) \geq \mathcal{M}(fv, gv, t)$, using Lemma 1.2, we have $fv = gv$.
Again $\mathcal{R}$- weakly commuting of type (P), we have $\mathcal{M}(ffv, ggv, t) \geq \mathcal{M}\left(fv, gv, \frac{t}{\mathcal{R}}\right)$=1,
which gives $ffv = ggv$. Therefore, $ffv = ggv = gfv$ =ggv.
Next we claim that $fv = ffv$. Putting $x = v$ and $y = fv$, $\lambda = 1$ in (C2), we have

$$\mathcal{M}(fv, ffv, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(gv, gfv, t), \mathcal{M}(gv, ffv, t)\\ \mathcal{M}(fv, gv, t), \mathcal{M}(fv, gfv, t)\\ \mathcal{M}(ffv, gfv, t)\end{Bmatrix}\right),$$
$$= \phi\left(min\begin{Bmatrix}\mathcal{M}(fv, ffv, t), \mathcal{M}(fv, ffv, t)\\ \mathcal{M}(fv, fv, t), \mathcal{M}(fv, ffv, t)\\ \mathcal{M}(ffv, ffv, t)\end{Bmatrix}\right),$$

$\mathcal{M}(fv, ffv, kt) \geq \phi(\mathcal{M}(fv, ffv, t))$, then by property of ϕ, we have
$\mathcal{M}(fv, ffv, kt) \geq \mathcal{M}(fv, ffv, t)$, using Lemma 1.2, we have $fv = ffv$.
Hence $fv = ffv = gfv$ and fv is a common fixed point of f and g. Similarly, we prove if $lim_{n\to\infty} fgx_n = fv$. Uniqueness of the common fixed point theorem follows easily.

Now we prove a common fixed point theorem for weakly reciprocally continuous non compatible self-mappings in a fuzzy metric space.

Theorem 2.2 Let f and g be weakly reciprocally continuous non compatible self- mappings of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ satisfying (C_1) and the following conditions:

(C_6) $\quad \mathcal{M}(fx,fy,kt) \geq \phi\big(\mathcal{M}(gx,gy,t)\big)$

$$(C_7) \quad \mathcal{M}(fx,ffx,kt) \geq \phi\left(max\begin{Bmatrix}\mathcal{M}(gx,gfx,t),\mathcal{M}(fx,gx,(2-\lambda)t),\\ \mathcal{M}(ffx,gfx,\lambda t),\mathcal{M}(fx,gfx,t),\\ \mathcal{M}(gx,ffx,t)\end{Bmatrix}\right),$$

for all $x,y \in \mathfrak{D}$, where $k \in (0,1)$, $\lambda \in (0,2)$, $\phi \in \mathbf{\Phi}$ $t > 0$,

(C_8) If f and g are $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$ or $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$ or $\mathcal{R}$-weakly commuting of the type (P). Then f and g have a unique common fixed point.

Proof: Since f and g are non-compatible maps, then either $lim_{n\to\infty}\mathcal{M}(fgx_n,gfx_n,t) \neq 1$ or Limit does not exist, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = u$,for some $u \in \mathfrak{D}$. Since by (C_1), there exists another sequence $\{y_n\}$in $\mathfrak{D}$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = \lim_{n\to\infty} gy_n = u$.

Next we claim that $lim_{n\to\infty} fy_n = u$. Putting $x = x_n$ and $y = y_n$ in (C_6), we have

$$\mathcal{M}(fx_n,fy_n,kt) \geq \phi\big(\mathcal{M}(gx_n,gy_n,t)\big),$$

Letting limit as $n \to \infty$, we get

$\lim_{n\to\infty}\mathcal{M}(u,fy_n,kt) \geq \lim_{n\to\infty}\phi\big(\mathcal{M}(gx_n,gy_n,t)\big) = \phi\big(\mathcal{M}(u,u,t)\big) = \phi(1) = 1.$

Thus $lim_{n\to\infty} fy_n = u$. Hence $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = lim_{n\to\infty} fy_n = \lim_{n\to\infty} gy_n = u$.

Now suppose that f and g are $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$ and by weak reciprocal continuity of f and g implies that $lim_{n\to\infty} fgx_n = fu$ or $lim_{n\to\infty} gfx_n = gu$.

Let us first assume that $lim_{n\to\infty} gfx_n = gu$. Then by $\mathcal{R}$-weakly commuting of the type $(\mathcal{A}_g)$ implies that $\mathcal{M}(ffx_n,gfx_n,t) \geq \mathcal{M}\left(fx_n,gx_n,\frac{t}{\mathcal{R}}\right)$,

Taking limit as $n \to \infty$,we get

$\mathcal{M}(ffx_n,gu,t) \geq \mathcal{M}\left(fx_n,gx_n,\frac{t}{\mathcal{R}}\right) = \mathcal{M}\left(u,u,\frac{t}{\mathcal{R}}\right) =1$, then $lim_{n\to\infty} ffx_n = gu$.

Next we claim that $fu = gu$. Putting $x = fx_n$ and $y = u$ in (C_6), we have

$$\mathcal{M}(ffx_n,fu,kt) \geq \phi\big(\mathcal{M}(gfx_n,gu,t)\big),$$

Taking limit as $n\to\infty$,we get

$$\mathcal{M}(gu,fu,kt) \geq \phi\big(\mathcal{M}(gu,gu,t)\big) = \phi(1) = 1.$$

or $\mathcal{M}(gu,fu,kt) \geq 1$ gives $fu = gu$.

Again by $\mathcal{R}$-weakly commuting of the type $(\mathcal{A}_g)$ implies that

$\mathcal{M}(ffu,gfu,t) \geq \mathcal{M}\left(fu,gu,\frac{t}{\mathcal{R}}\right) =1$, gives $ffu = gfu$.Then $ffu = gfu = fgu = ggu$. Next we claim that $fu = ffu$. If $fu \neq ffu$,then by putting $x = u$ and $\lambda = 1$ in (C7), we have

$$\mathcal{M}(fu,ffu,kt) \geq \phi\left(max\begin{Bmatrix}\mathcal{M}(gu,gfu,t),\mathcal{M}(fu,gu,t),\\ \mathcal{M}(ffu,gfu,t),\mathcal{M}(fu,gfu,t),\\ \mathcal{M}(gu,ffu,t)\end{Bmatrix}\right),$$

$$= \phi\left(max\begin{Bmatrix}\mathcal{M}(fu,ffu,t),\mathcal{M}(fu,gu,t),\\ \mathcal{M}(ffu,gfu,t),\mathcal{M}(fu,ffu,t),\\ \mathcal{M}(fu,ffu,t)\end{Bmatrix}\right),$$

$$= \phi\left(max\begin{Bmatrix}\mathcal{M}(fu,ffu,t),1,\\ 1,\mathcal{M}(fu,ffu,t),\\ \mathcal{M}(fu,ffu,t)\end{Bmatrix}\right) = \phi(1),$$

$\mathcal{M}(fu,ffu,kt) \geq \phi(1) = 1.$

Thus $\mathcal{M}(fu,ffu,kt) \geq 1$, which is a contradiction. Hence $fu = ffu = gfu$ and fu is a common fixed point of f and g. Similarly, we prove if $lim_{n\to\infty} fgx_n = fu$.

Similarly, we prove this theorems for $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$or (P).

Theorem 2.3 Let f and g be weakly reciprocally continuous non compatible self- mappings of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ satisfying (C_1), (C_6) and the following condition:

(C_9) $\mathcal{M}(fx,ffx,t) \geq \phi\big(\mathcal{M}(gx,ggx,t)\big)$, whenever $fx \neq ffx$, for all $x,y \in \mathfrak{D}$, where , $\phi \in \mathbf{\Phi}$ $t > 0$. Then f and g have a unique common fixed point.

Proof: Since f and g are non-compatible maps, then either $lim_{n\to\infty}\mathcal{M}(fgx_n,gfx_n,t) \neq 1$ or Limit does not exist, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = w$,for some $w \in \mathfrak{D}$. Since by (C_1), there exists another sequence $\{y_n\}$in $\mathfrak{D}$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = \lim_{n\to\infty} gy_n = w$.

Next we claim that $lim_{n\to\infty} \mathcal{f}y_n = w$. Putting $x = x_n$ and $y = y_n$ in (C_6), we have

$$\mathcal{M}(\mathcal{f}x_n, \mathcal{f}y_n, kt) \geq \phi(\mathcal{M}(gx_n, gy_n, t)),$$

Letting limit as $n \to \infty$, we get

$\lim_{n\to\infty} \mathcal{M}(w, \mathcal{f}y_n, kt) \geq \lim_{n\to\infty} \phi(\mathcal{M}(gx_n, gy_n, t)) = \phi(\mathcal{M}(w, w, t)) = \phi(1) = 1.$

Thus $lim_{n\to\infty} \mathcal{f}y_n = w$. Hence $lim_{n\to\infty} \mathcal{f}x_n = lim_{n\to\infty} gx_n = lim_{n\to\infty} \mathcal{f}y_n = \lim_{n\to\infty} gy_n = w$.

Now suppose that $\mathcal{f}$ and g are $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$ and by weak reciprocal continuity of $\mathcal{f}$ and g implies that $lim_{n\to\infty} \mathcal{f}gx_n = \mathcal{f}w$ or $lim_{n\to\infty} g\mathcal{f}x_n = gw$.

Let us first assume that $lim_{n\to\infty} g\mathcal{f}x_n = gw$. Then by $\mathcal{R}$-weakly commuting of the type $(\mathcal{A}_g)$ implies that $\mathcal{M}(\mathcal{f}\mathcal{f}x_n, g\mathcal{f}x_n, t) \geq \mathcal{M}\left(\mathcal{f}x_n, gx_n, \frac{t}{\mathcal{R}}\right)$,

Taking limit as $n \to \infty$, we get

$\mathcal{M}(\mathcal{f}\mathcal{f}x_n, gw, t) \geq \mathcal{M}\left(\mathcal{f}x_n, gx_n, \frac{t}{\mathcal{R}}\right) = \mathcal{M}\left(w, w, \frac{t}{\mathcal{R}}\right) = 1$, then $lim_{n\to\infty} \mathcal{f}\mathcal{f}x_n = gw$.

Next we claim that $\mathcal{f}w = gw$. Putting $x = \mathcal{f}x_n$ and $y = w$ in (C_6), we have

$$\mathcal{M}(\mathcal{f}\mathcal{f}x_n, \mathcal{f}w, kt) \geq \phi(\mathcal{M}(g\mathcal{f}x_n, gw, t)),$$

Taking limit as $n \to \infty$, we get

$$\mathcal{M}(gw, \mathcal{f}w, kt) \geq \phi(\mathcal{M}(gw, gw, t)) = \phi(1) = 1.$$

or $\mathcal{M}(gw, \mathcal{f}w, kt) \geq 1$ gives $\mathcal{f}w = gw$.

Again by $\mathcal{R}$-weakly commuting of the type $(\mathcal{A}_g)$ implies that

$\mathcal{M}(\mathcal{f}\mathcal{f}w, g\mathcal{f}w, t) \geq \mathcal{M}\left(\mathcal{f}w, gw, \frac{t}{\mathcal{R}}\right) = 1$, gives $\mathcal{f}\mathcal{f}w = g\mathcal{f}w$. Then $\mathcal{f}\mathcal{f}w = g\mathcal{f}w = \mathcal{f}gw = ggw$. Next we claim that $\mathcal{f}w = \mathcal{f}\mathcal{f}w$, then by putting $x = w$ (C7), we have

$\mathcal{M}(\mathcal{f}w, \mathcal{f}\mathcal{f}w, kt) \geq \phi(\mathcal{M}(\mathcal{f}w, \mathcal{f}\mathcal{f}w, t))$, then by property of ϕ, we have

$\geq \mathcal{M}(\mathcal{f}w, \mathcal{f}\mathcal{f}w, t)$, by Lemma 1.2, we have

$$\mathcal{f}w = \mathcal{f}\mathcal{f}w.$$

Hence $\mathcal{f}u = \mathcal{f}\mathcal{f}u = g\mathcal{f}u$ and $\mathcal{f}u$ is a common fixed point of $\mathcal{f}$ and g. Similarly, we prove if $lim_{n\to\infty} \mathcal{f}gx_n = \mathcal{f}u$.

Similarly, we prove this theorems for $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$or (P).

III. APPLICATION

In 2002 Branciari [1] obtained a fixed point theorem for a single mapping satisfying an analogue of a Banach contraction principle for integral type inequality. Now we prove the following theorem as an application of Theorem 2.1 and Theorem 2.2.

Theorem 3.1 Let $\mathcal{f}$ *and* g are self mappings of a complete fuzzy metric space $(\mathfrak{D}, \mathcal{M}, *)$ satisfying the conditions (C1), (C3) and the following:

(C10) $$\int_0^{\mathcal{M}(\mathcal{f}x, \mathcal{f}y, kt)} \varphi(t)\, dt \leq \int_0^{\sigma(x,y)} \varphi(t)\, dt$$

$$\sigma(x, y) = \phi\left(min \begin{Bmatrix} \mathcal{M}(gx, gy, t), \mathcal{M}(gx, \mathcal{f}y, (2-\lambda)t) \\ \mathcal{M}(\mathcal{f}x, gx, t), \mathcal{M}(\mathcal{f}x, gy, \lambda t) \\ \mathcal{M}(\mathcal{f}y, gy, t) \end{Bmatrix} \right)$$

for all $x, y \in \mathfrak{D}$, $k \in (0,1)$, $\lambda \in (0,2)$, $\phi \in \mathbf{\Phi}$ $t > 0$, and where $\phi \in \mathbf{\Phi}$ and $\varphi : R^+ \to R^+$ is a "Lebesgue-integrable over R^+function" which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$. Then $\mathcal{f}$ and g have a unique common fixed point.

Proof. The proof of the theorem follows on the same lines of the proof of the Theorem 2.1. on setting φ (t) = 1.

Theorem 3.2 Let $\mathcal{f}$ and g be weakly reciprocally continuous non compatible self- mappings of a fuzzy metric space $(\mathfrak{D}, \mathcal{M}, *)$ satisfying (C_1) *and* (C_6) and the following condition:

(C10) $$\int_0^{\mathcal{M}(\mathcal{f}x, \mathcal{f}\mathcal{f}x, kt)} \varphi(t)\, dt \leq \int_0^{\sigma(x,y)} \varphi(t)\, dt$$

$$\sigma(x, y) = \phi\left(max \begin{Bmatrix} \mathcal{M}(gx, g\mathcal{f}x, t), \mathcal{M}(\mathcal{f}x, gx, (2-\lambda)t), \\ \mathcal{M}(\mathcal{f}\mathcal{f}x, g\mathcal{f}x, \lambda t), \mathcal{M}(\mathcal{f}x, g\mathcal{f}x, t), \\ \mathcal{M}(gx, \mathcal{f}\mathcal{f}x, t) \end{Bmatrix} \right)$$

for all $x, y \in \mathfrak{D}$, $k \in (0,1)$, $\lambda \in (0,2)$, $\phi \in \mathbf{\Phi}$ $t > 0$, and where $\phi \in \mathbf{\Phi}$ and $\varphi : R^+ \to R^+$ is a "Lebesgue-integrable over R^+function" which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)dt > 0$. Then $\mathcal{f}$ and g have a unique common fixed point.

Proof. The proof of the theorem follows on the same lines of the proof of the Theorem 2.2. on setting φ (t) = 1.

Remark 3.1. Every contractive condition of integral type automatically includes a corresponding contractive condition not involving integrals, by setting φ (t) = 1.

Conclusion: In this paper, we prove some common fixed point theorems for $\mathcal{R}$-weakly commuting of type $(\mathcal{A}_g)$, type $(\mathcal{A}_f)$ and type (P) using control function in Fuzzy metric space. At the last we provide an application in support of our theorems.

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