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**Research Paper**

# Analysis of Primes in Arithmetical Progressions 5*n* + *k* up to a Trillion

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***ABSTRACT:-*** *Owing to the seemingly most irregular distribution of prime numbers amongst the sequence of positive integers, their occurrence patterns have always been under critical review. The search for superior arithmetic progression covering them with dominance is on. As part of continued contribution to this endeavor, this work analyzes prime numbers with their distribution in the arithmetical progressions 5n + k. The more and less gaps between them, their successive partners, their density variation in blocks of* $10^i$ *and few more facts are analyzed and presented in this work.*

***Keywords:-*** *Arithmetical progressions, block-wise distribution, prime, prime density, prime spacing.*
***Mathematics Subject Classification 2010:*** *11A41, 11N05, 11N25.*

## I. INTRODUCTION

A prime number *p*, briefly called as prime, is, by definition, a positive integer greater than 1 which has only two positive divisors, namely, 1 and *p*. It is long known that there are infinitely many primes [1].

## II. PRIME DISTRIBUTIONS

The distribution of prime numbers within the sequence of positive integers is quite irregular, at least to this point of time when the regularity, if any, in their pattern of occurrence remains a mystery. There are ample of twin primes, those successive prime pairs with very small spacing of 2 and at the same time there are also arbitrarily large gaps between successive prime pairs.

The notation of $\pi(x)$ is in use for representing number of primes less than or equal to a positive value *x*. This $\pi(x)$ as yet lacks a precise formula.

## III. PRIME DISTRIBUTIONS IN ARTIMETICAL PROGRESSIONS

Relevant peculiar properties of primes are listed in [2]. The very first property stated there that 2 is the only even prime makes all remaining prime numbers the members of arithmetical progression $2n + 1$.

There is no other arithmetical progression containing all odd primes. But there are arithmetical progressions containing infinite number of primes. The credit of this discovery goes to Dirichlet [3]. He, in fact, characterized all such arithmetical progressions containing infinitely many primes. They are $an + b$ where *a* and *b* are coprime. The converse of Dirichlet Theorem is also true that if *a* and *b* are not coprime, then $an + b$ does not contain infinite number of primes.

## IV. PRIME DISTRIBUTIONS IN ARTIMETICAL PROGRESSIONS 5*n* + *k*

The basic property of integer division gives one of the numbers $0, 1, 2, \cdots, m - 1$ as remainders after dividing any positive integer by positive integer *m*. We consider $m = 5$ here, so that the possible values of remainders in the process of division by 5 are 0, 1, 2, 3, and 4. Since every positive integer after dividing by 5has to give as remainder one and only one amongst these values, it must be of either of the forms $5n + 0 = 5n$ or $5n + 1$ or $5n + 2$ or $5n + 3$ or $5n + 4$, which form arithmetical progressions.

First few numbers of the form $5n$ are

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, · · ·

Each of these is perfectly divisible by 5. Except the first member, viz., 5, none of these is prime. Thus this sequence contains only one prime 5 and its all other members are composite numbers. It is clear also by seeing $5n$ as arithmetical progression $5n + 0$, where $\gcd(5, 0) = 5 > 1$ and by Dirichlet's Theorem, this cannot contain many primes.

First few numbers of the form 5*n* + 1 are
1, 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, · · ·
This does contain infinitely many primes as gcd(5, 1) is 1 as per requirement of Dirichlet's Theorem.
First few numbers of the form 5*n* + 2 are
2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, · · ·
This sequence also does contain infinitely many primes as gcd(5, 2) is 1 as per requirement of Dirichlet's Theorem.
First few numbers of the form 5*n* + 3 are
3, 8, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58, · · ·
This one also contains infinitely many primes as gcd(5, 3) is 1 as per requirement of Dirichlet's Theorem.
First few numbers of the form 5*n* + 4 are
4, 9, 14, 19, 24, 29, 34, 39, 44, 49, 54, 59, · · ·
This sequence also contains infinitely many primes as gcd(5, 4) is 1 as per requirement of Dirichlet's Theorem.

There are independent proofs about infinitude of primes in other arithmetical progressions [4], and similar can be tracked in the cases here also.

We present here a comparative analysis of the primes occurring in arithmetical progressions 5*n* + 1, 5*n* + 2, 5*n* + 3 and 5*n* + 4.

## V. PRIME NUMBER RACE

For a fixed positive integer $a$ and all positive integers $b < a$, all the arithmetical progressions $an + b$ which contain infinitely many primes are compared to check which one amongst them contains more number of primes. This is term popularly known as prime number race [5].

Here we have compared the number of primes of form 5*n* + 1, 5*n* + 2, 5*n* + 3 and 5*n* + 4 for abundance till one trillion, i.e., 1,000,000,000,000 ($10^{12}$). The huge database was made available by using a smart choice amongst the algorithms compared in [6]. Java Programming Language, with its simple and lucid power highlighted in [7], was used on many electronic computers to analyze complete prime range.

**Table 1. Number of Primes of form 5n + k In First Blocks of 10 Powers.**

| Sr. No. | Range 1-*x* (1 to *x*) | Number of Primes of Form | | | |
|---|---|---|---|---|---|
| | | 5*n* + 1 ($\pi_{5,1}(x)$) | 5*n* + 2 ($\pi_{5,2}(x)$) | 5*n* + 3 ($\pi_{5,3}(x)$) | 5*n* + 4 ($\pi_{5,4}(x)$) |
| **1.** | 1-10 | 0 | 2 | 1 | 0 |
| **2.** | 1-100 | 5 | 7 | 7 | 5 |
| **3.** | 1-1,000 | 40 | 47 | 42 | 38 |
| **4.** | 1-10,000 | 306 | 309 | 310 | 303 |
| **5.** | 1-100,000 | 2,387 | 2,412 | 2,402 | 2,390 |
| **6.** | 1-1,000,000 | 19,617 | 19,622 | 19,665 | 19,593 |
| **7.** | 1-10,000,000 | 166,104 | 166,212 | 166,230 | 166,032 |
| **8.** | 1-100,000,000 | 1,440,298 | 1,440,496 | 1,440,474 | 1,440,186 |
| **9.** | 1-1,000,000,000 | 12,711,386 | 12,712,315 | 12,712,499 | 12,711,333 |
| **10.** | 1-10,000,000,000 | 113,761,519 | 113,764,040 | 113,765,625 | 113,761,326 |
| **11.** | 1-100,000,000,000 | 1,029,517,130 | 1,029,518,338 | 1,029,509,448 | 1,029,509,896 |
| **12.** | 1-1,000,000,000,000 | 9,401,960,980 | 9,401,997,001 | 9,401,979,904 | 9,401,974,132 |

Since all primes, except 5, are of only of one of these forms, their quantity seems quite averagely distributed. The deviation from respective averages is plotted separately.

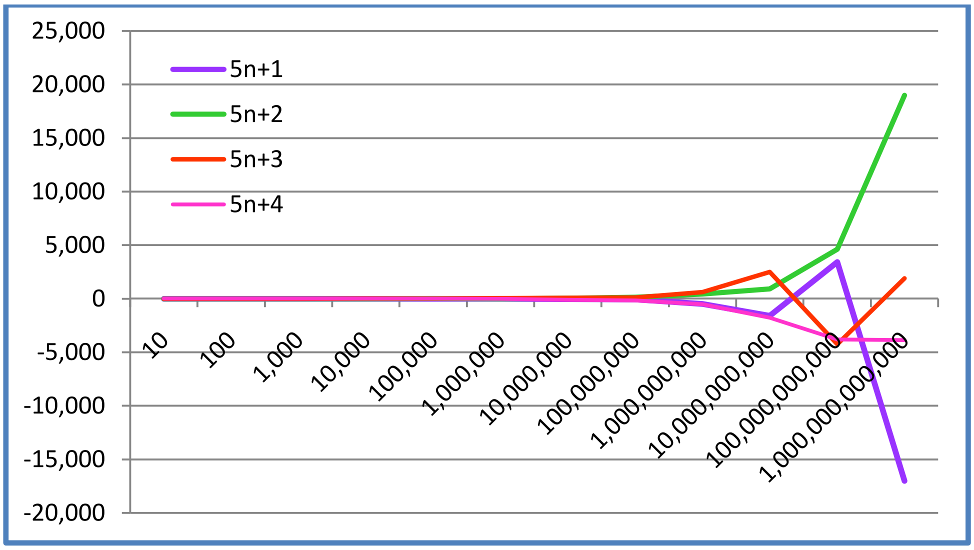



**Figure 1.** ***Deviation of $\pi_{5,k}(x)$ from Average***

The number of primes of the form 5*n* + 2 and 5*n* + 3 seems most of the times ahead of the average up to $10^{12}$ in discrete blocks of 10 powers. This trend is a subject matter of future explorations.

## VI. BLOCK-WISE DISTRIBUTION OF PRIMES

There is no formula to capture all primes in one go, nor are the primes finite in number to consider them all together. So, for understanding their random-looking distribution, we adopted an approach of considering all primes up to a certain limit, viz., one trillion ($10^{12}$) and dividing this complete number range under consideration in blocks of powers of 10 each :

1-10, 11-20, 21-30, 31-40, · · ·
1-100, 101-200, 201-300, 301-400, · · ·
1-1000, 1001-2000, 2001-3000, 3001-4000, · · ·
⋮

A rigorous analysis is done on many fronts. Since selected range is 1-$10^{12}$, there are $10^{12-i}$ number of blocks of $10^i$ size for each $1 \le i \le 12$.

### VI. 1. THE FIRST AND THE LAST PRIMES IN THE FIRST BLOCKS OF 10 POWERS

The first and the last prime in each first block of 10 powers till the range of $10^{12}$occurring there are determined. The first prime of first power of 10, when it occurs naturally continues ahead for all higher blocks.

***Table 2.First Primes of form 5n+k in First Blocks of 10 Powers.***

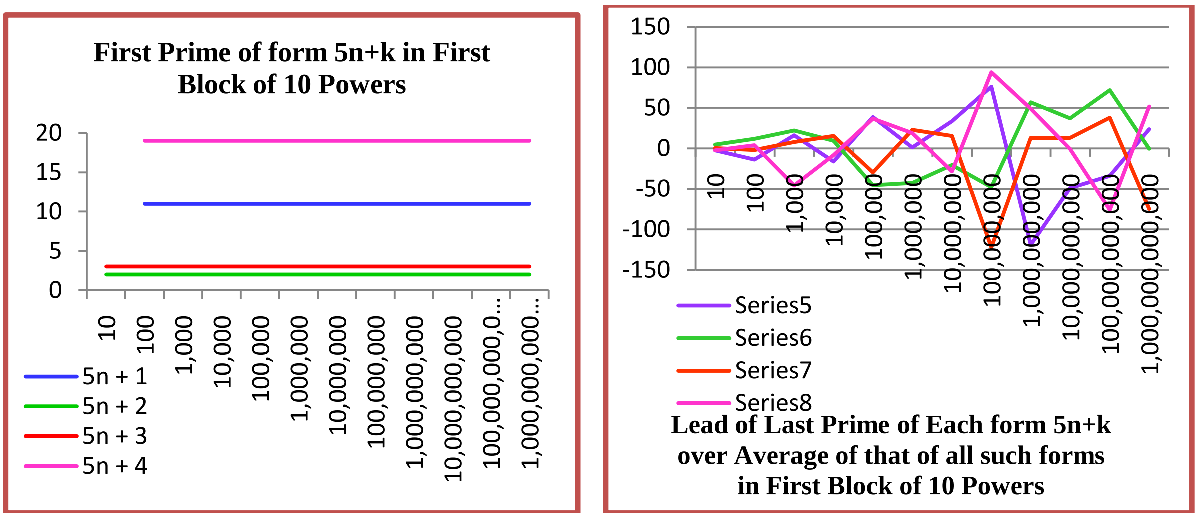
| Sr. No. | Blocks of Size (of 10 Power) | First Prime in the First Block | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | NOT FOUND | 2 | 3 | NOT FOUND |
| **2.** | 100 | 11 | 2 | 3 | 19 |
| **3.** | 1,000 | 11 | 2 | 3 | 19 |
| **4.** | 10,000 | 11 | 2 | 3 | 19 |
| **5.** | 100,000 | 11 | 2 | 3 | 19 |
| **6.** | 1,000,000 | 11 | 2 | 3 | 19 |
| **7.** | 10,000,000 | 11 | 2 | 3 | 19 |
| **8.** | 100,000,000 | 11 | 2 | 3 | 19 |
| **9.** | 1,000,000,000 | 11 | 2 | 3 | 19 |
| **10.** | 10,000,000,000 | 11 | 2 | 3 | 19 |
| **11.** | 100,000,000,000 | 11 | 2 | 3 | 19 |
| **12.** | 1,000,000,000,000 | 11 | 2 | 3 | 19 |

Of particular interest are the last primes in first blocks of 10 powers.

**Table 3. Last Primes of form 5n+k in First Blocks of 10 Powers.**

| Sr. No. | Blocks of Size (of 10 Power) | Last Prime in the First Block | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | NOT FOUND | 7 | 3 | NOT FOUND |
| **2.** | 100 | 71 | 97 | 83 | 89 |
| **3.** | 1,000 | 991 | 997 | 983 | 929 |
| **4.** | 10,000 | 9,941 | 9,967 | 9,973 | 9,949 |
| **5.** | 100,000 | 99,991 | 99,907 | 99,923 | 99,989 |
| **6.** | 1,000,000 | 999,961 | 999,917 | 999,983 | 999,979 |
| **7.** | 10,000,000 | 9,999,991 | 9,999,937 | 9,999,973 | 9,999,929 |
| **8.** | 100,000,000 | 99,999,971 | 99,999,847 | 99,999,773 | 99,999,989 |
| **9.** | 1,000,000,000 | 999,999,761 | 999,999,937 | 999,999,893 | 999,999,929 |
| **10.** | 10,000,000,000 | 9,999,999,881 | 9,999,999,967 | 9,999,999,943 | 9,999,999,929 |
| **11.** | 100,000,000,000 | 99,999,999,871 | 99,999,999,977 | 99,999,999,943 | 99,999,999,829 |
| **12.** | 1,000,000,000,000 | 999,999,999,961 | 999,999,999,937 | 999,999,999,863 | 999,999,999,989 |

While the first primes in all the first blocks, whenever found, have respective fixed values, the deviation of the last primes of these forms in the first blocks have quite a zigzag trend.




**Figures 2. First & Last Primes of form 5n+k in First Blocks of 10 Powers.**

## VI. 2. MINIMUM NUMBER OF PRIMES IN BLOCKS OF 10 POWERS

Considering all blocks of each 10 power from $10^1$ to $10^{12}$ till $10^{12}$, the minimum number of primes occurring in each 10 power block is determined for primes of all forms under consideration.

**Table 4.Minimum Number of Primes of form 5n + k in Blocks of 10 Powers**

| Sr. No. | Blocks of Size (of 10 Power) | Minimum Number of Primes in Blocks | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 0 | 0 | 0 | 0 |
| **2.** | 100 | 0 | 0 | 0 | 0 |
| **3.** | 1,000 | 0 | 0 | 0 | 0 |
| **4.** | 10,000 | 50 | 48 | 50 | 49 |
| **5.** | 100,000 | 795 | 801 | 803 | 794 |
| **6.** | 1,000,000 | 8,748 | 8,772 | 8,789 | 8,734 |
| **7.** | 10,000,000 | 89,851 | 89,846 | 89,904 | 89,846 |
| **8.** | 100,000,000 | 903,380 | 903,712 | 903,467 | 903,526 |
| **9.** | 1,000,000,000 | 9,046,766 | 9,046,962 | 9,046,777 | 9,046,857 |
| **10.** | 10,000,000,000 | 90,495,945 | 90,493,875 | 90,493,544 | 90,494,057 |
| **11.** | 100,000,000,000 | 906,486,613 | 906,481,722 | 906,472,632 | 906,483,465 |
| **12.** | 1,000,000,000,000 | 9,401,960,980 | 9,401,997,001 | 9,401,979,904 | 9,401,974,132 |

The block-wise deviation of minimum number of primes found there from respective averages is as appears in the figure ahead.

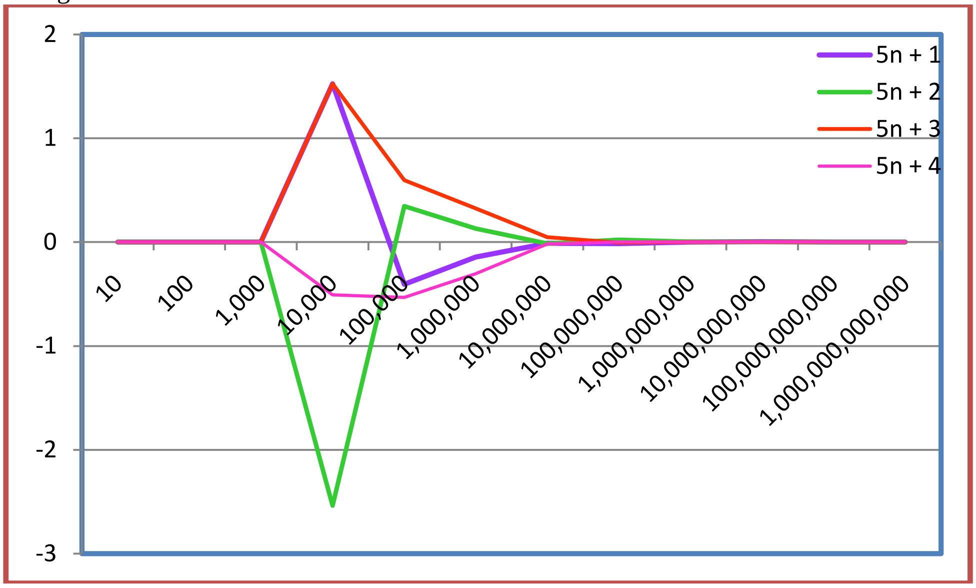



**Figure 3.% Deviation in Minimum Number of Primes of form 5n+k in Blocks of 10 Powers from Average**

The first blocks in our range of one trillion with minimum number of primes of these four forms in them are determined.

**Table 5.First Blocks of 10 Powers with Minimum Number of Primes of form 5n+k**

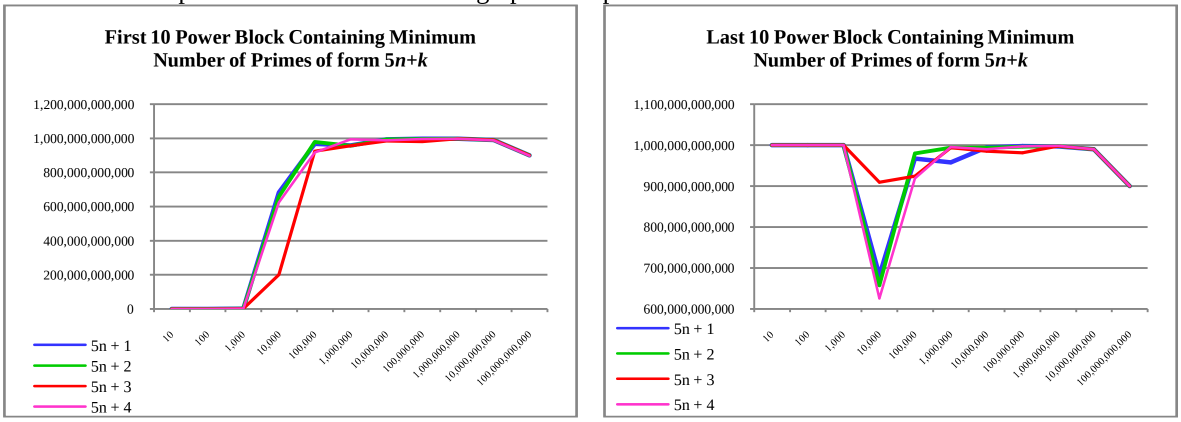
| Sr. No. | Blocks of Size (of 10 Power) | First Block with Minimum Number of Primes | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 0 | 20 | 30 | 0 |
| **2.** | 100 | 10,400 | 8,900 | 13,200 | 13,500 |
| **3.** | 1,000 | 1,992,636,000 | 2,174,469,000 | 1,054,256,000 | 1,036,101,000 |
| **4.** | 10,000 | 681,769,270,000 | 657,874,630,000 | 200,077,450,000 | 625,725,710,000 |
| **5.** | 100,000 | 967,423,100,000 | 979,846,600,000 | 924,727,600,000 | 918,734,500,000 |
| **6.** | 1,000,000 | 957,750,000,000 | 957,617,000,000 | 956,012,000,000 | 995,465,000,000 |
| **7.** | 10,000,000 | 994,560,000,000 | 994,120,000,000 | 985,230,000,000 | 989,830,000,000 |
| **8.** | 100,000,000 | 997,800,000,000 | 996,300,000,000 | 981,100,000,000 | 997,000,000,000 |
| **9.** | 1,000,000,000 | 997,000,000,000 | 998,000,000,000 | 998,000,000,000 | 999,000,000,000 |
| **10.** | 10,000,000,000 | 990,000,000,000 | 990,000,000,000 | 990,000,000,000 | 990,000,000,000 |
| **11.** | 100,000,000,000 | 900,000,000,000 | 900,000,000,000 | 900,000,000,000 | 900,000,000,000 |

The last blocks in our range of one trillion with minimum number of primes of these four forms in them are also determined.

**Table 6. First Blocks of 10 Powers with Minimum Number of Primes of form 5n+k**

| Sr. No. | Blocks of Size (of 10 Power) | Last Block with Minimum Number of Primes | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 999,999,999,990 | 999,999,999,990 | 999,999,999,990 | 999,999,999,990 |
| **2.** | 100 | 999,999,999,800 | 999,999,999,300 | 999,999,999,900 | 999,999,999,700 |
| **3.** | 1,000 | 999,945,413,000 | 999,936,675,000 | 999,969,741,000 | 999,928,156,000 |
| **4.** | 10,000 | 681,769,270,000 | 657,874,630,000 | 909,482,100,000 | 625,725,710,000 |
| **5.** | 100,000 | 967,423,100,000 | 979,846,600,000 | 924,727,600,000 | 918,734,500,000 |
| **6.** | 1,000,000 | 957,750,000,000 | 993,599,000,000 | 994,187,000,000 | 995,465,000,000 |
| **7.** | 10,000,000 | 994,560,000,000 | 994,120,000,000 | 985,230,000,000 | 989,830,000,000 |
| **8.** | 100,000,000 | 997,800,000,000 | 996,300,000,000 | 981,100,000,000 | 997,000,000,000 |
| **9.** | 1,000,000,000 | 997,000,000,000 | 998,000,000,000 | 998,000,000,000 | 999,000,000,000 |
| **10.** | 10,000,000,000 | 990,000,000,000 | 990,000,000,000 | 990,000,000,000 | 990,000,000,000 |
| **11.** | 100,000,000,000 | 900,000,000,000 | 900,000,000,000 | 900,000,000,000 | 900,000,000,000 |

The comparative trend does deserve graphical representations.




**Figures4. First and Last Blocks of 10 Powers with Minimum Number of Primes of form 5n+k.**

Now we determine the frequency of minimum occurrence of primes of 5*n* + *k* forms within the blocks.

**Table 7.Frequency of Minimum Number of Primes of form 5n + k in Blocks of 10 Powers**

| Sr. No. | Blocks of Size (of 10 Power) | No. of Times Minimum No. of Primes Occurring in Blocks | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 90,598,039,020 | 90,598,003,000 | 90,598,020,096 | 90,598,025,868 |
| **2.** | 100 | 3,549,112,098 | 3,549,105,296 | 3,549,128,343 | 3,549,101,467 |
| **3.** | 1,000 | 18,529 | 18,534 | 18,764 | 18,709 |
| **4.** | 10,000 | 1 | 1 | 3 | 1 |
| **5.** | 100,000 | 1 | 1 | 1 | 1 |
| **6.** | 1,000,000 | 1 | 2 | 2 | 1 |

For rest 6 blocks of all, the frequency is only 1.

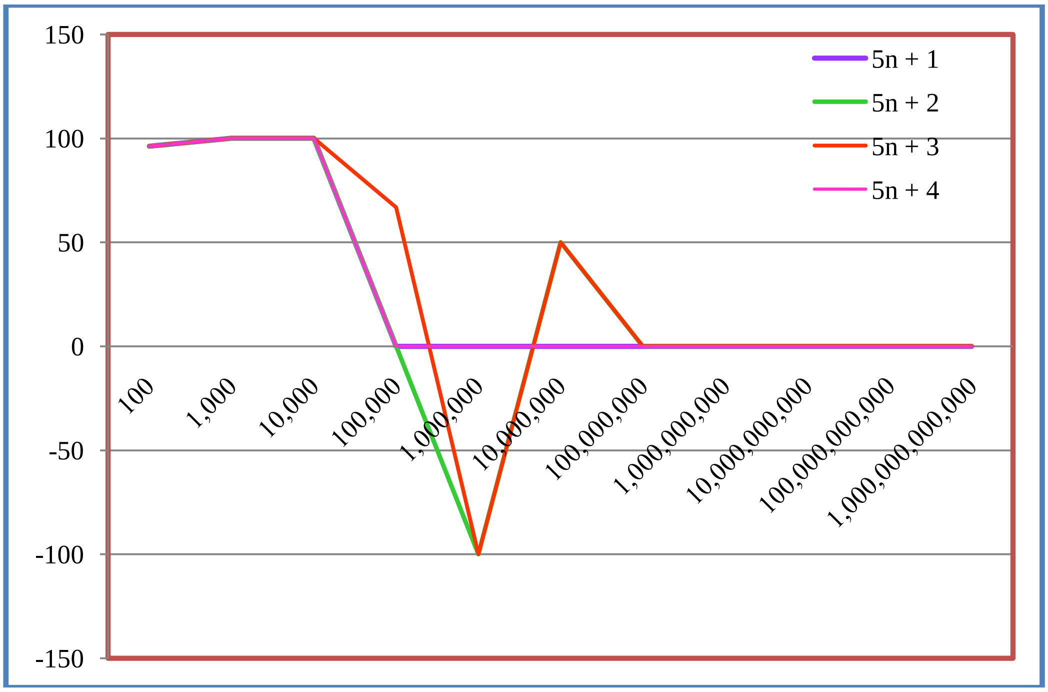



**Figure 5.% Decrease in Occurences of Minimum Number of Primes of form 5*n* + *k* in Blocks of 10 Powers.**

## VI. 3. MAXIMUM NUMBER OF PRIMES IN BLOCKS OF 10 POWERS

All blocks of 10 powers from $10^1$ to $10^{12}$ have been analyzed for the maximum number of primes found in each of them.

| Sr. No. | Blocks of Size (of 10 Power) | Maximum Number of Primes in Blocks | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 1 | 2 | 1 | 1 |
| **2.** | 100 | 7 | 7 | 7 | 7 |
| **3.** | 1,000 | 40 | 47 | 42 | 38 |
| **4.** | 10,000 | 306 | 309 | 310 | 303 |
| **5.** | 100,000 | 2,387 | 2,412 | 2,402 | 2,390 |
| **6.** | 1,000,000 | 19,617 | 19,622 | 19,665 | 19,593 |
| **7.** | 10,000,000 | 166,104 | 166,212 | 166,230 | 166,032 |
| **8.** | 100,000,000 | 1,440,298 | 1,440,496 | 1,440,474 | 1,440,186 |
| **9.** | 1,000,000,000 | 12,711,386 | 12,712,315 | 12,712,499 | 12,711,333 |
| **10.** | 10,000,000,000 | 113,761,519 | 113,764,040 | 113,765,625 | 113,761,326 |
| **11.** | 100,000,000,000 | 1,029,517,130 | 1,029,518,338 | 1,029,509,448 | 1,029,509,896 |
| **12.** | 1,000,000,000,000 | 9,401,960,980 | 9,401,997,001 | 9,401,979,904 | 9,401,974,132 |

Analyzing deviation from average, it is found that the primes of form 5*n* + 2 and 5*n* + 3 lie on upper side in major cases.

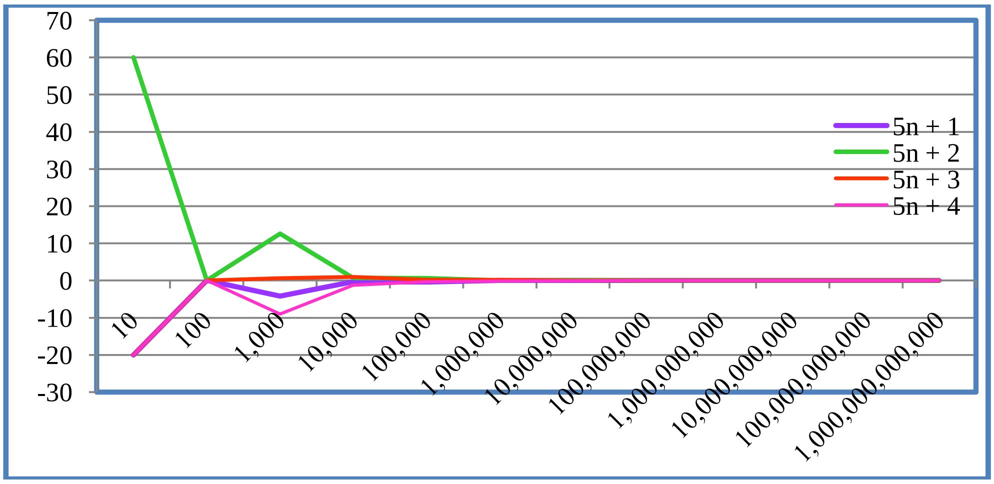



**Figure 6. % Deviation in Maximum Number of Primes of form 5n + k in Blocks of 10 Powers from Average.**

The first blocks in our range of one trillion with maximum number of primes of forms 5*n* + *k*in them are found.

**Table 8.***First Blocks of 10 Powers with Maximum Number of Primes of form 5n + k.*

| Sr. No. | Blocks of Size (of 10 Power) | First Block with Maximum Number of Primes | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 10 | 0 | 0 | 10 |
| **2.** | 100 | 8,056,200 | 0 | 0 | 21,169,600 |
| **3.** | 1,000 | 0 | 0 | 0 | 0 |
| **4.** | 10,000 | 0 | 0 | 0 | 0 |
| **5.** | 100,000 | 0 | 0 | 0 | 0 |
| **6.** | 1,000,000 | 0 | 0 | 0 | 0 |
| **7.** | 10,000,000 | 0 | 0 | 0 | 0 |
| **8.** | 100,000,000 | 0 | 0 | 0 | 0 |
| **9.** | 1,000,000,000 | 0 | 0 | 0 | 0 |
| **10.** | 10,000,000,000 | 0 | 0 | 0 | 0 |
| **11.** | 100,000,000,000 | 0 | 0 | 0 | 0 |
| **12.** | 1,000,000,000,000 | 0 | 0 | 0 | 0 |

The last blocks in our range of one trillion with maximum number of primes of forms 5*n* + *k*in them are also determined.

**Table 9. Last Blocks of 10 Powers with Maximum Number of Primes of form 5n + k.**

| Sr. No. | Blocks of Size (of 10 Power) | Last Block with Maximum Number of Primes | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 999,999,999,960 | 0 | 999,999,999,860 | 999,999,999,980 |
| **2.** | 100 | 996,503,865,600 | 998,658,215,200 | 999,318,647,900 | 998,726,687,000 |
| **3.** | 1,000 | 0 | 0 | 0 | 0 |
| **4.** | 10,000 | 0 | 0 | 0 | 0 |
| **5.** | 100,000 | 0 | 0 | 0 | 0 |
| **6.** | 1,000,000 | 0 | 0 | 0 | 0 |
| **7.** | 10,000,000 | 0 | 0 | 0 | 0 |
| **8.** | 100,000,000 | 0 | 0 | 0 | 0 |
| **9.** | 1,000,000,000 | 0 | 0 | 0 | 0 |
| **10.** | 10,000,000,000 | 0 | 0 | 0 | 0 |
| **11.** | 100,000,000,000 | 0 | 0 | 0 | 0 |
| **12.** | 1,000,000,000,000 | 0 | 0 | 0 | 0 |

Prime density decreases for higher range of numbers. So for larger block sizes, the first as well as the last occurrences of maximum number of primes in them starts in the first block after 0.

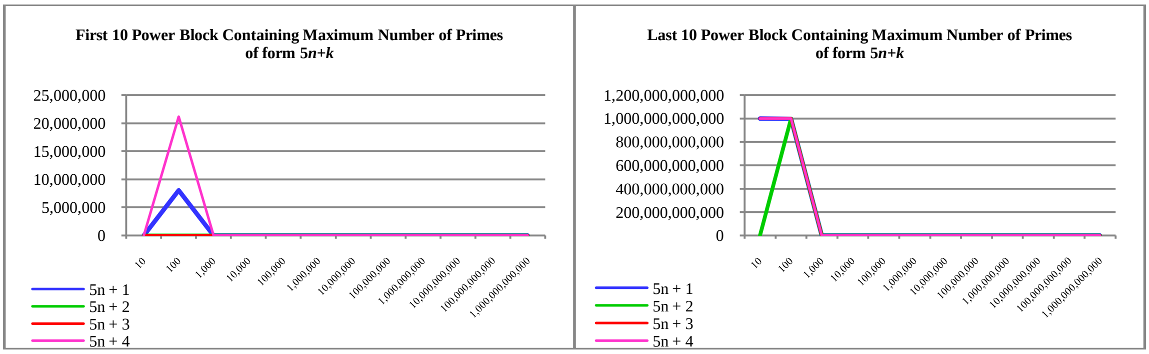



**Figure 7. First and last blocks of 10 powers with maximum number of primes of form 5n + k.**

The maximum number of primes within blocks cannot occur frequently for higher sized blocks. This endorses the reducing prime frequency property.

**Table 10.Frequency of Maximum Number of Primes of form 5n + k In Blocks of 10 Powers.**

| Sr. No. | Blocks of Size (of 10 Power) | No. of Times Maximum No. of Primes Occurring in Blocks | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 9,401,960,980 | 1 | 9,401,979,904 | 9,401,974,132 |
| **2.** | 100 | 1,158 | 1,139 | 1,266 | 1,241 |
| **3.** | 1,000 | 1 | 1 | 1 | 1 |
| **4.** | 10,000 | 1 | 1 | 1 | 1 |
| **5.** | 100,000 | 1 | 1 | 1 | 1 |
| **6.** | 1,000,000 | 1 | 1 | 1 | 1 |
| **7.** | 10,000,000 | 1 | 1 | 1 | 1 |
| **8.** | 100,000,000 | 1 | 1 | 1 | 1 |
| **9.** | 1,000,000,000 | 1 | 1 | 1 | 1 |
| **10.** | 10,000,000,000 | 1 | 1 | 1 | 1 |
| **11.** | 100,000,000,000 | 1 | 1 | 1 | 1 |
| **12.** | 1,000,000,000,000 | 1 | 1 | 1 | 1 |

The case of 5*n* + 2 is special for the block of 10 as there is unique occurrence of maximum number of primes in this block in contrast with the many such occurrences for other forms 5*n* + *k*. So the case of 5*n* + 2 for the block of 100 is not considered for graph as it is exceptional rise.

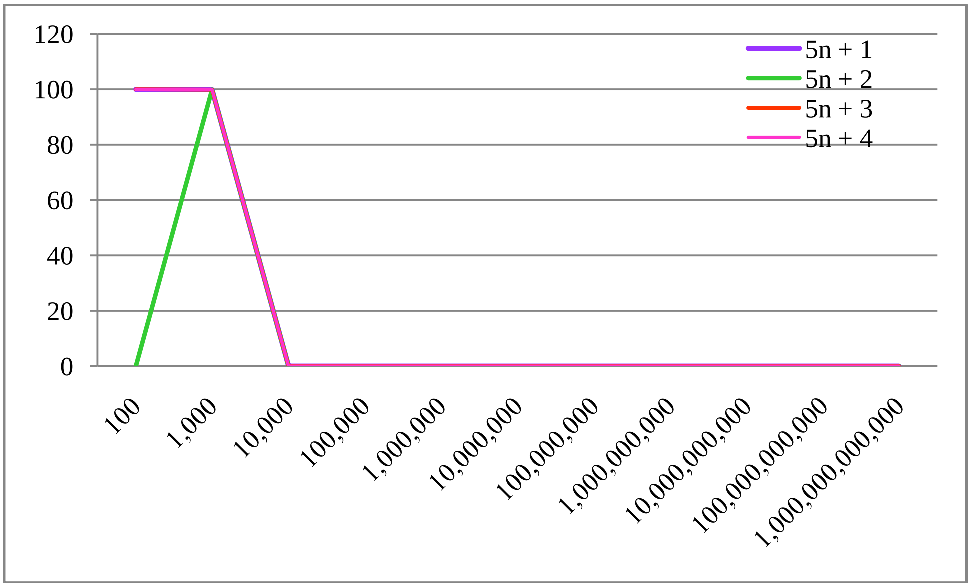



**Figure 8.% Decrease in Occurences of Maximum Number of Primes of form 5n + k in Blocks of 10 Powers.**

## VII. SPACINGS BETWEEN PRIMES OF FORM 5*n* + *k* IN BLOCKS OF 10 POWERS

### *VII. 1. MINIMUM SPACING BETWEEN PRIMES IN BLOCKS OF 10 POWERS*

Exempting prime-empty blocks, the minimum spacing between primes of form 5*n* + 1, 5*n* + 2, 5*n* + 3 and 5*n* + 4 in blocks of 10 powers are determined to be 10 for all forms except for 5*n* + 2, for which one only it is 5. So, excluding the form 5*n* + 2, these spacings are not found in blocks of 10; they start to appear with the block-size of 100 onwards. Since for larger block sizes, the minimum spacing value cannot increase, it remains same ahead for all blocks of all higher powers of 10 in all ranges, even beyond our range of a trillion, virtually till infinity!

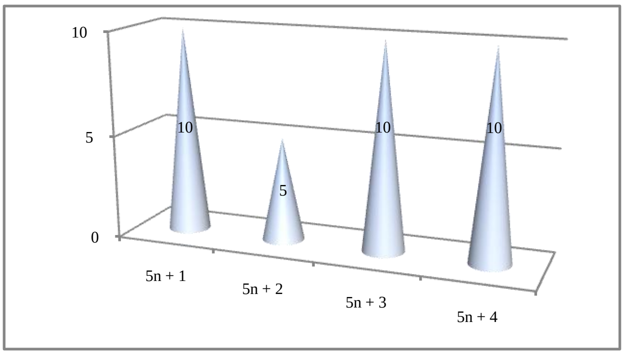



**Figure 9. Minimum Block Spacing between Primes of form 5n+k.**

The smallest considered block-size 10 is special here as the minimum block happens to be 10 only except for primes of form 5*n* + 2 and it cannot occur within the blocks of 10(!). It does so occur for primes of form 5*n* + 2 both first and last at 2.

***Table 11. First Starters of Minimum Block Spacing between Primes of form 5n + k in Blocks of 10$^{i}$.***

| Sr. No. | Blocks of Size (of 10 Power) | First Prime with Respective Minimum Block Spacing | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | NOT FOUND | 2 | NOT FOUND | NOT FOUND |
| **2.** | 100 & All higher 10 Power Blocks ≤ $10^{12}$ | 31 | 2 | 3 | 19 |

The last primes in the 10 power blocks with minimum block spacing are also more or less uniform.

**Table 12.First Starters of Minimum Block Spacing between Primes of form 5n + k in Blocks of 10i.**

| Sr. No. | Blocks of Size (of 10 Power) | Last Prime with Respective Minimum Block Spacing | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | NOT FOUND | 2 | NOT FOUND | NOT FOUND |
| **2.** | 100 | 999,999,998,761 | 2 | 999,999,999,133 | 999,999,999,589 |
| **3.** | 1,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **4.** | 10,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **5.** | 100,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **6.** | 1,000,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **7.** | 10,000,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **8.** | 100,000,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **9.** | 1,000,000,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **10.** | 10,000,000,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **11.** | 100,000,000,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |
| **12.** | 1,000,000,000,000 | 999,999,999,091 | 2 | 999,999,999,133 | 999,999,999,589 |

Graphically they form following patterns in comparison.

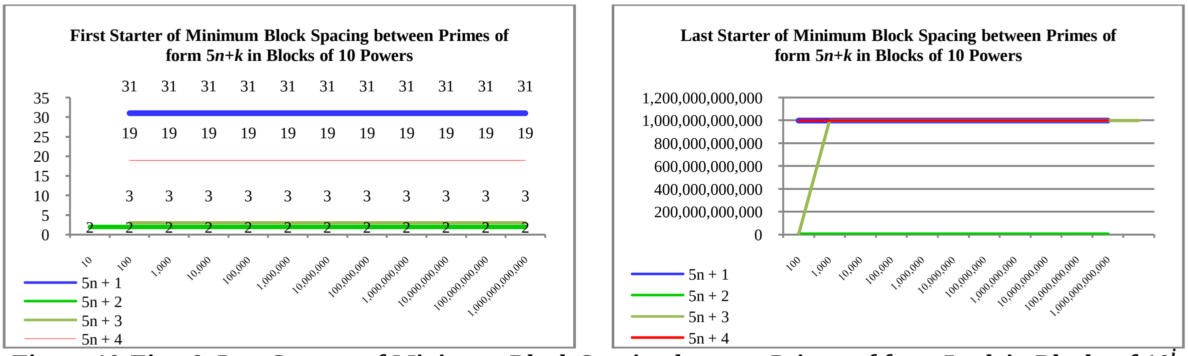



**Figure 10.*First & Last Starters of Minimum Block Spacing between Primes of form 5n+k in Blocks of $10^i$***

It is important to know how many times this minimum block spacing occurs between primes of all forms 5*n* + 1, 5*n* + 2, 5*n* + 3 and 5*n* + 4.

**Table 13. Frequency of minimum block spacings between primes of form 5n + k.**

| Sr. No. | Blocks of Size (of 10 Power) | Number of Times Minimum Block Spacing Occurring for Primes | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | 0 | 1 | 0 | 0 |
| **2.** | 100 | 561,179,421 | 1 | 561,166,888 | 561,155,858 |
| **3.** | 1,000 | 617,292,749 | 1 | 617,280,565 | 617,271,782 |
| **4.** | 10,000 | 622,905,283 | 1 | 622,889,099 | 622,880,699 |
| **5.** | 100,000 | 623,466,642 | 1 | 623,450,492 | 623,442,758 |
| **6.** | 1,000,000 | 623,522,885 | 1 | 623,506,667 | 623,499,063 |
| **7.** | 10,000,000 | 623,528,485 | 1 | 623,512,262 | 623,504,599 |
| **8.** | 100,000,000 | 623,529,047 | 1 | 623,512,837 | 623,505,126 |
| **9.** | 1,000,000,000 | 623,529,102 | 1 | 623,512,894 | 623,505,180 |
| **10.** | 10,000,000,000 | 623,529,111 | 1 | 623,512,899 | 623,505,188 |
| **11.** | 100,000,000,000 | 623,529,112 | 1 | 623,512,899 | 623,505,188 |
| **12.** | 1,000,000,000,000 | 623,529,112 | 1 | 623,512,899 | 623,505,188 |

There is increase in the number of times the minimum spacing occurs for primes of all forms except 5*n* + 2. This is due to accommodation of cross-over cases for small blocks within the larger blocks.

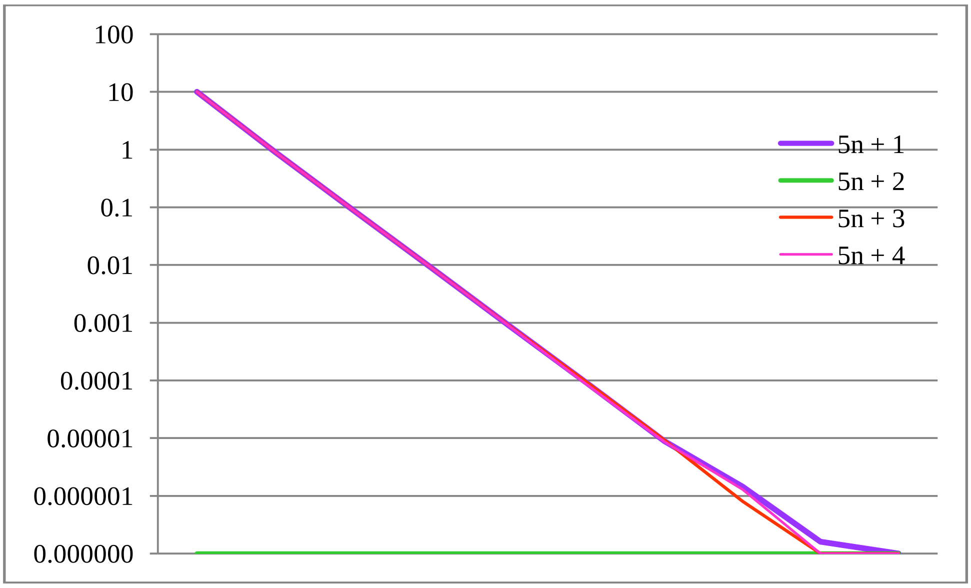



**Figure 11. % Increase in Occurences of Minimum Block Spacing between Primes of form 5n+k in Blocks of $10^i$.**

### VII. 2. MAXINIMUM SPACING BETWEEN PRIMES IN BLOCKS OF 10 POWERS

The maximum spacing in these blocks goes on increasing with increase in the block size. This is unlike the minimum spacing between primes in blocks of 10 powers.

**Table 14.Maximum Block Spacing between Primes of form 5n + k.**

| Sr. No. | Blocks of Size (of 10 Power) | Maximum Spacing (in Blocks of 10 Powers) Between Primes | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | NOT FOUND | 5 | NOT FOUND | NOT FOUND |
| **2.** | 100 | 90 | 90 | 90 | 90 |
| **3.** | 1,000 | 990 | 990 | 990 | 990 |
| **4.** | 10,000 & All higher 10 Power Blocks ≤ $10^{12}$ | 2,070 | 2,150 | 2,200 | 2,450 |

Till our ceiling of one trillion, the following trend of increase and settling is seen.

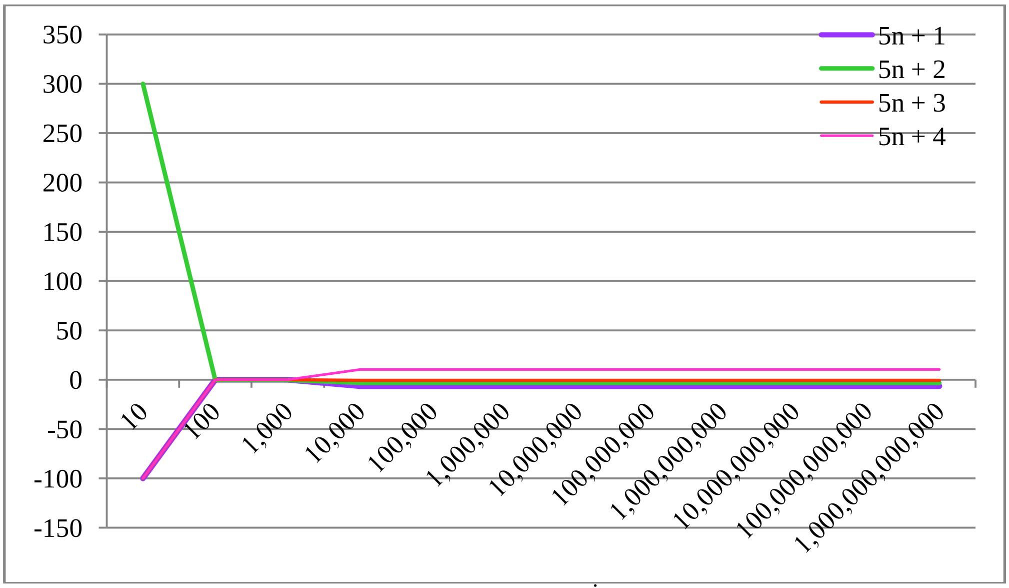



**Figure 12.% Deviation of Max Block Spacing in Blocks of $10^i$ between Primes of form 5n+k from Average**

**Table 15.*First Primes with Maximum Block Spacings.***

| Sr. No. | Blocks of Size (of 10 Power) | First Prime with Respective Maximum Block Spacing | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | NOT FOUND | 2 | NOT FOUND | NOT FOUND |
| **2.** | 100 | 12,301 | 41,507 | 7,103 | 5,309 |
| **3.** | 1,000 | 10,897,791,001 | 4,995,555,007 | 2,540,083,003 | 14,533,219,009 |
| **4.** | 10,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **5.** | 100,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **6.** | 1,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **7.** | 10,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **8.** | 100,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **9.** | 1,000,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **10.** | 10,000,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **11.** | 100,000,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **12.** | 1,000,000,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |

**Table 16. Last Primes with Maximum Block Spacings.**

| Sr. No. | Blocks of Size (of 10 Power) | Last Prime with Respective Maximum Block Spacing | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | NOT FOUND | 2 | NOT FOUND | NOT FOUND |
| **2.** | 100 | 999,999,982,901 | 999,999,981,907 | 999,999,989,803 | 999,999,981,109 |
| **3.** | 1,000 | 998,163,162,001 | 998,947,570,007 | 999,925,508,003 | 997,034,224,009 |
| **4.** | 10,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **5.** | 100,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **6.** | 1,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **7.** | 10,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **8.** | 100,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **9.** | 1,000,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **10.** | 10,000,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **11.** | 100,000,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |
| **12.** | 1,000,000,000,000 | 670,429,921,241 | 911,012,180,597 | 807,173,675,533 | 236,455,710,509 |

Here follows thecomparative trend from graphical representation.

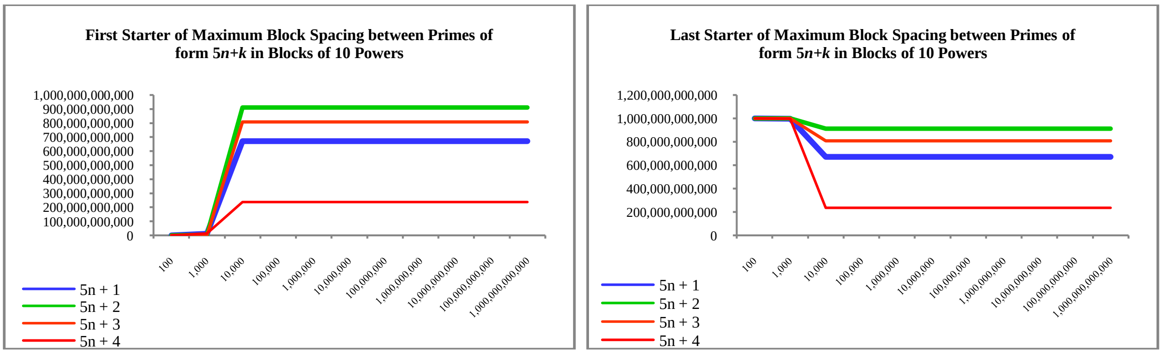



**Figure 13.*First & Last Primes with Maximum Block Spacings.***

The frequency of occurrence of maximum block spacing primes of these forms is also determined.

| Sr. No. | Blocks of Size (of 10 Power) | Number of Times Maximum Block Spacing Occurs for Primes | | | |
|---|---|---|---|---|---|
| | | Form 5*n* + 1 | Form 5*n* + 2 | Form 5*n* + 3 | Form 5*n* + 4 |
| **1.** | 10 | NOT FOUND | 1 | NOT FOUND | NOT FOUND |
| **2.** | 100 | 61,573,038 | 61,570,520 | 61,571,881 | 61,569,175 |
| **3.** | 1,000 | 385 | 410 | 407 | 412 |
| **4.** | 10,000 & All higher 10 Power Blocks ≤ $10^{12}$ | 1 | 1 | 1 | 1 |

## VIII. UNITS PLACE & TENS PLACE DIGITS IN PRIMES OF FORM 5*n* + *k*

As is well known prime numbers can have only 6 different digits in units place.

**Table 17.Number of Primes of form 5n + k with Different Units Place Digits till One Trillion.**

| Sr. No. | Digit in Units Place | Number of Primes of form | | | |
|---|---|---|---|---|---|
| | | 5*n* + 1 | 5*n* + 2 | 5*n* + 3 | 5*n* + 4 |
| **1.** | 1 | 9,401,960,980 | 0 | 0 | 0 |
| **2.** | 2 | 0 | 1 | 0 | 0 |
| **3.** | 3 | 0 | 0 | 9,401,979,904 | 0 |
| **4.** | 5 | 0 | 0 | 0 | 0 |
| **5.** | 7 | 0 | 9,401,997,000 | 0 | 0 |
| **6.** | 9 | 0 | 0 | 0 | 9,401,974,132 |

There are in confirmation of following properties.

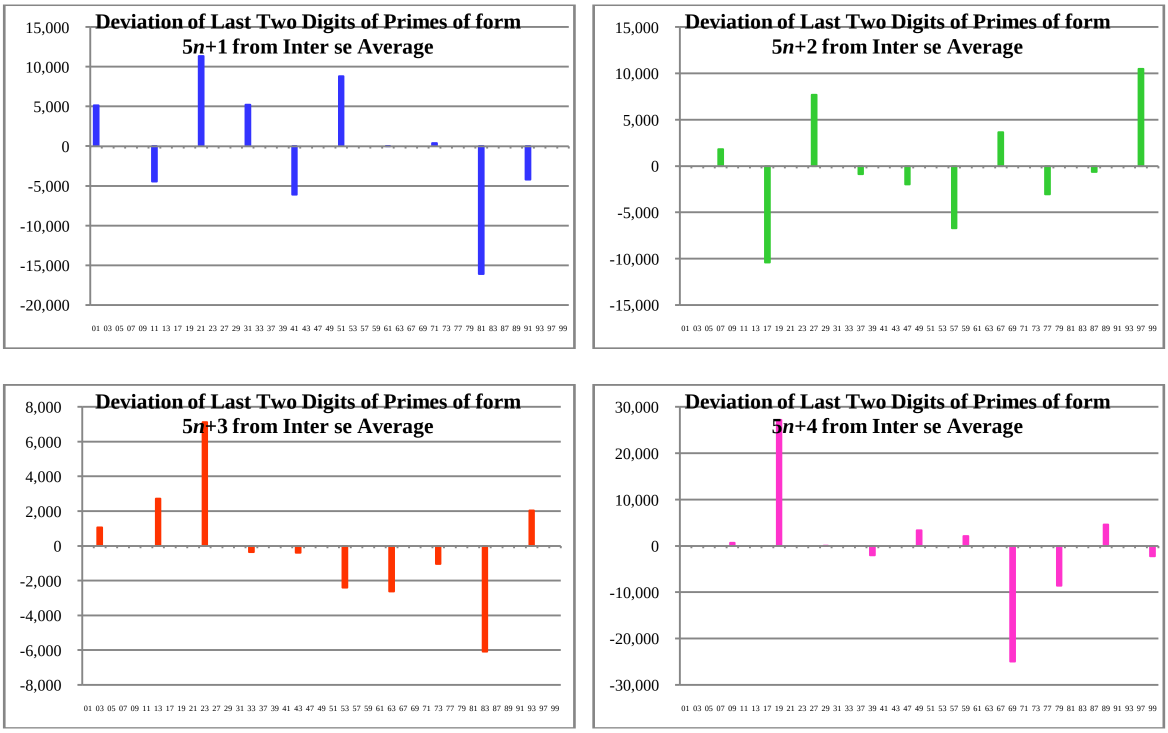
**Theorem 1** : Each digit from 0 to 9 occurs as units place digit in at most one form of prime 5*n* + *k*.
*Proof.*In the standard decimal system, there are 10 digits – 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Those even numbers with 0, 4, 6, 8 in units place are divisible by non-trivial divisor 2 and hence are never prime. These digits do not occur in units place of any prime of form 5*n* + *k*.
The only even prime 2 is the only number with units place digit 2 and is a prime of form 5*n* + 2.
5 is the only prime with units place digit 5 and it doesn't occur in any form 5*n* + *k*, for non-zero *k*.
The remaining odd numbers are with odd digits 1, 3, 7, 9 in units place. These happen to be of form 5*n* + 1, 5*n* + 3, 5*n* + 2 and 5*n* + 4, respectively. This completes the proof.
**Theorem 2** : Each form 5*n* + *k*, with the exception of 5*n* + 2, contains primes with a fixed digit in units place.
*Proof.*The classical division algorithm asserts that 5 on division gives either of the digits 0, 1, 2, 3 or 4 as remainder and accordingly all integers fit in one and only one of forms 5*n*, 5*n* + 1, 5*n* + 2, 5*n* + 3, and 5*n* + 4.
Form 5*n* contains unique prime 5.
Form 5*n* + 1 contains only those numbers with units place digits 1 and 6. Out of these, numbers with units place digit 6 are never prime and hence form 5*n* + 1 contains all and only primes with units place digit 1.
The exception form 5*n* + 2 contains only those numbers with units place digits 2 and 7. Out of these numbers, the only even prime 2 is with units place digit 2 and all other primes of this form are with units place digit 7.
Form 5*n* + 3 contains only those numbers with units place digits 3 and 8. Out of these, numbers with units place digit 8 are never prime and hence form 5*n* + 3 contains all and only primes with units place digit 3.
Form 5*n* + 4 contains only those numbers with units place digits 4 and 9. Out of these, numbers with units place digit 4 are never prime and hence form 5*n* + 4 contains all and only primes with units place digit 9.
This completes the proof.

**Table 18.Number of Primes of form 5n + k with Different Tens and Units Place Digits till One Trillion.**

| Sr. No. | Digits in Tens & Units Place | Number of Primes of form | | | |
|---|---|---|---|---|---|
| | | 5*n* + 1 | 5*n* + 2 | 5*n* + 3 | 5*n* + 4 |
| **1.** | 01 | 940,201,224 | 0 | 0 | 0 |
| **2.** | 02 | 0 | 1 | 0 | 0 |
| **3.** | 03 | 0 | 0 | 940,199,042 | 0 |
| **4.** | 05 | 0 | 0 | 0 | 0 |
| **5.** | 07 | 0 | 940,201,524 | 0 | 0 |
| **6.** | 09 | 0 | 0 | 0 | 940,198,037 |
| **7.** | 11 | 940,191,631 | 0 | 0 | 0 |
| **8.** | 13 | 0 | 0 | 940,200,704 | 0 |
| **9.** | 17 | 0 | 940,189,305 | 0 | 0 |
| **10.** | 19 | 0 | 0 | 0 | 940,224,567 |
| **11.** | 21 | 940,207,451 | 0 | 0 | 0 |
| **12.** | 23 | 0 | 0 | 940,205,113 | 0 |
| **13.** | 27 | 0 | 940,207,372 | 0 | 0 |
| **14.** | 29 | 0 | 0 | 0 | 940,197,429 |
| **15.** | 31 | 940,201,296 | 0 | 0 | 0 |
| **16.** | 33 | 0 | 0 | 940,197,634 | 0 |
| **17.** | 37 | 0 | 940,198,836 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| **18.** | 39 | 0 | 0 | 0 | 940,195,363 |
| **19.** | 41 | 940,190,006 | 0 | 0 | 0 |
| **20.** | 43 | 0 | 0 | 940,197,593 | 0 |
| **21.** | 47 | 0 | 940,197,732 | 0 | 0 |
| **22.** | 49 | 0 | 0 | 0 | 940,200,776 |
| **23.** | 51 | 940,204,880 | 0 | 0 | 0 |
| **24.** | 53 | 0 | 0 | 940,195,587 | 0 |
| **25.** | 57 | 0 | 940,192,995 | 0 | 0 |
| **26.** | 59 | 0 | 0 | 0 | 940,199,522 |
| **27.** | 61 | 940,196,110 | 0 | 0 | 0 |
| **28.** | 63 | 0 | 0 | 940,195,366 | 0 |
| **29.** | 67 | 0 | 940,203,357 | 0 | 0 |
| **30.** | 69 | 0 | 0 | 0 | 940,172,444 |
| **31.** | 71 | 940,196,489 | 0 | 0 | 0 |
| **32.** | 73 | 0 | 0 | 940,196,947 | 0 |
| **33.** | 77 | 0 | 940,196,643 | 0 | 0 |
| **34.** | 79 | 0 | 0 | 0 | 940,188,826 |
| **35.** | 81 | 940,180,003 | 0 | 0 | 0 |
| **36.** | 83 | 0 | 0 | 940,191,900 | 0 |
| **37.** | 87 | 0 | 940,199,054 | 0 | 0 |
| **38.** | 89 | 0 | 0 | 0 | 940,202,008 |
| **39.** | 91 | 940,191,890 | 0 | 0 | 0 |
| **40.** | 93 | 0 | 0 | 940,200,018 | 0 |
| **41.** | 97 | 0 | 940,210,182 | 0 | 0 |
| **42.** | 99 | 0 | 0 | 0 | 940,195,160 |

Neglecting the case of those digit combinations with no primes, the deviation from average is as shown below.




**Figures14.** ***Deviation of Last Two Digits of Primes of form 5n + k from Inter se Average.***

## IX. ANALYSIS OF SUCCESSIVE PRIMES OF FORM 5*n* + *k*

We have determined the number of successive primes of each form 5*n* + *k* within our range of 1-$10^{12}$.

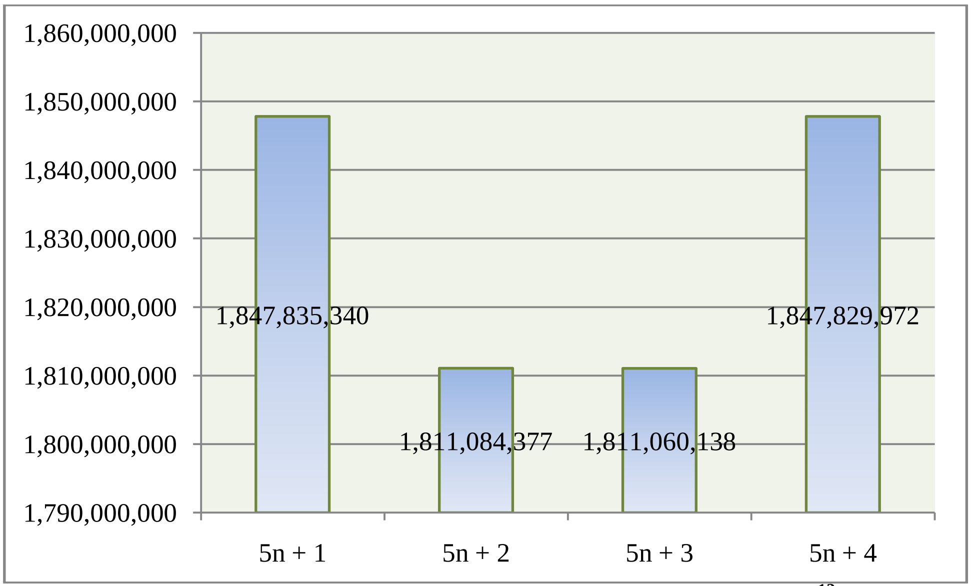
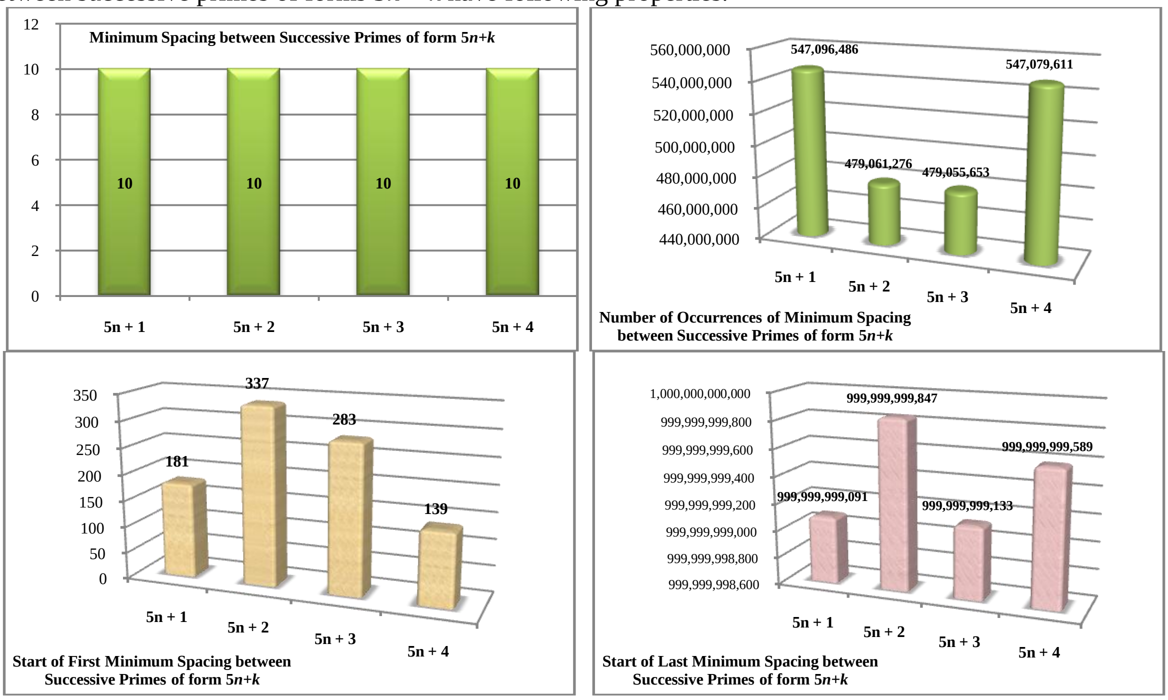
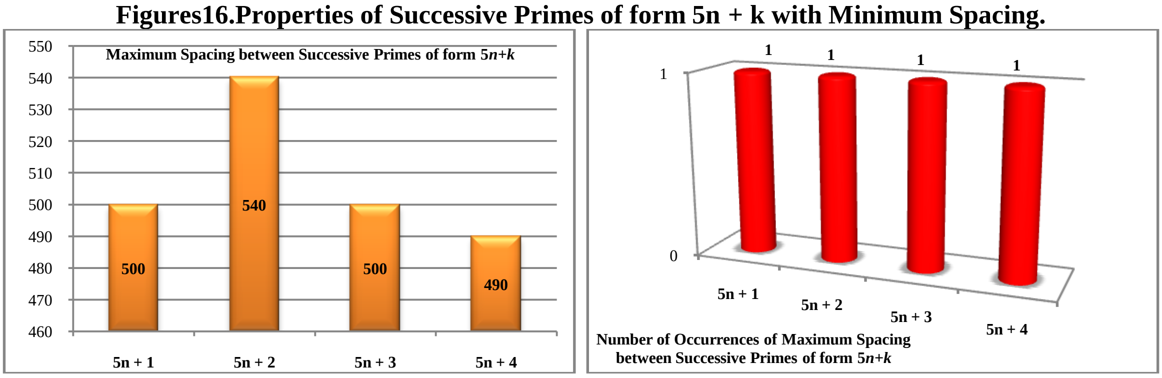


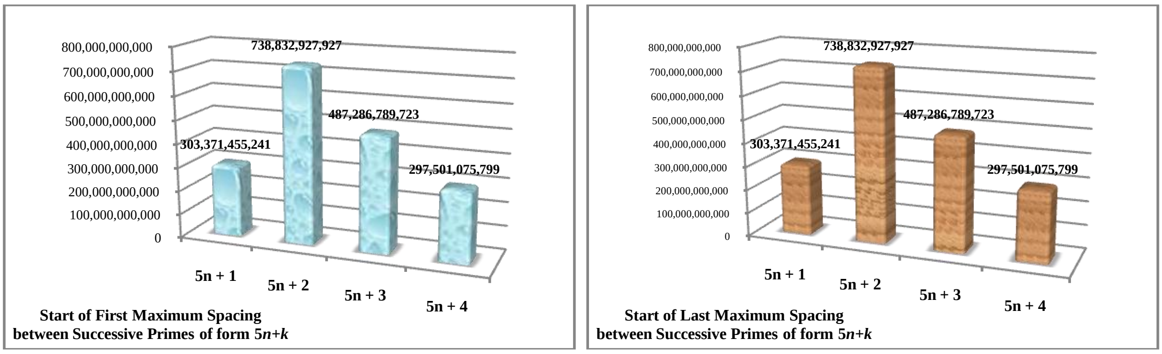

**Figure 15. Number of Successive Primes of form 5n + k □ $10^{12}$.**

We have also analyzed these cases from various perspectives.The minimum spacing and maximum spacing between successive primes of forms 5*n* + *k* have following properties.




**Figures16.Properties of Successive Primes of form 5n + k with Minimum Spacing.**






**Figures17.Properties of Successive Primes of form 5n + k with Maximum Spacing.**

The random like distribution of primes is quite eligible candidate for undertaking detail study. The analysis done here is an addition to that with respect to a specific linear pattern of $5n + k$. The author is sure that the availability of rigorous analysis like this will help give a deeper insight into prime distribution.

## ACKNOWLEDGEMENTS

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The extensive continuous use of the Computer Laboratory of Mathematics & Statistics Department of the affiliation institution for several continuous months has a great deal of credit in materializing the analysis achieved. The power support extended by the Department of Electronics of the institute has allowed run the computer programmes without interruption and is duly acknowledged.

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