Quest Journals Journal of Research in Applied Mathematics Volume 2~ Issue 6 (2016) pp: 01-15 ISSN(Online) : 2394-0743 ISSN (Print):2394-0735 **www.questjournals.org**

A Comparative Study of One-Sample t-Test Under Fuzzy Environments

S. Parthiban^{1*} and P. Gajivaradhan²

¹Research Scholar, Department of Mathematics, Pachaiyappa's College, Chennai-600 030, Tamil Nadu, India. ²Department of Mathematics, Pachaiyappa's College, Chennai-600 030, Tamil Nadu, India.

Received 17 February, 2016; **A**ccepted 25 February, 2016 © The author(s) 2015. Published with open access at **www.questjournals.org**

ABSTRACT:- This paper proposes a method for testing hypotheses over one sample t-test under fuzzy environments using trapezoidal fuzzy numbers (tfns.). In fact, trapezoidal fuzzy numbers have many advantages over triangular fuzzy numbers as they have more generalized form. Here, we have approached a new method where trapezoidal fuzzy numbers are defined in terms of alpha level of trapezoidal interval data and based on this approach, the test of hypothesis is performed. More over the proposed test is analysed under various types of trapezoidal fuzzy models such as Alpha Cut Interval, Membership Function, Ranking Function, Total Integral Value and Graded Mean Integration Representation. And two numerical examples have been illustrated. Finally a comparative view of all conclusions obtained from various test is given for a concrete comparative study.

Keywords:- Trapezoidal Fuzzy Numbers (tfns./TFNS.), Alpha Cut, Test of Hypothesis, Confidence Limits, One-sample t-Test, Ranking Function, Total Integral Value (TIV), Graded Mean Integration Representation (GMIR).

AMS Mathematics Subject Classification (2010): 62A86, 62F03, 97K80

I. INTRODUCTION

Test of hypotheses concerning some population parameters are an important part of statistical analysis. In traditional mean testing, the observation of sample is generally assumed to be a crisp value and to satisfy some relevant assumptions. However, in the real life, the data sometimes cannot be recorded or collected precisely. The statistical hypotheses testing under fuzzy environments has been studied by many authors using the fuzzy set theory concepts introduced by Zadeh [38]. Viertl [33] investigated some methods to construct confidence intervals and statistical tests for fuzzy data. Wu [37] proposed some approaches to construct fuzzy confidence intervals for the unknown fuzzy parameter. A new approach to the problem of testing statistical hypotheses is introduced by Chachi et al. [15]. Asady [9] introduced a method to obtain the nearest trapezoidal approximation of fuzzy numbers. Gajivaradhan and Parthiban analysed one sample t-test using alpha cut interval method using trapezoidal fuzzy numbers [17]. Abhinav Bansal [5] explored some arithmetic properties of arbitrary trapezoidal fuzzy numbers of the form (a, b, c, d). Moreover, Liou and Wang ranked fuzzy numbers with total integral value [23]. Wang et al. presented the method for centroid formulae for a generalized fuzzy number [35]. Iuliana Carmen B RB CIORU dealt with the statistical hypotheses testing using membership function of fuzzy numbers [21]. Salim Rezvani analysed the ranking functions with trapezoidal fuzzy numbers [28]. Wang arrived some different approach for ranking trapezoidal fuzzy numbers [35]. Thorani et al. approached the ranking function of a trapezoidal fuzzy number with some modifications [29]. Salim Rezvani and Mohammad Molani presented the shape function and Graded Mean Integration Representation for trapezoidal fuzzy numbers [27]. Liou and Wang proposed the Total Integral Value of the trapezoidal fuzzy number with the index of optimism and pessimism [23].

In this paper, we propose a new statistical fuzzy hypothesis testing of one-sample t-test in which the designated samples are in terms of fuzzy (trapezoidal fuzzy numbers) data. Another idea in this paper is, when we have some vague data about an experiment, what can be the result when the centroid point/ranking grades of those imprecise data are employed in hypothesis testing? For this reason, we have used the centroid/ranking grades of trapezoidal fuzzy numbers (tfns.) in hypothesis testing. In fact, we would like to counter an argument that the -cut interval method can be general enough to deal with one-sample t-test under fuzzy environments. And for better understanding, the proposed fuzzy hypothesis testing technique is illustrated with two numerical examples at each models. Finally a tabular form of all conclusions obtained from various test is given for a concrete comparative study. And the same concept can also be used when we have samples in terms of triangular fuzzy numbers [10]. A *Comparative Study of One-Sample t-Test under Fuzzy Environments*

that the cutt interval method can be general enough to deal with one-sample t-test under fuzzy environments.

And it reducts and concrete comparative su **EXECUTE:** A Comparative Study of One-Sample

that the cut interval method can be general enough to deal with one-samples at each models. Finally a tabular form of all conclusions obta

concrete comparative study. And the A Comparative Study of One-Sample t-Test under Fuzzy Environments

² --cut interval method can be general enough to deal with one-sample t-ets under fuzzy environments.

In the term understanding, the proposed fuzzy hyp A *Comparative Study of One-Sample t-Test under Fuzzy Environments*
 \sim -cut inneval method can be general enough to deal with one-sample t-stat under fuzzy environments.

In the thermal method can be general enough to *A Comparative Study of One-Sample*
 i -cut interval method can be general enough to deal with one-samples

the vector understanding, the proposed fuzzy hypothesis testing technic

the comparative study. And the same co

II. PRELIMINARIES

Definition 2.1. Generalized fuzzy number

-
-
-
-
-
-

A Computative Study of One-Sample r-Test under Fargy Environments

that the -cutt interval method can be general encouply to local with one-sample t-test under fargy print

Anal for better understanding, the proposed (acc And the better interestinants the propose large hypothesis restant entergy bullenting with two summations of the measuring for the same conserpt can analy a bullent for the measuring term in the propose large for a summar It $\mu_{\tilde{\chi}}(x) = 0$, for all $x \in (-\infty, a]$.

It is $\mu_1(x) = \frac{1}{\tilde{\chi}}(x)$ is strictly increasing on [a, b],
 $\therefore \mu_2(x) = \frac{1}{\tilde{\chi}}(x) = \frac{1}{\tilde{\chi}}(x)$ is strictly identerial on $\left[\frac{\tilde{\chi}}{\tilde{\chi}}(x)\right]$,
 $\therefore \mu_2(x) = 0$, iii. $\mu_1(x) = L_{\overline{A}}(x)$ is sariedly increasing on [a, b],

iv. $\mu_{\overline{A}}(x) =$, for all [b, c], as is a constant and $0 < \le 1$,
 $\nu_1(x) = R_{\overline{A}}(x)$ is siredly decreasing on [c, d],
 $\nu_1(x) = R_{\overline{A}}(x) = 0$, for all $x \$ w . $\mu_{\overline{X}}(x) =$, for all [b, e], as is a constant and $0 <$ ≤1,
 v . $\mu_{R}(x) = R_{\overline{X}}(x)$ is strictly decreasing on [c, d],
 \therefore $\mu_{R}(x) = R_{\overline{X}}(x) = 0$, for all $x \in [d, \infty)$ where a, b, c, d are real numbers s $0 < \le 1$,

numbers such that $a < b \le c < d$.

exists an element (member) 'x' such that
 $convex$ fuzzy set if
 $x, x_2 \in X$ and $\in [0, 1]$. The set
 \tilde{A} .
 $v = \tilde{A}$.
 \tilde{A} .
 \tilde{A} .
 \tilde{A} \tilde{A} .
 \tilde{A} \tilde{A} v. $H_X(X) = K_X(X)$ is strictly decreasing on [x, y, z] where a, b, c, d are real numbers such that $a < b \le c < d$.
 Definition 2.2.A fuzzy set \tilde{A} is called *normal* fuzzy set if there exists an element (member) 'x' such **Definition 2.2.** A thory set \tilde{A} is called normal fuzzy set if there exists an element (member) 'x' such that $\mu_{\tilde{A}}(x) = 1$. A fuzzy set \tilde{A} is called **convex** fuzzy set if $\mu_{\tilde{A}}(x) + (1 - x) x_1 \ge \min \{ \mu_{\til$ 'x' such that
set if
The set
 \mathbf{x}) such that
supper semi-
is the closure
 $\in \mathbb{R}$ and two
ng and non-
 $c \le x \le d$
of the fuzzy
nown that the
 $\left(\bigcup_{e[0, 1]} \widetilde{A}\right)$,
number is a
number is a
where \mathbf{x}

operator. **Definition 2.2.** A fuzzy set A is called *normals* fuzzy set if there exists an element (member) 'x' such that $\lim_{\lambda \to \infty} \lambda \leq \max$ set \tilde{A} is called *convex* fuzzy set if $\lambda = \frac{1}{2}$ and $\alpha = \frac{1}{2}$ and $\alpha = \frac{$ fuzzy set if

[0, 1]. The set
 $\mu_{\tilde{A}}(x)$ such that
 $\mu_{\tilde{A}}(x)$ is upper semi-

and 'cl' is the closure

b, c, $d \in \mathbb{R}$ and two

decreasing and non-

d) for $c \le x \le d$
 ght side of the fuzzy

it is known that $\mu_{\overline{X}}(x) = 1$. A fuzzy set \overrightarrow{X} is called convex fuzzy set if $\mu_{\overline{X}}(x_1 + (1 - \)x_2) \ge \min{\{\mu_{\overline{X}}(x_1), \mu_{\overline{X}}(x_2)\}}$ where $x_1, x_2 \in X$ and $\in [0, 1]$. The set $\overrightarrow{A} = \{x \in X/\mu_{\overline{X}}(x) \ge \}$ is said to be $\mu_{\tilde{x}}(x_1) = (1^2 - 3x_2) \ge 0$ and the best close of a fuzzy scheme λ_1 , $\lambda_2 \ge 1$ and $\lambda_1 = 2^2$ ($x \ge 1$ and $\lambda_2 = 2^2$) and the set of $\lambda_1 = \lambda_2$ ($x \ge 1$). Hence, $\lambda_1 = \lambda_2 = \lambda_1$ and $\lambda_2 = \lambda_1$ and $\lambda_3 = \$ function $\mu_{\tilde{A}}(x)$ such that

convex, $\mu_{\tilde{A}}(x)$ is upper semi-
 $(x) > 0$ and 'cl' is the closure

mbers a, b, c, d $\in \mathbb{R}$ and two

are non-decreasing and non-

ows:
 $(x-d)/(c-d)$ for $c \le x \le d$
 eft and *right s* **Definition 2.3.** A facey subset \overline{A} of the red line \mathbb{R} with *membership function* $\mu_{\tilde{A}}(x)$ such that $\mu_{\tilde{A}}(x): \mathbb{R} \rightarrow [0, 1]$, is called a fuzzy number if \tilde{A} is normal, \tilde{A} is fuzzy convex

increasing functions respectively. And its membership function is defined as follows:

 ∞

 $\int_{-\infty}^{\infty} \widetilde{A}(x) dx < +\infty$ and it is known that the

- cut of a fuzzy number is
$$
\widetilde{A} = \{x \in \mathbb{R}/\mu_{\widetilde{A}}(x) \ge \}
$$
, for $\in (0, 1]$ and $\widetilde{A}_0 = \text{cl} \left(\bigcup_{\in (0, 1]} \widetilde{A} \right)$,

R and two

; and non-

≤ x ≤ d

of the fuzzy

wn that the
 $\bigcup_{\in (0, 1]} \widetilde{A}$,

number is a

where

∴

Nation of

*Corresponding Author: S. Parthiban 2 | Page

A Comparative Study of One-Sample t-Test under Fuzzy Enviromments
\nthe fuzzy number
$$
\tilde{A}
$$
 are strictly monotone, obviously, \tilde{A}_L and \tilde{A}_U are inverse functions of $L_{\tilde{A}}(x)$ and
\n $R_{\tilde{A}}(x)$ respectively. Another important type of fuzzy number was introduced in [11] as follows:
\nLet a, b, c, d $\in \mathbb{R}$ such that $a < b \le c < d$. A fuzzy number \tilde{A} defined as $\mu_{\tilde{A}}(x): \mathbb{R} \rightarrow [0, 1]$,
\n $\mu_{\tilde{A}}(x) = \left(\frac{x-a}{b-a}\right)^n$ for $a \le x \le b$; 1 for $b \le x \le c$; $\left(\frac{d-x}{d-c}\right)^n$ for $c \le x \le d$; 0 otherwise where
\n $n > 0$, is denoted by $\tilde{A} = (a, b, c, d)_n$. And $L(x) = \left(\frac{x-a}{b-a}\right)^n$, $R(x) = \left(\frac{d-x}{d-c}\right)^n$ can also be termed as
\nleft and right spread of the tfn. [Dubois and Prade in 1981].
\nIf $\tilde{A} = (a, b, c, d)_n$, then [1-4],
\n $\tilde{A} = \left[\tilde{A}_L(\) , \tilde{A}_U(\)\right] = \left[a + (b-a)\sqrt[n]{\ } , d - (d-c)\sqrt[n]{\ }]; \in [0, 1]$.
\nWhen n = 1 and b = c, we get a triangular fuzzy number. The conditions $r = 1$, $a = b$ and $c = d$ imply

 $b - a$ ^{$d - c$} left and right spread of the tfn. [Dubois and Prade in 1981].

$$
\widetilde{A} = \left[\widetilde{A}_L(), \widetilde{A}_U()\right] = \left[a + (b-a)\sqrt[m]{,} d - (d-c)\sqrt[m]{,} \right]; \in [0, 1].
$$

A Comparative Study of One-Sample t-Test under Fuzzy Environments

the fuzzy number \tilde{A} are strictly monotone, obviously, \tilde{A}_L and \tilde{A}_U are inverse functions of $L_{\tilde{A}}(x)$ and
 $R_{\tilde{A}}(x)$ respectively **A Comparative Study of One-Sample t-Test under Fuzzy Environments**
the fuzzy number \overline{A} are strictly monotone, obviously, \overline{A}_L and \overline{A}_U are inverse functions of $L_{\overline{X}}(x)$ and $R_{\overline{X}}(x)$ respectively the closed interval and in the case $r = 1$, $a = b = c = d = t$ (some constant), we can get a crisp number 't'. A Comparative Study of One-Sample t-Test under Fuzzy Environments

the fuzzy number \tilde{A} are strictly monotone, obviously, \tilde{A}_L and \tilde{A}_U are inverse functions of $L_x(x)$ and

Let a, b, c, d $\in \mathbb{R}$ such th $F^T(\mathbb{R})$. Now, for $n = 1$ we have a normal trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d)$ and the A Comparative Study of One-Sample r-Test under Fuzzy Environments

In Euzzy number \overline{A} are strictly monotone, obviously, \overline{A}_V and \overline{A}_V are inverse functions of $L_{\overline{A}}(x)$ and
 $R_{\overline{A}}(x)$ respectively. corresponding - cut is defined by
 $\widetilde{A} = \begin{bmatrix} a + (b - a), d - (d - c) \end{bmatrix}; \in [0, 1] - - (2.4)$. And we need the following results which be fuzzy number \overline{A} are strictly monotone, obviously, \overline{A}_L and \overline{A}_U are inverse functions of $L_{\chi}(x)$ and $R_{\chi}(x)$ respectively. Another important type of fuzzy number was introduced in [11] as follows:
L Let a, b, c, d e R such that $a < b \le c < d$. A fuzzy number \tilde{A} defined as $\mu_{\tilde{A}}(x)$: $\mathbb{R} \rightarrow [0, 1]$,
 $\mu_{\tilde{A}}(x) = \left(\frac{x-a}{b-a}\right)^a$ for $a \le x \le b$; 1 for $b \le x \le c$; $\left(\frac{d-x}{d-c}\right)^a$ for $c \le x \le d$; 0 otherwise where **R** ⇒ 0, is denoted by $\tilde{A} = \{a, b, c, d\}_a$. And $L(x) = \left(\frac{x-a}{b-a}\right)^a$, $R(x) = \left(\frac{d-x}{d-c}\right)^a$ can also be termed as

Left and right spread of the tfn. [Dubois and Prude in 1981].

If $\tilde{A} = \{a, b, c, d\}_a$, then $[1.4]$,
 EVALUATION THE CONDUCTED THE CONDUCTED THE SET FOR SINGLE MEAN FIRE SET EVALUATION THE SAMPLE CHEST FOR SINGLE MEAN FIRE SOMETHER I FOR SHOWING THE SAMPLE CHEST FOR SINGLE MEAN FIRE CONDUCTED THE CONDUCTED THE CONDUCTED T EF (R). Now, for n = 1 ve have a normal trapezoidal fuzzy number $\tilde{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Now, for n = 1 ve have a normal trapezoidal fuzzy number $\tilde{A} = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}$ and the contents of n = 1 ve ha $\begin{bmatrix} (4 \cdot c) \\ -(4 \cdot c) \end{bmatrix}$
 $= (4, b, c, d)_a$. And $L(x) = \left(\frac{4 \cdot c}{b + a}\right)^n$, $R(x) = \left(\frac{d-x}{d+c}\right)^n$ can also be termed as

te the [Dubeis and Prade in 1981].

(a) $\left[1 + 1\right]$.
 $\left[1\right] = \left[a + (b + a) \sqrt{b'}\right]$, $d - (d - c) \sqrt{b'}\right]$;

can be found in [20, 22].

 \mathbb{R} .

III. ONE – SAMPLE t-TEST FOR SINGLE MEAN

from a normal population with a specified mean, say μ_0 or (ii) if the sample mean differs significantly from the hypothetical value μ_0 of the population mean, then under the null hypothesis H₀: nd B = [c, d] be in D. Then A = B if a = c and b = d.

ONE – SAMPLE t-TEST FOR SINGLE MEAN

f a random (small) sample x_i ($i = 1, 2, ..., n$) of size $n < 3$

a specified mean, say μ_0 or (ii) if the sample mean differs sig $B = [c, d]$ be in D. Then A = B if a

NE – SAMPLE t-TEST FOR SINC

candom (small) sample x_i ($i = 1, 2, ...$

ecified mean, say μ_0 or (ii) if the samp

ation mean, then under the null hypothe:

from the population with me **IONE - SAMPLE t-TEST FOR SINGLE M**
 IONE - SAMPLE t-TEST FOR SINGLE M

if a random (small) sample x_i ($i = 1, 2, ..., n$) of

i a specified mean, say μ_0 or (ii) if the sample mean

sopulation mean, then under the null s n n - 1 Result 2.1. Let $D = \{[a, b], a \le b \text{ and } a, b \in \mathbb{R}\}$, the s

R.

Result 2.2. Let $A = [a, b]$ and $B = [c, d]$ be in D . The

III. ONE – SAMPLE t-TES

In case if we want to test (i) if a random (small) sample x

from a normal popu

- (a) The sample has been drawn from the population with mean μ_0 or
- (b) There is no significant difference between the sample mean X and the population mean μ_0 , in this case, **the test statistic** is given by

$$
t = \frac{x - \mu_0}{s \sqrt{n}} \text{ where } \overline{x} = \frac{1}{n} \sum_{i=1}^n x_i \text{ and } s^2 = \frac{1}{n-1} \sum_{i=1}^n \left(x_i - \overline{x} \right)^2 \text{ follows } \text{Student's } t\text{-distribution with}
$$

 $(n - 1)$ degrees of freedom.

We now compare the calculated value of 't' with the tabulated value at certain level of significance. Let t be the tabulated value at level of significance and 't' be the calculated value and we set the null hypothesis as

 H_0 may be accepted (one tailed test) at the level of significance adopted. If $|t| < t$ \Rightarrow the null hypothesis st under Fuzzy Environments

gion

tailed test]

tailed test]

ailed test]
 $|t| < t \implies$ the null hypothesis
 $|t| < t \implies$ the null hypothesis

oonding to the given sample are H_0 is accepted (two tailed test).

 $H_A: \mu > \mu_0$
 $H_A: \mu < \mu_0$
 $H_A: \mu \neq \mu_0$
 $t \leq t_{n-1}$

That is, if $|t| > t$ ⇒ The null hypothesis H_0 is rejected (one tailed test)
 H_0 may be accepted (one tailed test) at the level of significance adopted
 H_0

IV. TEST OF HYPOTHESIS FOR INTERVAL DATA
Let $\{[x_i, y_i]$; $i = 1, 2, ..., n\}$ be a random sample of size $n(< 30)$ A Comparative Study of One-Sample t-Test under Fuzzy Environments
 Alternative Hypothesis
 $\frac{1}{2}$, $\frac{1}{2$ A Comparative Study of One-Sample t-Test under Fuzzy Environments
 $\frac{H_x\mu > \mu_0}{H_x\mu > \mu_0}$ $t \ge t_{-n}$. [Upper aited test]
 $\frac{H_x\mu < \mu_0}{H_x\mu < \mu_0}$ $|t| \ge t_{-n}$. [Upper aited test]

That is, if $|t| > t \Rightarrow$ The null hypot with the population means and μ and the sample means μ_0 and μ_0 of the sample respectively. Then the **IVE :** $\frac{1}{2}$ if $\frac{1$ H_A: $\mu > \mu_0$ $t \ge$

H_A: $\mu < \mu_0$ $t \le$

H_A: $\mu < \mu_0$ $|t| \ge$

That is, if $|t| > t$ \Rightarrow The null hypothesis H₀ is rejected (one tailed

H₀ may be accepted (one tailed test) at the level of significance

H₀ is a and $\mu < \mu_0$; H_A : $\left[\right]$, μ $>$ $\left[\right]$ $\left[\right]$, μ_0 , μ_0 , that is μ_0 and $\mu > \mu_0$ $t \geq t_{n+1}$ [Upper tailed test]
 $t \leq -t_{n+1}$ [Lower tailed test]
 $|t| \geq t_{n+1}$ [Two tailed test]
 $|t| \geq t_{n+1}$ [Two tailed test]

significance adopted. If $|t| < t \Rightarrow$ the null hypothesis

significance adopted. If $|t$ H_A: $\mu \leq \mu_0$
 $H_A: \mu \neq \mu_0$
 $H_A: \mu \neq \mu_0$
 $H_A: \mu \neq \mu_0$
 $H_B: \mu = \sum_{i=1}^n H_i$ (Figure 1 and text)
 H_0 may be accepted (one tailed test) at the level of significance adopted. If $|x| < H$
 H_0 is accepted (one t or $\mu \neq \mu_0$. We now consider the random sample of lower value That is, if $\left|z\right| > 1$ \Rightarrow The null hypothesis H_0 is rejected (one tailed test) and if $\left|1\right| < t \Rightarrow$ the null hypothesis H_0 may be accepted (one tailed test) at the level of significance adopted. If $\left|z\right| < t \Rightarrow$ is rejected (one tailed test) and if $|t| < t \Rightarrow$ the
vel of significance adopted. If $|t| < t \Rightarrow$ the
r the population mean μ corresponding to the gi
 $\left(\frac{s}{\sqrt{n}}\right)$.
OTHESIS FOR INTERVAL DATA
pe a random sample of size Let $\{\lfloor x_i, y_i\rfloor; i = 1, 2, ..., n\}$ be a random sample of size $n(< 30)$ such that $\{x_i; i = 1, 2, ..., n\}$ and $\{y_i; i = 1, 2, ..., n\}$ are the random samples from two distinct normal population with the population means and μ and the sample means σ_0 and μ_0 of the sample respectively. Then the null hypothesis $H_0: [\, \, \mu] = [\, \, \sigma, \, \mu_0]$, that is, σ_0 and $\mu = \mu_0$ (n) and the random samples from two distinct no

sample means σ_0 and μ_0 of the sample respections is, $\mu_0 = \sigma_0$ and $\mu = \mu_0$ and the alternative hypother $\mu < \mu_0$; $H_A: [\sigma, \mu] > [\sigma_0, \mu_0]$, that is $> \mu \neq \mu_0$. tevel of significance adopted. If $|t| < t_{\bigtriangleup}$ ⇒ the null hypothesis

or the population mean μ corresponding to the given sample are
 $\left(\frac{s}{\sqrt{n}} \right)$.
 POTHESIS FOR INTERVAL DATA

be a random sample of size n(<30 at the level of significance adopted. If $|I| < I_{\frac{1}{2}}$ = the null hypothesis

limits for the population mean μ corresponding to the given sample are
 $+ I_{\frac{1}{2},n-1} \left(\frac{s}{\sqrt{n}} \right)$.
 F HYPOTHESIS FOR INTERVAL DATA
 For the population mean μ corresponding to the given sa
 $\left[\frac{s}{\sqrt{n}}\right]$.
 POTHESIS FOR INTERVAL DATA

be a random sample of size $n(<30)$ sue

., n} are the random samples from two distinct normal pc

sample means $\$ $\binom{1}{2}$, $\binom{1}{3}$, $\binom{1}{4}$, $\binom{1}{5}$, $\binom{1}{4}$, $\binom{1}{5}$, $\binom{1}{5}$, $\binom{1}{1}$, $\binom{1}{2}$, $\binom{1}{3}$, $\binom{1}{4}$, $\binom{1}{5}$ i $\binom{1}{4}$, $\binom{1}{5}$ i $\binom{1}{4}$, $\binom{1}{5}$ i $\binom{1}{4}$, $\binom{1}{5}$ i **HESIS FOR INTERVAL DATA**

a random sample of size $n(< 30)$ such that
\nare the random samples from two distinct normal population
\nble means 0 and μ_0 of the sample respectively. Then the
\n $= 0$ and $\mu = \mu_0$ and the alternative hypotheses
\n μ_0 ; H_A : $[\, \cdot \, \mu] > [\, \cdot \, \cdot \, \mu_0],$ that is > 0 and $\mu > \mu_0$
\n μ_0 . We now consider the random**NTERVAL DATA**
ple of size $n(*30*)$ such that
ples from two distinct normal population
 μ_0 of the sample respectively. Then the
 μ_0 and the alternative hypotheses
 $[\circ, \mu_0]$, that is $> \circ$ and $\mu > \mu_0$
onsider t IV. TEST OF HYPOTHESIS FOR INTERVAL

{[x₁, y₁]; i = 1, 2, ..., n} be a random sample of size

2, ..., n} and {y_i; i = 1, 2, ..., n} are the random samples from two

lation means and μ and the sample means o and μ **HESIS FOR INTERVAL DATA**

a random sample of size $n(< 30)$ such that
\nare the random samples from two distinct normal population
\nple means 0 and μ_0 of the sample respectively. Then the
\n $= 0$ and $\mu = \mu_0$ and the alternative hypotheses
\n μ_0 ; H_A : $[\, \cdot \, \mu] > [\, \cdot \, \cdot \, \mu_0],$ that is > 0 and $\mu > \mu_0$
\n μ_0 : H_A **NTERVAL DATA**

ple of size $n(*30*)$ such that

mples from two distinct normal population
 μ_0 of the sample respectively. Then the
 μ_0 and the alternative hypotheses
 $\begin{bmatrix} 0, \mu_0 \end{bmatrix}$, that is > 0 and $\$ ${[\{x_i, y_i\}; i = 1, 2, ..., n\}$ be a random sample of size

2, ..., n} and {y_i; i = 1, 2, ..., n} are the random samples from two

lation means and μ and the sample means ₀ and μ₀ of the sam

is H₀:[, μ] = [₀, μ₀], random sample of size $n(< 30)$ such the
\nthe random samples from two distinct normal population
\nmeans 0 and μ_0 of the sample respectively. Then it
\n $= 0$ and $\mu = \mu_0$ and the alternative hypotheses
\n $\{H_A: [\, \mu] > [\, \, 0, \, \mu_0],$ that is > 0 and $\mu > \mu_0$
\n $\{h_A: [\, \mu] > [\, \, 0, \, \mu_0],$ that is > 0 and $\mu > \mu_0$
\n $\{h_A$ ble of size $n(< 30)$ such that

uples from two distinct normal population
 μ_0 of the sample respectively. Then the
 μ_0 and the alternative hypotheses
 $\begin{bmatrix} 0, \mu_0 \end{bmatrix}$, that is > 0 and $\mu > \mu_0$

usider the

respectively. The sample mean of S_L and S_U are S_x and S_y respectively.

n $\sqrt{\sqrt{n}}$

The rejection region of the alternative hypothesis for level of significance is given by

If $|t^L| < t_{n, n-1}$ (One tailed test) and $|t^U| < t_{n, n-1}$ (One tailed test)

hypothesis H_A is accepted

If
$$
|t^L| < t_{\frac{1}{2}, n-l}
$$
 (Two tailed test) and $|t^U| < t_{\frac{1}{2}, n-l}$ (Two tailed test)

A Comparative Study of One-Sample t-Test under Fuzzy Environments
 \Rightarrow The difference between $[\ , \ \mu]$ and $[\ , \ \mu_0]$ is not significant at level. Otherwise the alternative

Also, the 100(1 -)% confidence limits for the hypothesis H_A is accepted.

are given below:

Also, the 100 1 - α % confidence limits for the population mean η, μ corresponding to the given sample x x y y α α α α ,n-1 ,n-1 ,n-1 ,n-1 2 2 2 2 s s x - t , y - t η, μ x + t , y + t n n n n

Decision table:

Partial acceptance of null hypothesis H_0 at the intersection of certain level of α at both upper level and lower level models can be taken into account for the acceptance of the null hypothesis H_0 .

Example-1

The department of career and placement of an inspection committee of a University claims that the placement percentage of the students from its affiliated institution is between 50 and 65. Only 12 colleges in that zone are selected at random. The colleges are observed and the placement percentages of each of the colleges are recorded. Due to some limited resources, the minimum and maximum of the placement percentage of each of the colleges can only be observed. Therefore, the placement percentages of the colleges are taken to be 'intervals' as follows [19]: ϵ [0,1]

If H₀^t is rejected for all α, or H₀^t is rejected
 ϵ [0,1] ϵ [0,1] ϵ [0,1] ϵ [0,1]

Partial acceptance of null hypothesis H₀ at the intersection

evel models can be taken into account fo ejected for all α_s and H_0^0 is accepted for all α_s then \overline{H}_0 is rejected for all α_s

ejected for all α_s or H_0^0 is rejected for all α_s then \overline{H}_0 is rejected for all α_s

eigected for all $\in [0,1]$

or H₀^U is rejected for all α,
 $\in [0,1]$

or H₀^U is rejected for all α,
 $\in [0,1]$

SH₀ at the intersection of certain level of α at both upp

nt for the acceptance of the null hypothesis H₀.

and ϵ [0,1]

certain level of α at both upper level and lower

null hypothesis H̃o.

ction committee of a University claims that the

tion is between 50 and 65. Only 12 colleges in

and the placement percentages of each o Partial acceptance of mil hypothesis H₃ at the intersection of certain level of α at both upper level and lower
tevel models can be taken into account for the acceptance of the mull hypothesis H₃.
 Example-1

The d is career and placement of an inspection committee of a Univerentum statis est
sudents from its affiliated institution is between 50 and 65. Candom. The colleges are observed and the placement percentate or
only be observ n committee of a University claims that the
i is between 50 and 65. Only 12 colleges in
d the placement percentages of each of the
m and maximum of the placement percentage
ment percentages of the colleges are taken to
.

[44, 53], [40, 38], [61, 69], [52, 57], [32, 46], [44, 39], [70, 73], [41, 48], [67, 73], [72, 74] [53, 60], [72, 78].

Solution

level of significance. Now, the sample mean value of the lower and upper interval values are

$$
\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
$$
 $\implies \overline{x}^L = 54$ and $\overline{x}^U = 59$

And the sample S.D. of the lower and upper interval values are $s^2 = \frac{1}{s} \sum_{i=1}^{s} (x_i - x_i)$ 2 - $1 \frac{n}{\sum (x - \overline{x})^2}$ i Λ Λ $i=1$ $=\frac{1}{n-1}\sum_{i=1}^{n} (x_i - \overline{x})^2$

$$
\Rightarrow
$$
 S^L = 14.06 and S^U = 14.30.

The test statistic:

EXECUTE:
\n**10.** The test statistic:
\n
$$
t^{L} = \frac{\begin{pmatrix} -L & 0 \\ x^{L} - \mu_{0}^{L} \end{pmatrix}}{s^{L}/\sqrt{n}} = 0.9855 \text{ and } t^{U} = \frac{\begin{pmatrix} -U & 0 \\ x^{U} - \mu_{0}^{U} \end{pmatrix}}{s^{U}/\sqrt{n}} = -1.4535
$$
\nSince $|t^{L}| < T$ and $|t^{U}| < T$, the null hypothesis " the population mean $\mu = \mu_{0}$ " is accepted and the 95% confidence limits for the population mean μ is
\n
$$
\begin{bmatrix} -L & 0 \\ x^{L} - t & 0 \\ y^{n-1} & y^{N} \end{bmatrix} = \frac{1}{\sqrt{n}} \begin{bmatrix} -L & 0 \\ -L & 0 \\ y^{N} & y^{N} \end{bmatrix} \begin{bmatrix} -L & 0 \\ -L & 0 \\ z^{N} & z^{N} \end{bmatrix} = \frac{1}{\sqrt{n}} \begin{bmatrix} -L & 0 \\ -L & 0 \\ y^{N} & z^{N} \end{bmatrix} \begin{bmatrix} -L & 0 \\ -L & 0 \\ z^{N} & z^{N} \end{bmatrix}
$$

e Study of One-Sample t-Test under Fuzzy Environne
 $\frac{1}{x}$ $\left(\frac{1}{x} - \mu_0^U\right)$
 $= -1.4535$

sis " the population mean $\mu = \mu_0$ " is accepted and the varative Study of One-Sample t-Test under Fuzzy Environments
 $t^U = \frac{\begin{pmatrix} -U & \mathbf{u}^U \ \mathbf{x}^U - \mathbf{\mu}_0^U \end{pmatrix}}{s^U \sqrt{n}} = -1.4535$

ypothesis " the population mean $\mu = \mu_0$ " is accepted and the 95%
 μ is
 $|\mathbf{x}| \leq \begin{bmatrix}$ dy of One-Sample t-Test under Fuzzy Environments
 $\left(\frac{1}{0}\right)^{U}$
 $= -1.4535$
 $\frac{1}{0}$

he population mean $\mu = \mu_0$ " is accepted and the 95% Since $|t^L| < T$ and $|t^U| < T$, the null hypothesis " the population mean $\mu = \mu_0$ " is accepted and the 95% confidence limits for the population mean μ is

t T and t T , the null hypothesis " the population mean μ = μ⁰ L U L U α α α α ,n-1 ,n-1 ,n-1 ,n-1 2 2 2 2 s s s s x - t , x - t η, μ x + t , x + t n n n n 45.0707, 49.9183 η, μ 62.9293, 68.0817 Let a trapezoidal fuzzy number be A a, b, c, d , then the fuzzy interval [31] in terms of α - cut A a + b - a α, d - d - c α ; 0 α 1 --- [5.1]

V. TESTING HYPOTHESIS FOR FUZZY DATA USING TFNS. Trapezoidal Fuzzy Number to Interval

interval is defined as follows:

$$
\widetilde{A} = \begin{bmatrix} a + (b - a) , d - (d - c) \end{bmatrix}; 0 \leq s \leq 1 - [5.1]
$$

Suppose that the given sample is a fuzzy data that are trapezoidal fuzzy numbers and if we want to test the hypothesis about the population mean, then we can transfer the given fuzzy data into interval data by using the relation (5.1).

Example-2

The marketing department for a tire and rubber company wants to claim that the average life of a tire, the company recently developed exceeds the well-known average tire life of a competitive brand, which is known to be 32000 miles. Only 24 new tires were tested because the tests are tedious and take considerable time to complete. Six cars of a particular model and brand were used to test the tires. Car model and brand were fixed so that other car-related aspects did not come into play. The situation was that the life of the tires were not known exactly. The obtained life of the tire was around a number. Therefore the tire life numbers were $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} < \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7293, 68.0817]
 THESIS FOR FUZZY DATA USING TFNS.
 THESIS FOR FUZZY DATA USING TFNS.

$$
\begin{bmatrix}\nx^6 - t_{\frac{\ell}{2},\text{B-1}}\left(\frac{s^6}{\sqrt{n}}\right), \frac{1}{x} - t_{\frac{\ell}{2},\text{B-1}}\left(\frac{s^6}{\sqrt{n}}\right)\right] < \left[\frac{x}{x} + t_{\frac{\ell}{2},\text{B-1}}\left(\frac{s^6}{\sqrt{n}}\right), \frac{1}{x} - t_{\frac{\ell}{2},\text{B-1}}\left(\frac{s^6}{\sqrt{n}}\right)\right]
$$
\n
$$
\Rightarrow [45,0707, 49.9183] < \left[\frac{1}{x} + \frac{1}{x}\right] < [62.9293, 68.0817]\n\text{Trapezoids Huzzy Number to Interval ESING TENS.\nTrapezoidal fuzzy Number $\overline{A} = [a, b, c, d)$, then the fuzzy interval [31] in terms of $-$ cut interval is defined as follows:\n
$$
\widetilde{A} = [a, b, c, d], \text{ then the fuzzy number is an fuzzy interval is defined as follows:\n
$$
\widetilde{A} = [a, b, c, d], \text{ then the fuzzy number is an fuzzy data into interval data by using the relation (51).\n
$$
\text{Example 2}
$$
\n**Example 2**\n**Example 3**\n**Example 4**\n**Example 4**\n**Example 5**\n**Example 6**\n**Example 7**\n**Example 8**\n**Example 9**\n**Example 1**\n**Example 1**\n**Example 1**\n**Example 1**\n**Example 2**\n**Example 3**\n**Example 4**\n**Example 4**\n**Example 5**\n**Example 6**\n**Example 1**\n**Example 1**\n**Example 2**\n**Example 3**\n**Example 4**\n**Example 5**\n**Example 5**\n**Example 6**\n**Example 6**\n**Example 7**\n**Example 9**\n**Example 10**\n**Example 11**\n**Example 12**\n**Example 22**\n**Example 3**\n**Example 3**\n**Example 4**\n**Example 5**
$$
$$
$$

"*linguistic data*". Therefore,

A Comparative Study of One-Sample t-Test under Fuzzy Environments

H₀: $\tilde{\mu} \approx 32000$ and \tilde{H}_{Λ} : $\tilde{\mu} > 32000$ where 32000 means "around 32000", which regarded as a

H₀: The average life of the tire is aroun A Comparative Study
 $\widetilde{H}_0 : \widetilde{\mu} \approx 32000$ and $\widetilde{H}_A : \widetilde{\mu} > 32000$ where 3200
 Hinguistic data". Therefore,
 \widetilde{H}_0 : The average life of the tire is around 32000.
 \widetilde{H}_A : The average life of the ti A Comparative Study of One-Sample t-Test under Fuzzy Environments
 $\widetilde{H}_0: \widetilde{\mu} \cong 32000$ and $\widetilde{H}_A: \widetilde{\mu} > 32000$ where 32000 means "around 32000", which regarded as a

"linguistic data". Therefore,
 \widetilde{H}_0

Example 1. The sample n = 32 000 and \tilde{H} s: $\tilde{\mu}$ > 32000 where 32000 means "**around 32000**", which regarded as a "*linguistic data*". Therefore,
 \tilde{H} s: The average life of the tire is around 32000.
 $\tilde{$ unknown sample standard deviation.
Using relation (5.1), we convert the fuzzy data into interval data and are listed below: **17.** $\vec{u} = \frac{3}{2}$ **30.00** and **17.** $\vec{u} = \frac{3}{2}$ **60.00** were $\vec{u} = \frac{3}{2}$ standard devation.

sample standard devation

(5.1), we convert the fuzzy data into interval data and are listed below

1266 + 405 , 34889 - 712], $\tilde{x}_1 = [31720 + 423, 33497 - 678, 806 + 220, 33908 - 562]$, $\tilde{x}_8 = [31977 +$ a into interval data and are listed below:
 $\tilde{x}_2 = [32093 + 520, 33255 - 106]$,
 $\tilde{x}_4 = [31720 + 423, 33497 - 678]$,
 $\tilde{x}_6 = [31977 + 260, 33034 - 217]$,
 $\tilde{x}_8 = [31943 + 360, 33212 - 159]$,
 $\tilde{x}_{10} = [32169 + 786, 33968 - 820$ $\begin{bmatrix}\n1 & 1 & 1 \\
1 & 2 & 3\n\end{bmatrix}\n= [32185 + 538, 33186 - 226], \quad \tilde{x}_{16} = [31187 + 1775] = [33208 + 594, 34771 - 513], \quad \tilde{x}_{18} = [33208 + 1975] = [31639 + 842, 33542 - 516], \quad \tilde{x}_{20} = [30945 - 1215] = [31511 + 527, 33064 - 307], \quad \tilde{x}_{22} = [30826 \tilde{x}_3 = [32585 + 300, 33787 - 572], \tilde{x}_4 = [31720 + 423, 33498 - 562], \tilde{x}_6 = [31977 + 260, 3308 - 562], \tilde{x}_6 = [31977 + 260, 3308 - 562], \tilde{x}_8 = [31943 + 360, 332, \tilde{x}_9 = [30743 + 582, 32460 - 466], \tilde{x}_{10} = [32169 + 786, 3398 - 52460 - 52466], \til$ ($\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{3}$, 712], $\bar{x}_2 = [32093 + 520, 33255 - 106]$,

572], $\bar{x}_4 = [31720 + 423, 33497 - 678]$,

562], $\bar{x}_6 = [31977 + 260, 33034 - 217]$,

138], $\bar{x}_8 = [31943 + 360, 33212 - 159]$,

466], $\bar{x}_{10} = [32169 + 786, 33968 - 820]$,

467], $\$ $x_3 = [32000 + 220, 33906 - 302]$, $x_8 = [31977 + 200, 33034 - 217]$,
 $\bar{x}_2 = [33065 + 280, 34131 - 138]$, $\bar{x}_8 = [31943 + 360, 33212 - 159]$,
 $\bar{x}_9 = [30743 + 582, 32460 - 466]$, $\bar{x}_{10} = [32169 + 786, 33968 - 820]$,
 $\bar{x}_{11} = [3241$ x₂ = [33065 + 280 , 34131 - 138], x₈ = [31943 + 360 , 33212 - 159],

x₂ = [30743 + 582 , 32460 - 466], x₁₀ = [32169 + 786 , 33968 - 820],

x₁₁ = [32415 + 686 , 34072 - 117], x₁₂ = [32900 + 552 , 34335 -= [31943 + 360 , 33212 - 159],

= [32169 + 786 , 33968 - 820],
 $a =$ [32169 + 786 , 33968 - 820],
 $a =$ [30327 + 648 , 31445 - 150],
 $a =$ [31187 + 249 , 32237 - 238],
 $a =$ [31187 + 249 , 32237 - 238],
 $a =$ [30945 $\bar{x}_0 = \begin{bmatrix} 30743 + 582 \\ 37440 - 466 \end{bmatrix}$, $\bar{x}_{10} = \begin{bmatrix} 22169 + 786 \\ 33968 - 820 \end{bmatrix}$,
 $\bar{x}_{11} = \begin{bmatrix} 32415 + 686 \\ 34072 - 117 \end{bmatrix}$, $\bar{x}_{12} = \begin{bmatrix} 32000 + 552 \\ 34335 - 462 \end{bmatrix}$,
 $\bar{x}_{13} = \begin{bmatrix} 32087 + 508 \\ 39318$ 2415 + 686 , 34072 - 117], x₁₂ = [32900 + 552 , 34335 - 462].

2687 + 508 , 33908 - 477], x₁₄ = [30327 + 648 , 31445 - 150],

3143 + 538 , 33186 - 220], x_n = [3187 + 249 , 32237 - 238],

3423 + 594 , 34711 -3423 + 594 , 34771 - 513], $\tilde{x}_{18} = [33208 + 65$

1639 + 842 , 33542 - 516], $\tilde{x}_{20} = [30945 + 65]$

1511 + 527 , 33064 - 307], $\tilde{x}_{22} = [30826 + 75]$

3063 + 699 , 34449 - 268], $\tilde{x}_{24} = [33464 + 5$

obtain the mean

 L and L and L \sim U and L and L and L And the sample S. D. of the lower and upper interval values are:

$$
S^{L} = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^{n} \left(x_i^L - \overline{x}^L \right)^2 \right]} \implies S^{L} = \sqrt{33316.02^{-2} - 80004.54 + 838201.99}
$$

and
$$
S^{U} = \sqrt{740506.58^{-2} - 169304.62 + 815219.02}
$$

Now, we set the null hypothesis: H_0 : $\tilde{\mu} = \tilde{\mu}_0$ and the alternative hypothesis: H_A : $\tilde{\mu} > \tilde{\mu}_0$ (Upper tailed test) And the interval representation of $\tilde{\mu}_0$ is given by 1639 + 842 , 33542 - 516], $\bar{x} = [30945 + 689, 32739 - 414]$,

1511 + 527 , 33064 - 307], $\bar{x} = [30826 + 784, 32913 - 507]$,

30663 + 699 , 34449 - 268], $\bar{x}_4 = [33464 + 598, 34974 - 586]$.

obtain the mean value of the lo

And therefore,

$$
t^{L} = \frac{\overline{x}^{L} - \mu_{0}^{L}}{s^{L}/\sqrt{n}} = \begin{cases} 11.6042 \; ; & = 0\\ 3.8429 \; ; & = 1 \end{cases}
$$
\n
$$
\Rightarrow t^{L} > T = 1.714 \text{ for all } , 0 \leq \leq 1.
$$

And,

$$
A\text{ Comparative Study of One-Sample t-Test under Fuzzy En}
$$
\n
$$
t^{U} = \frac{\overline{x}^{U} - \mu_{0}^{U}}{s^{U}/n} = \begin{cases}\n-2.1134 \quad ; & = 0 \\
1.8105 \quad ; & = 0.53 \\
1.8757 \quad ; & = 0.54 \\
4.2402 \quad ; & = 1\n\end{cases}
$$
\n
$$
\Rightarrow t^{U} > T \text{ for } 0.53 \leq 1
$$
\n
$$
0.53 \leq 1, t^{L} > T \text{ and } t^{U} > T \text{, the alternative hypothesis is accepted. Therefore}
$$

A Comparative Study of One-Sample t-Test under Fuzzy Environments
 $t^U = \frac{\overline{x}^U - \mu_0^U}{s^U/\sqrt{n}} = \begin{cases} .2.1134 \ , & = 0 \\ 1.8105 \ , & = 0.53 \\ 1.8757 \ , & = 0.54 \end{cases}$
 $\Rightarrow t^U > T$ for $0.53 \leq \leq 1$

celusion
 $0.53 \leq \leq 1$, $t^L > T$ **5.1. Conclusion**
Since for $0.53 \le \le 1$, $t^L > T$ and $t^U > T$, the alternative hypothesis is accepted. Therefore, \widetilde{H}_A : The A Comparative Study of One-Sample t-Test under Fuzzy Environments
 $t^U = \frac{\overline{x}^U - \mu_0^U}{s^U - \mu_0^U} = \begin{cases} -2.1134 &; = 0 \\ 1.8105 &; = 0.53 \\ 1.8757 &; = 0.54 \\ 4.2402 &; = 1 \end{cases}$
 $\Rightarrow t^U > T$ for $0.53 \le \le 1$

Since for $0.53 \le \le 1$ A Comparative Study of One-Sample t-Test under
 $\begin{cases}\n-2.1134 : 0 \\
1.8105 : 0.53 \\
1.8757 : 0.54 \\
4.2402 : 1\n\end{cases} = 0.54$
 $t^L > T$ and $t^U > T$, the alternative hypothesis is accepted.
 $t^L > T$ and $t^U > T$, the alternative hypoth mparative Study of One-Sample t-Test under Fuzzy Environments
 $\begin{aligned}\n &\; = 0 \\
 &\; = 0.53 \\
 &\; = 1\n\end{aligned}$
 $\begin{aligned}\n &\text{t}^U > T \text{, the alternative hypothesis is accepted. Therefore, } \widetilde{H}_A : \text{The number of the original system is given by } \begin{cases}\n &\text{t}^U > T \text{, the alternative hypothesis is accepted.} \\
 &\text{t}^U > T \text{, the alternative hypothesis is accepted.} \\
 &\text{$ average tire life is approximately greater than 32000 is accepted based on the given fuzzy data with condition A Comparative Study of One-Sample t-Test under Fuzzy Environ
 $t^U = \frac{x^U - \mu_0^U}{s^U / \sqrt{n}} =\begin{cases} -2.1134 &; = 0 \\ 1.8105 &; = 0.53 \\ 1.8757 &; = 0.54 \\ 4.2402 &; = 1 \end{cases}$

5.1. **Conclusion**

5.1. **Conclusion**

5.1. **Conclusion**

5.1. A Comparative Study of One-Sample t-Test under Fuzzy Environments
 $t^V = \frac{x^V - \mu_0^V}{s^V/\sqrt{n}} = \begin{cases} 2.1134 : 1005 : 0.535 \ 1.8757 : 100535 : 0.535 \end{cases}$
 $t^V = \frac{x^V - \mu_0^V}{s^V/\sqrt{n}} = 0.535 \ 1.8757 : 100535 \le 1.1$
 $t^V > T$ for $0.$ A Comparative Study of One-Sample t-Test under Fuzzy Environments
 $\left[1^{\frac{1}{2}}\right]_{\text{S}} = \frac{1}{\sqrt{n}} \begin{cases} -2.1134 \div 0.5 & = 0.53 \\ 1.8757 \div 0.53 \div 0.577 \div 0.53 \div 0.54 \end{cases} = 0.54 \end{cases}$
 $\Rightarrow t^{\frac{11}{2}} \times T$ for $0.53 \le \frac{1}{21}$,

Wang et al. [35] found that the centroid formulae proposed by Cheng are incorrect and have led to some misapplications such as by Chu and Tsao. They presented the correct method for centroid formulae for a

A *Comparative Study of One-Sample t-test under Fuzzy Environments*
\n
$$
t^{U} = \frac{x^{U} - \mu_{0}^{U}}{s^{U} \sqrt{n}} = \begin{cases} 2.1134 \div 0 & 0 \\ 1.8105 \div 0 & 0.53 \\ 1.8757 \div 0 & 0.53 \le 1 \end{cases}
$$
\n5.1. *Conclusion*
\nSince for 0.53 ≤ ≤1, t¹ > T and t^U > T, the alternative hypothesis is accepted. Therefore, \widetilde{H}_{A} : The
\n $\Rightarrow t^{U} > T$ for 0.53 ≤ ≤1
\n5.1. *Conclusion*
\nSince for 0.53 ≤ ≤1, t¹ > T and t^U > T, the alternative hypothesis is accepted. Therefore, \widetilde{H}_{A} : The
\n $0.53 ≤ ≤1$.
\n5.2. *Remark*
\nThe results obtained from example-2 differ by 0.01 level of lower limit of $(0.53 ≤ ≤1)$ when compared
\nwith the results in Wu [37]. *Chachi et al.* [15] which is $(0.54 ≤ ≤1)$.
\nWang *et al.* [35] found that the centroid formulae proposed by *Cheng* are incorrect and have led to
\nsome misapplications such as by Chu and Taso. They presented the correct method for centroid formulae for a
\ngeneralized fuzzy number $\widetilde{A} = (a, b, c, d; w)$ as
\n
$$
(\overline{x}_0, \overline{y}_0) = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} \left((a + b + c + d) - \begin{bmatrix} dc - ab \\ (d + c) - (a + b) \end{bmatrix} \right) \left(\frac{w}{3} \right) \left(1 + \begin{bmatrix} c - b \\ (d + c) - (a + b) \end{bmatrix} \right) \left) \right) \left(-1 - (6.1) \right)
$$
\nAnd the ranking function associated with \widetilde{A} is $\mathbb{R}(\widetilde{A}) = \sqrt{\widetilde{x}_0 + \widetilde{y}_0} =$ --- (6.2)
\nFor a normalized *thn*, we put $w = 1$ in equations (6.1) so we have,
\n $$

And the ranking function associated with A is R

$$
\left(\widetilde{A}\right) = \sqrt{\widetilde{x}_0^2 + \widetilde{y}_0^2} - \cdots - (6.2)
$$

For a normalized tfn., we put $w = 1$ in equations (6.1) so we have,

$$
\left(\overline{x}_0, \overline{y}_0\right) = \left[\frac{1}{3}\left((a+b+c+d) - \left(\frac{dc - ab}{(d+c)-(a+b)}\right)\right), \left(\frac{1}{3}\right)\left(1 + \left(\frac{c-b}{(d+c)-(a+b)}\right)\right)\right] - \dotsb \tag{6.3}
$$

And the ranking function associated with ^A is $R(\widetilde{A}) = \sqrt{x_0^2 + y_0^2}$ ---(6.4). 5.2. **Remark**

The results obtained from example-2 differ by 0.01 level of lower limit of $(0.53 \leq \leq 1)$ when cor

with the results in Wu [37], Chachi et al. [15] which is $(0.54 \leq \leq 1)$.

Wang et al. [15] found that Let \widetilde{A}_i and \widetilde{A}_j be two fuzzy numbers, (i) $R(\widetilde{A}_i) > R(\widetilde{A}_j)$ then $\widetilde{A}_i > \widetilde{A}_j$ (ii) $R(\widetilde{A}_i) > R(\widetilde{A}_j)$ The results obtained from example-2 differ by 0.01 level of lower limit of $(0.53 \le$

with the results in Wu [37], Chachi et al. [15] which is $(0.54 \le \le 1)$.

Wang et al. [35] found that the centroid formulae proposed by then $\widetilde{A}_i > \widetilde{A}_j$ and (iii) $R(\widetilde{A}_i) = R(\widetilde{A}_j)$ then $\widetilde{A}_i = \widetilde{A}_j$.

Example 6.1. Let we consider example 2, using the above relations (6.3) and (6.4), we obtain the ranks of tfns. which are tabulated below:

And the calculated value of 't' is $t = 3.998$. The tabulated value of 't' at 5% level of significance with 23 degree **of** *A Comparative Study of One-Sample t-Test under Fuzzy Environments*
And the calculated value of 't' is t = 3.998. The tabulated value of 't' at 5% level of significance with 23 degree
of freedom is T =1.714. Here, t the average life of the tire is greater than 32000.

VII. REZVANI'S RANKING FUNCTION OF TFNS.

The centroid of a trapezoid is considered as the balancing point of the trapezoid. Divide the trapezoid into three plane figures. These three plane figures are a triangle (APB), a rectangle (BPQC) and a triangle A Comparative Study of One-Sample 1-Test under Fuzzy Environments

And the calculated value of 't' is t = 3.998. The tabulated value of 't' at 5% level of significance with 23 degree

of freedom is T = 1.714. Here, t > T **Example 12** A Comparative Study of One-Sample 1-Test under Fuzzy Environments

And the calculated value of 't' is t = 3.998. The tabulated value of 't' at 5% level of significance with 23 degree

of freedom is T = 1.714. trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point are *balancing points* of each individual plane figure and the incenter of these centroid points is much more balancing point for a generalized trapezoidal fuzzy number. Therefore, this point would be a better reference point than the centroid point of the trapezoid. of freedom is $T = 1.714$. Here, $t > T$. So, the null hypothesis \vec{H}_0 is rejected. Therefore, we conclude that
the the werenge ific of the time is greater than 32000.
The centroid of a trapezoid is considered as the bal **VII. REZVANTS RANKING FUNCTION OF TFNS.**

The centroid of a taxpecoid is considered as the balancing point of the tensoroid. Divide the trapecoid

thro three planes figures are a triangle (APB), a recensity. (PDC) and a werage life of the tire is greater than 32000.

The centroid of a trapezoid is considered as the balancing point of the trapezoid. Divide the trapezoid

three plane figures. These three plane figures are a triangle (APB), into three plane ingures. These three plane ingures

(CQD) respectively. Let the centroids of the three plane

of these centroids G_1 , G_2 and G_3 is taken as the

trapezoidal fuzzy numbers. The reason for selecting Fracta are a transpect (APP), an accolage (BPQC) and a transpect

plane figures be G_1 , G_2 and G_3 respectively. The incenter

is the point of reference to define the making of generalized

ing this point as a poin

are:

$$
G_1 = \left(\frac{a+2b}{3}, \frac{w}{3}\right)
$$
, $G_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right)$ and $G_3 = \left(\frac{2c+d}{3}, \frac{w}{3}\right) - - - (7.1)$

 $y = \frac{w}{q}$ and G_2 does not lie on the line G_1G_3 . Therefore, G_1, G_2 and $\frac{1}{3}$ and G_2 does not lie on the line G_1G_3 . T of these centroids G_1 , G_2 and G_3 is the mean the point of reference to define the making of generalized
trapposital fuzzy numbers. The reason for selecting this point of the incenter of these centroid points is m are non-collinear and they form a triangle. We define the incenter $I(\overline{x}_0, \overline{y}_0)$ of the triangle with vertices Exploration traces that the control of the binary interesting the generalized fuzzy humber. Therefore, this point well plains is much more
than diagram for a generalized tupeworld fuzzy number. Therefore, this point would

^A ⁰ ⁰ a+2b b+c 2c+d w w w α β γ α β γ 3 2 3 3 2 3 I x , y , (7.2) α + β + γ α + β + γ 2 2 2 2 2 c - 3b + 2d w 2c + d - a - 2b 3c - 2a - b w where α ,β ,γ 6 3 6 And ranking function of the trapezoidal fuzzy number A= a, b, c, d; w which maps the set of all fuzzy numbers to a set of all real numbers i.e. R: A is defined as a+2b 1 b+c 1 2c+d 1 G , , G , and G , (7.4) 3 3 2 2 3 3

 $-(-7.2)$
 $+\sqrt{2}$

the set of all fuzzy
 $\frac{2}{0} + \frac{-2}{y_0} - -(-7.3)$

fn, we put w = 1 in centroids of the three plane figures

1)
 G_3 . Therefore, G_1 , G_2 and G_3
 \overline{y}_0 of the triangle with vertices

28]
 $+\left(\frac{w}{3}\right)\right]$
 $--(7.2)$
 $\overline{c \cdot 2a \cdot b)^2 + w^2}$

6

(which maps the set of all fuzzy
 $R\$ which is the Euclidean distance from the incenter of the centroids. For a normalized tfn, we put $w = 1$ in equations (7.1) , (7.2) and (7.3) so we have, where $= \frac{\sqrt{(c-3b+2d)^2 + w^2}}{6}$, $= \frac{\sqrt{(2c+d-a-2b)^2}}{3}$

and ranking function of the trapezoidal fuzzy number $\widetilde{A} = (a, b, c)$

imbers to a set of all real numbers $\left[i.e. R: \left[\widetilde{A} \right] \rightarrow \mathbb{R} \right]$ is definite is the Euc

$$
G_1 = \left(\frac{a+2b}{3}, \frac{1}{3}\right), G_2 = \left(\frac{b+c}{2}, \frac{1}{2}\right) \text{ and } G_3 = \left(\frac{2c+d}{3}, \frac{1}{3}\right) \text{---}(7.4)
$$

*Corresponding Author: S. Parthiban 9 | Page

A *Comparative Study of One-Sample t-Test under Fuzzy Enviroments*
\n
$$
I_{\overline{\lambda}}(\overline{x}_0, \overline{y}_0) = \left[\frac{(\overline{a}+2b)}{3} + \frac{(\overline{b}+2)}{2} + \frac{(\overline{2c}+d)}{3}, \frac{(\overline{a})}{3} + \frac{(\overline{a})}{2} + \frac{(\overline{a})}{3} + \frac{(\overline{a})}{3} - \frac{(\overline{a})}{3} + \frac{(\overline{a})}{3} + \frac{(\overline{a})}{3} + \frac{(\overline{a})}{3} + \frac{(\overline{a})}{3} + \frac{(\overline
$$

 $R(\widetilde{A}) = \sqrt{\overline{X_0}^2 + \overline{y}_0^2}$ --- (7.6).

One-sample t-test using Rezvani's ranking function

We now analyse the one-sample t-test by assigning rank for each normalized trapezoidal fuzzy numbers and based on the ranking grades the decisions are observed.

Example 7.1. Let we consider example 2, using the above relations (7.4), (7.5) and (7.6), we obtain the ranks of tfns. which are tabulated below:

the calculated value of 't' is t = 2.572. The tabulated value of 't' at 5% level of significance with 23 degree of the average life of the tire is greater than 32000. $\frac{33186}{333186}$ $\frac{33313}{33313}$ $\frac{32382.50}{32627}$
 $\frac{32527}{33669}$ $\frac{32841.5}{32678}$ $\frac{33971.50}{33971.50}$
 $\frac{32678}{32678}$ $\frac{31717.50}{31717.50}$ $\frac{34225}{34225}$
ations (7.4), (7.5) and (7.6), we obtain 33669 32841.5 33971.50
32678 31717.50 34225
34225
dtions (7.4), (7.5) and (7.6), we obtain the calculated popu
of 't' is t = 2.572. The tabulated value of 't' at 5% level c
14. Here, $t > T$. So, the null hypothesis $\overline{H}_$

VIII. GRADED MEAN INTEGRATION REPRESENTATION (GMIR)

defined by
$$
P(\widetilde{A}) = \int_{0}^{w} h \left[\frac{L^{1}(h) + R^{1}(h)}{2} \right] dh / \int_{0}^{w} h dh
$$
.

Example 12. $\frac{3392.44}{23285}$ 13.679.50 $\frac{34137.50}{34300}$ $\frac{34177.50}{34300}$
 $\frac{332481}{32485}$ $\frac{32525}{32481}$ $\frac{32382.58}{32482.50}$
 $\frac{32569}{32699}$ $\frac{32441.5}{32491.5}$ $\frac{32387.50}{32699}$
 $\frac{32569}{$ $P(\widetilde{A}) = \frac{(a+d)}{2} + \frac{n}{2n+1}(b-a-d+c)$. **3305** 33628 32753.50

33481 3366 32481

33186 33481 33662.50 31980

33186 33313 3282.50

3366 3362.50 3284.5 33971.50

3369 3284.5 33281.5 32908

32678 31717.50 3284.5 33971.50

Using the above relations (7.4), (7.5) and $\frac{32481}{33862.50}$ $\frac{32481}{33862.50}$ $\frac{32882.7}{328727}$ $\frac{32882.7}{32672}$ $\frac{32882.7}{32678}$ $\frac{32882.7}{32678}$ $\frac{32882.7}{32678}$ $\frac{32882.7}{32678}$ $\frac{32881.5}{32071.5}$ $\frac{32882.7}{32678}$ $\frac{32881.5}{3271$ $\begin{array}{r} \n\frac{3.3924}{3281} & \frac{316550}{33600} & \frac{3447.50}{3447.00} \\ \n\frac{32881}{33051.50} & \frac{336528}{33978.50} & \frac{37553.50}{21980} \\ \n\frac{32481}{32862.50} & \frac{338628}{33971.50} \\ \n\frac{3257}{32600} & \frac{3113}{32678} & \frac{32841.5}{31717.50} & \frac{3$

A Comparative Study of One-Sample t-Test under Fuzzy Environments
 Proof :For a trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$ and
 $R(x) = \left(\frac{d - x}{d - c}\right)^n$ Then, $h = \left(\frac{x - a}{b - a}\right)^n \Rightarrow L^{-1$ ler Fuzzy Environments

L(x) = $\left(\frac{x - a}{b - a}\right)^n$ and $b - a$ zzy Environments
= $\left(\frac{x-a}{b-a}\right)^n$ and $(x) = \frac{a - h}{1}$ Then, $h = \frac{h - h}{1}$ \Rightarrow I **A Comparative Study of C**
 Proof :For a trapezoidal fuzzy number $\widetilde{A} = (a, b, c,$
 $R(x) = \left(\frac{d-x}{d-c}\right)^n$ Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^{-1}(h) = a + (b-a)(b-a)^n$
 $h = \left(\frac{d-x}{d-c}\right)^n \Rightarrow R^{-1}(h) = d - (d - c)h^{1/n}$
 $\therefore P(\widetilde{A}) = \left(\frac{1}{2}\int_0^1 h$ $d-c$ $(b-a)$ A Comparative Study of One-Sample t-Test under Fuzzy Environments

:For a trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$ and
 $= \left(\frac{d - x}{d - c}\right)^n$ Then, $h = \left(\frac{x - a}{b - a}\right)^n \Rightarrow L^1(h) = a + (b - a)h^{\$ n
 -1 (c)

(c)
 $\frac{1}{2}$ A Comparative Study of One-Sample t-Test under Fuzzy Environments

dal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$ and
 $h = \left(\frac{x - a}{b - a}\right)^n \Rightarrow L^{-1}(h) = a + (b - a)h^{\frac{1}{h}};$
 $h = d - (d - c)h^{\frac{1}{h}}$
 $(h - a)h^{\$ \mathbf{b} - a \mathbf{a} A Comparative Study of One-Sample t-Test under Fuzzy Environments

fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$ and
 $\left(\frac{x - a}{b - a}\right)^n \Rightarrow L^1(h) = a + (b - a)h^{\frac{1}{n}}$;
 $1 - (d - c)h^{\frac{1}{n}}$
 $a)h^{\frac{1}{n}} + (d - ($ $(h) = d - (d - c)h^{n}$ n and a structure of the structure $\frac{1}{\sqrt{2}}$ A Comparative Study of One-Sample t-Test under
 Proof :For a trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have L(
 $R(x) = \left(\frac{d-x}{d-c}\right)^n$ Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^{-1}(h) = a + (b-a)h^{1/n}$;
 $h = \left(\frac{d-x}{d-c}\right)^n \Rightarrow R^{-1}(h) = d - ($ $d - c$ d $\left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right)$ A Comparative Study of One-Sample t-Test under Fuzzy Environments

of :For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; 1)_a$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^a$ and
 $k = \left(\frac{d - x}{d - c}\right)^a$ Then, $h = \left(\frac{x - a}{b - a}\right)^a \Rightarrow L^1(h) = a + (b - a)$ A Comparative Study of One-Sample t-Test under Fuzzy

1 trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\sum_{n=1}^{\infty} \int_{0}^{n} \text{Then, } h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^1(h) = a + (b-a)h^{\frac{1}{n}};$
 $\Rightarrow R^{-1}(h) = d - (d - c)h^{\frac{1}{n}}$
 \int A Comparative Study of One-Sample t-Test under Fuzzy

u trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{c}{b-a}\right)^n$

Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^1(h) = a + (b-a)h^{\frac{1}{h}}$;
 $\Rightarrow R^{-1}(h) = d - (d - c)h^{\frac{1}{h}}$
 $\int_0^1 h$ A Comparative Study of One-Sample t-Test under Fuzzy Environments
 oof :For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; 1)_a$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$ and
 $(x) = \left(\frac{d - x}{d - c}\right)^n$ Then, $h = \left(\frac{x - a}{b - a}\right)^n \Rightarrow L^1(h) = a + ($ $\therefore P(\widetilde{A}) = \left(\frac{1}{2}\int_a^1 h \left[\left(a + (b-a)h^{\frac{1}{n}} \right) + \left(d - (d-c)h^{\frac{1}{n}} \right) \right] dh \right] / \int_a^1 h dh$ A Comparative Study of One-Sample t-Test under Fuzzy Environments

For a trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_a$, we have $L(x) = \left(\frac{x-a}{b-a}\right)^n$ and
 $\left(\frac{d-x}{d-c}\right)^n$ Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^+(h) = a + (b-a)h^{\frac{1}{2}}$;
 A Comparative Study of One-Sample t-Test under Fuzzy Environments
 Proof :For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x-a}{b-a}\right)^n$ and
 $R(x) = \left(\frac{d-x}{d-c}\right)^n$ Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^1(h) = a + (b-a)$ $(a+d)$ n $(a+b)(1)$ $(b-a-d+c)$ | $/(1/2)$ A Comparative Study of One-Sample t-T.

a trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we
 $\left(\frac{-x}{-c}\right)^n$ Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^1(h) = a + (b-a)h^{\frac{1}{n}}$;
 $\Rightarrow R^{-1}(h) = d - (d - c)h^{\frac{1}{n}}$
 $\frac{1}{2} \int_0^1 h \left[\left(a + (b-a)h^{\frac{$ **Example 1:** For a trapezoidal fuzzy number $\tilde{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. We have $L(x) = \left(\frac{x-a}{b-a}\right)^n$
 Proof : For a trapezoidal fuzzy number $\tilde{A} = \begin{pmatrix} a & b & c \\ d & c & d \end{pmatrix}$, we have $L(x) = \left(\frac{x-a}{b-a}\right)^n$
 $R(x) = \left(\frac{d-x}{d$ A Comparative Study of One-Sample t-Test under Fuzzy Environme

a trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$
 $\left(\frac{-x}{c}\right)^n$ Then, $h = \left(\frac{x - a}{b - a}\right)^n \Rightarrow L^1(h) = a + (b - a)h^{\frac{1}{2}}$;
 $\Rightarrow R^{-1}($ A Comparative Study of One-Sample t-Test under Fuzzy Environments

r a trapezoidal fuzzy number $\overline{A} = (a, b, c, d; 1)_0$, we have $L(x) = \left(\frac{x-a}{b-a}\right)^0$ and
 $\frac{1-x}{1-c}\right)^n$ Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^1(h) = a + (b - a)h^{\frac{1}{2}}$;
 \sum A Comparative Study of One-Sample t-Test under Fuzzy Environments

For a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; 1)_a$, we have $L(x) = \left(\frac{x-a}{b-a}\right)^a$ and
 $\left(\frac{d-x}{d-c}\right)^n$ Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^1(h) = a + (b-a)h^{\frac{1}{\alpha}}$;
 Thus, $P(\widetilde{A}) = \frac{(a+d)}{2} + \frac{n}{2n+1}(b-a-d+c)$ Hence the proof. A Comparative Study of One-Sample t-Test under Fu:

Proof :For a trapezoidal fuzzy number $\overline{A} = (a, b, c, d; 1)_n$, we have $L(x)$
 $R(x) = \left(\frac{d-x}{d-c}\right)^n$ Then, $h = \left(\frac{x-a}{b-a}\right)^n \Rightarrow L^1(h) = a + (b-a)h^{1/2}$;
 $h = \left(\frac{d-x}{d-c}\right)^n \Rightarrow R^{-1}(h)$ A Comparative Study of One-Sample t-Test under Fuzzy Environments

a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$ and
 $\sum_{c}^{\infty} \int_{a}^{b}$ Then, $h = \left(\frac{x - a}{b - a}\right)^n \Rightarrow L^+(h) = a + (b - a)h^{\frac{1}{2$ e Study of One-Sample t-Test under Fuzzy Environments
 $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$ and
 $\widetilde{A} = (b - a)h^{\frac{1}{n}}$;
 $\widetilde{A} = (b - a)h^{\frac{1}{n}}$;
 $\widetilde{A} = (b - a)h^{\frac{1}{n}}$
 $\widetilde{B} = (b - a)h^{\frac{1}{n}}$
 $=\left(\frac{1}{2}\left[\frac{(a+d)}{2}+\frac{n}{2n+1}(b-a-d+c)\right]\right)/\left(\frac{1}{2}\right)$

Thus, $P(\overline{A})=\frac{(a+d)}{2n+1}(b-a-d+c)$ Hence the proof.
 Result 8.I.If $\alpha = 1$ in the above theorem, we have $P(\overline{A})=\frac{a+2b+2c+d}{6}$
 One-sample 1-4 the simple GMIR of frie $\sqrt{1 - \left(\frac{1}{2}\int_{0}^{1} \left[(a + (b-a))b^{1/2} \right] \cdot \left(d - (d - c)b^{1/2} \right) \right] dh$ $\left| \int_{0}^{1} \left[bdh \right]$
 $-\left(\frac{1}{2} \left[\frac{(a - 0)}{2} - \frac{n}{2n + 1} (b - a - d + c) \right] \right] / (\frac{1}{2})$

10.8. $\left| \left(\frac{n}{2} \right) \right| = \frac{(n + d)}{2n + 1} (b - a - d - c)$ Hence the proof.
 ⇒ L⁻¹ (h) = a + (b - a) h^{1/n};

→ L⁻¹ (h) = a + (b - a) h^{1/n};

(d - (d - c) h^{1/n}) dh) / (hdh

- d + c) d) / (1/2)

- d + c) d) / (1/2)

- d + c) Hence the proof.

we have P(\tilde{A}) = $\frac{a + 2b + 2c + d}{6}$

s.

P

Result 8.1.If n =1 in the above theorem, we have $P(\widetilde{A}) = \frac{a + 2b + 2c + d}{6}$

One-sample t-test using GMIR of tfns.

Example 8.1. Let us consider example 2, using the result-8.1 from above theorem-8.1, we get the GMIR of each tfns. \tilde{x}_i which are tabulated below:

Thus, P(\tilde{A}) = $\left(\frac{1}{2}\right)\left(-\frac{2}{2} + \frac{1}{2n+1}\left(b-a-d+c\right)\right)$ Hence the proof.
 Result 8.1. If $n = 1$ in the above theorem, we have P(\tilde{A}) = $\frac{a+2b+2c+d}{6}$
 One-sample 1.6t us consider example 2, using the greater than 32000. Fig. 1. The membership grades for a normalized tfn. A $=$ (a, b, c, d; 1) is calculated by the membership grades for a normalized time is a state of the relation of $\frac{P(\tilde{x}_1)}{32975.16}$ and $\frac{3412}{33205.30}$ and $\frac{$ $\begin{array}{|r|l|l|} \hline &33243&33307.83&32350\\ \hline &32519.83&31052&31961\\ \hline &33645.33&32789.50&33899\\ \hline &32644.50&31715.67&3422\\ \hline \end{array}$

alt-8.1, we obtain population mean $\mu_0 = 32333.33$

ated value of 't' at 5% level of signi Using the above result-8.1, we obtain population mean $\mu_0 = 32333$.

Using the above result-8.1, we obtain population mean $\mu_0 = 32333$.
 $t = 3.368$. The tabulated value of 't' at 5% level of significance with

Here, t **Example 9.1. Let us consider example 2,** the total integral value of first entry x 33266, 33671, 34177, 34889 ¹ will be 368. The tabulated value of 't' at 5% level of significance with 23 degree of f
 $t > T$. So, the null hypothesis \tilde{H}_0 is rejected. Therefore, we conclude that the are than 32000.

IX. ONE-SAMPLE t-TEST USING TOTAL INT 329/2.16 34926.16 34926.16 34926.7

32812 33095.16 34909.7

322613 33481.16 22099.17

32324 33307.33 3239.38

3244.5 32894 3307.33 3239.88.7

3244.5 32789.50 33896.8

32644.50 31715.67 34223

3289.85 3289.967

the above (2) $2x+1$ (a) $y/2$)
 $y = 2x+1$ (b) $y/2$)
 $y = 2x+1$ (b) $y = 2x+1$ (b) times the primal.
 SSLIC a = in the above theorem, we have $P(\lambda) = \frac{a+2b+2c+d}{6}$
 SSLIC a = in the showe theorem, we have $P(\lambda) = \frac{a+2b+2c+d}{6}$

IX. ONE-SAMPLE t-TEST USING TOTAL INTEGRAL VALUE (TIV) OF TFNS.

$$
\int_{\text{Supp}(\widetilde{A})} \mu_{\widetilde{A}}(x) dx = \int_{a}^{b} \left(\frac{x-a}{b-a}\right) dx + \int_{b}^{c} dx + \int_{c}^{d} \left(\frac{x-d}{c-d}\right) dx --- (9.1)
$$

EXECUTE: (15.19), the half hypothesis **10** is rejected. Therefore, we conclude that the average greater than 32000.
\n**IX.** ONE-SAMPLE t-TEST USING TOTAL INTEGRAL VALUE (TIV) of
\nThe membership grades for a normalized tfn.
$$
\tilde{A} = (a, b, c, d; 1)
$$
 is calculated by
\n
$$
\int_{\text{Supp}(\tilde{A})} \mu_{\tilde{A}}(x) dx = \int_{a}^{b} \left(\frac{x-a}{b-a}\right) dx + \int_{b}^{c} dx + \int_{c}^{d} \left(\frac{x-d}{c-d}\right) dx --- (9.1)
$$
\n**Example 9.1.** Let us consider example 2, the total integral value of
\n
$$
\tilde{x}_1 = (33266, 33671, 34177, 34889)
$$
 will be
\n
$$
\int_{\text{Supp}(\tilde{x}_i)} \mu_{\tilde{x}_i}(x) dx = \int_{33266}^{33671} \left(\frac{x-33266}{405}\right) dx + \int_{33671}^{34177} dx + \int_{34177}^{34889} \left(\frac{x-34899}{-712}\right) dx = 1064.5 = I
$$
\nThe total integral value of remaining entries can be calculated in similar way, which have been t
\n*Corresponding Author: S. Parthiban

The total integral value of remaining entries can be calculated in similar way, which have been tabulated below:

				A Comparative Study of One-Sample t-Test under Fuzzy Environments
		$\mu_{\tilde{x}_i}(x)dx = I; i = 1, 2, , 24$		
	$Supp(\tilde{x}_i)$			
	1064.50	1193	794.50	
	849	996 1255.50	1044	
			1224	
	766			
	1226.50	928	1242.50	
	711	728.50	1121	
	818.50	719	1441.50	
	857	619	902.50	
	1009.50	806.50	918	
				Using the above relation (9.1), we obtain the calculated population mean $\mu_0 = 2500$. And the calculated value
				of 't' is $t = -34.83$. The tabulated value of 't' at 5% level of significance with 23 degree of freedom is
				$T = 1.714$. Here, $ t > T$. So, the null hypothesis H_0 is rejected. Therefore, we conclude that the average
life of the tire is greater than 32000.				
X.	LIOU AND WANG'S CENTROID POINT METHOD			Liou and Wang [23] ranked fuzzy numbers with total integral value. For a fuzzy number defined by

of 't' is $t = -34.83$. The tabulated value of 't' at 5% level of significance with 23 degree of freedom is life of the tire is greater than 32000.

X. LIOU AND WANG'S CENTROID POINT METHOD

A *Compartative Study of One-Sample t-Test under Fuzzy Enviromments*
\n
$$
\frac{4}{\sin(\lambda)} \left\{ \begin{array}{ll} \mu_{z_1}(x) dx = 1; i = 1, 2, ..., 24 \\ \frac{9\pi i(1)}{1064.50} & \frac{1093}{1096} & \frac{794.50}{1044} \\ \frac{726.50}{766} & \frac{1255}{108} & \frac{1293}{1024.50} \\ \frac{788.50}{760} & \frac{1293}{711} & \frac{1293}{7215} & \frac{121236}{7215} \\ \frac{88.50}{702} & \frac{1213}{7215} & \frac{1213}{7215} \\ \frac{88.50}{702} & \frac{1213}{7215} & \frac{1213}{7215} \\ \frac{88.50}{702} & \frac{1213}{7215} \\ \frac{128.50}{700} & \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} \\ \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} \\ \frac{128.5}{7221} & \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} \\ \frac{128.5}{7221} & \frac{128.5}{7221} & \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} & \frac{128.5}{721} \\ \frac{12
$$

decision maker's view point and is equal to the mean of right and left integral values. For a decision maker, the larger the value of is, the higher is the degree of optimism. 2, ..., 24 respectively and $0 \le$ ≤ 1.

1] is the **index of optimism** which represents the **degree of optimism** of a

11 is the **index of optimism** which represents a **pessimistic decision maker's view** p

11 value. (iii 1, 1 = 1, 2, ..., 24 Espectively and $0 \le 31$.

(a) = [0,1] is the **index of optimism** which represents the degree of optimism $= 0$, then the total value of integral represents a **pessimistic decision maker's vider** of 1] is the **index of optimism** which represents the **degree of optimism** of a

in the total value of integral represents a **pessimistic decision maker's view p**

1 value. (iii) If = 1, then the total integral value represe

One-sample t-test using LIOU and WANG'S centroid point method:

Example 10.1. Let us consider example 2, using the above equations (10.1), (10.2) and (10.3), we get the centroid point of first member as follows:

$$
I_{L}(\tilde{x}_{1}) = \int_{33266}^{33671} \left(\frac{x-33266}{405}\right) dx = 405/2 ; I_{R}(\tilde{x}_{1}) = \int_{34177}^{34889} \left(\frac{x-34889}{-712}\right) dx = 356
$$

Therefore I_T(\tilde{x}_{1}) = (307 +405)/2.

Similarly we can find $I_T(\tilde{x}_i)$; for $i = 2, ..., 24$. and the calculated values are tabulated below:

Using the above relations (10.1), (10.2) and (10.3), we obtain the calculated population mean

$$
t = \frac{(0.40)(10555 - 17649)}{\sqrt{170300217^{2} + 1486563634 + 325055007}}
$$

average life of the tire is around 32000.

XI. THORANI'S RANKING METHOD

As per the description in Salim Rezvani's ranking method, Thorani et al. [29] presented a different kind of centroid point and ranking function of thes. The incenter $I_{\tilde{A}}(\overline{x}_0, \overline{y}_0)$ of the triangle [Fig. 1] with vertices

G , G and G 1 2 3 of the generalized tfn. A= a, b, c, d; w is given by, ^A ⁰ ⁰ a+2b b+c 2c+d w w w α β γ α β γ 3 2 3 3 2 3 I x , y , (11.1) α + β + γ α + β + γ 2 2 2 2 2 c - 3b + 2d w 2c + d - a - 2b 3c - 2a - b w where α ,β ,γ 6 3 6 And the ranking function of the generalized tfn. A= a, b, c, d; w which maps the set of all fuzzy numbers to a set of real numbers is defined as R A x y (11.2) 0 0 . For a normalized tfn., we put w = 1 in

equations $(11. 1)$ and $(11. 2)$ so we have,

The tabuated value of T at 5% level of significance with 23 degree of freedom is
$$
1 = 1.714
$$
. Here,
\nt $\langle T, \forall , 0.5 \le 1$. So, the null hypothesis \tilde{H}_0 is accepted in this case. Therefore, we conclude that the average life of the tire is around 32000.
\nAs per the description in Salim Rezvan's ranking method, Thorani et al. [29] presented a different kind of centroid point and ranking function of this. The incenter $I_{\tilde{\lambda}}(\bar{x}_0, \bar{y}_0)$ of the triangle [Fig. 1] with vertices
\n G_1, G_2 and G_3 of the generalized tfin. $\tilde{A} = (a, b, c, d; w)$ is given by,
\n $I_{\tilde{\lambda}}(\bar{x}_0, \bar{y}_0) = \begin{bmatrix} \frac{4+2b}{3} + \frac{b+c}{2} + \frac{2c+d}{3} \\ \frac{b+c}{2} + \frac{c+d}{3} \end{bmatrix}$, $\frac{w}{3} + \frac{w}{2} + \frac{w}{3} \\ -a(1.1)$
\nwhere $= \frac{\sqrt{(c-3b+2d)^2 + w^2}}{6}$, $= \frac{\sqrt{(2c+d-a-2b)^2}}{3}$, $= \frac{\sqrt{(3c-2a-b)^2 + w^2}}{6}$
\nAnd the ranking function of the generalized tfin. $\tilde{A} = (a, b, c, d; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\tilde{A}) = x_0 \times y_0 = --(11.2)$. For a normalized trin, we put $w = 1$ in equations (11. 1) and (11. 2) so we have,
\n $I_{\tilde{\lambda}}(\bar{x}_0, \bar{y}_0) = \begin{bmatrix} \frac{(a+2b)}{3} + \frac{(b+c)}{2} + \frac{(2c+d)}{3} \\ \frac{(2c+d-a-2b)^2}{3} \\ + \end{bmatrix}$, $\frac{1}{3} + \frac{1}{4} \begin{bmatrix} \frac{1}{2} + \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} - -(-11.3)$
\nwhere $= \frac{\sqrt{(c-3b+2d)^2 + 1}}{6}$, $= \frac{\sqrt{(2c+d-a-2b)^2}}{3}$

One-sample t-test using Thorani's ranking method of tfns.

Example 11.1. Let us consider example 2, using the above relations (11.3) and (11.4), we get the ranks of each

the calculated value of 't' is $t = 2.57$. The tabulated value of 't' at 5% level of significance with 23 degree of the average life of the tire is greater than 32000.

XII. GENERAL CONCLUSION

The decisions obtained from various methods are tabulated below for the acceptance of null hypothesis.

Observing the decisions obtained from alpha cut interval method, for example-2, the null hypothesis is rejected A Comparative Study of One-Sample F-Test under Fuzzy Environments
 $R\left(\tilde{x}_1\right)$, $i = 1, 2, ..., 24$
 $\frac{14701.89}{13701.80}$
 $\frac{13701.83}{13701.80}$
 $\frac{13701.89}{13701.80}$
 $\frac{13827.56}{13701.80}$
 $\frac{13827.56}{1382.50}$ reliable result as it accepts the null hypotheses for all α . Also for example-2, the decisions obtained from Wang's ranking method, Rezvani's ranking method, GMIR, TIV and Thorani's ranking method provide parallel discussion.

REFERENCES

- [1]. S. Abbasbandy, B. Asady, The nearest trapezoidal fuzzy number to a fuzzy quantity, *Appl. Math. Comput.,* 156 (2004) 381-386.
- [2]. S. Abbasbandy, M. Amirfakhrian, The nearest approximation of a fuzzy quantity in parametric form, *Appl. Math. Comput.,* 172 (2006) 624-632.
- [3]. S. Abbasbandy, M. Amirfakhrian, The nearest trapezoidal form of generalized left right fuzzy number, *Internat. J. Approx. Reason.,* 43 (2006) 166-178.
- [4]. S. Abbasbandy, T. Hajjari, Weighted trapezoidal approximation-preserving cores of a fuzzy number, *Computers and Mathematics with Applications,* 59 (2010) 3066-3077.
- [5]. Abhinav Bansal, Trapezoidal Fuzzy Numbers (a, b, c, d): Arithmetic Behavior, *International Journal of Physical and Mathematical Sciences* (2011), 39-44.
- [6]. M. G. Akbari and A. Resaei, Bootstrap statistical inference for the variance based on fuzzy data. *Austrian Journal of Statistics*, 38 (2009), 121-130.
- [7]. M. Arefi and S. M. Taheri, Testing fuzzy hypotheses using fuzzy data based on fuzzy test statistic. *Journal of Uncertain Systems*, 5 (2011), 45-61.
- [8]. B. F. Arnold, Testing fuzzy hypotheses with crisp data, *Fuzzy Sets and Systems* 94 (1998), 323-333.
- [9]. Asady, B. Trapezoidal Approximation of a Fuzzy Number Preserving the Expected Interval and Including the Core, *American Journal of Operations Research*, 3 (2013), 299-306.
- [10]. E. BalouiJamkhaneh and A. NadiGhara, Testing statistical hypotheses for compare means with vague data*, International Mathematical Forum, 5 (2010) 615-620*.
- [11]. S. Bodjanova, Median value and median interval of a fuzzy number, *Inform. Sci. 172 (2005) 73-89.*
- James J. Buckley, Fuzzy Probability and Statistics, Springer-Verlag Berlin Heidelberg 2006.
- [13]. M. R. Casals, M. A. Gil and P. Gil, The fuzzy decision problem: an approach to the problem of testing statistical hypotheses with fuzzy information, *European Journal of Operational Research* 27 (1986), 371-382.
- [14]. M. R. Casals, M. A. Gil, A note on the operativeness of Neyman-Pearson tests with fuzzy information, *Fuzzy Sets and Systems* 30 (1989) 215-220.
- [15]. J. Chachi, S. M. Taheri and R. Viertl, Testing statistical hypotheses based on fuzzy confidence intervals, Forschungsbericht SM- 2012-2, TechnischeUniversitat Wien, Austria, 2012.
- [16]. D. Dubois and H. Prade, Operations on fuzzy numbers, *Int. J. Syst. Sci.,* 9 (1978) 613-626.
- [17]. P. Gajivaradhan and S. Parthiban, Statistical hypothesis testing through trapezoidal fuzzy interval data*, Int. Research J. of Engg. and Tech*. Vol. 02, Issue: 02, May 2015, pp. 251-258.
-
- [18]. P. Grzegorzewski, Testing statistical hypotheses with vague data, *Fuzzy sets and Systems*, 112 (2000), 501-510. [19]. S. C. Gupta and V. K. Kapoor, Fundamentals of Mathematical Statistics (A Modern Approach), *Sultan Chand & Sons*, New Delhi.
- [20]. R. R. Hocking, Methods and applications of linear models: regression and the analysis of variance, New York: *John Wiley & Sons,* 1996.
- [21]. Iuliana Carmen B RB CIORU, Statistical Hypothesis Testing Using Fuzzy Linguistic Variables, *FiabilitatesiDurabilitate-Fiability& Durability, Supplement,* 1 (2012*) Editura "AcademicaBrȃncusi", Tȃrgu Jiu, ISSN 1844-640X.*
- [22]. George J. Klir and Bo Yuan, Fuzzy sets and fuzzy logic, Theory and Applications, *Prentice-Hall*, New Jersey, 2008.
- [23]. T. S. Liou and M. J. Wang, Ranking Fuzzy Numbers with Integral Value, *Fuzzy Sets and Systems,* 50 (1992) 247-225.
- [24]. J. J. Saade and H. Schwarzlander, Fuzzy hypothesis testing with hybrid data, *Fuzzy Sets and Systems*, 35 (1990), 197-212.
- [25]. J. J. Saade, Extension of fuzzy hypothesis testing with hybrid data, *Fuzzy Sets and Systems*, 63 (1994), 57-71.
- [26]. S. Salahsour, S. Abbasbandy and T. Allahviranloo, Ranking Fuzzy Numbers using Fuzzy Maximizing-Minimizing points, *EUSFLAT-LFA: July 2011, Aix-les-Bains, France.*
- [27]. Salim Rezvani and Mohammad Molani, Representation of trapezoidal fuzzy numbers with shape function, to appear in *Annals of Fuzzy mathematics and Informatics.*
- [28]. Salim Rezvani, Ranking Generalized Trapezoidal Fuzzy Numbers with Euclidean Distance by the Incentre of Centroids, *Mathematica Aeterna,* 3 (2) (2013) 103-114.
- [29]. Y. L. P. Thorani, et al., Ordering Generalized Trapezoidal Fuzzy Numbers, *Int. J. Contemp. Math. Sciences,* 7(12) (2012) 555- 573.
- [30]. J. Ch. Son, I. Song and H. Y. Kim, A fuzzy decision problem based on the generalized Neymen-Pearson criterion, *Fuzzy Sets and Systems*, 47 (1992), 65-75.
- [31]. Superna Das and S. Chakraverty, Numerical Solution of Interval and Fuzzy Systems of Linear Equations, *Applications and Applied Mathematics: An International Journal* (AAM): Vol. 7, Issue 1 (June 2012), pp. 334-356.
- [32]. R. Viertl, Univariate statistical analysis with fuzzy data, *Computational Statistics and Data Analysis*, 51 (2006), 33-147.
- [33]. R. Viertl, Statistical methods for fuzzy data, *John Wiley and Sons*, Chichester, 2011.
- [34]. N. Watanabe and T. Imaizumi, A fuzzy statistical test of fuzzy hypotheses, *Fuzzy Sets and Systems*, 53 (1993) 167-178.
- [35]. Y. M. Wang et al., On the centroids of fuzzy numbers, *Fuzzy Sets and Systems*, 157 (2006) 919-926.
[36]. H. C. Wu, Statistical hypotheses testing for fuzzy data, *Information Sciences*, 175 (2005), 30-56.
- [36]. H. C. Wu, Statistical hypotheses testing for fuzzy data, *Information Sciences*, 175 (2005), 30-56.
- [37]. H. C. Wu, Statistical confidence intervals for fuzzy data, *Expert Systems with Applications*, 36 (2009), 2670-2676.
- [38]. Zadeh, L. Fuzzy Probabilities, *Information Processing and Management* 203 (1984), 363-372.