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Research Paper

Dense sets in Soft biminimal Spaces

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ABSTRACT: *The aim of this paper is to introduce the concept of dense sets in soft biminimal spaces and some of their simple properties.*
Keywords: *soft set, soft biminimal spaces, dense sets*

I. INTRODUCTION

Molodtsov (1999) initiated the theorey of soft sets as a new mathematical tool for dealing uncertainty, which is completely a new approach for modeling vagueness and uncertainties. Soft set theorey has a rich potential for application in solving practical problems in Economics, Social sciences, Medical sciences etc. Molodtsov successfully applied soft theory into several directions, such as Smoothness of func-tions , Game theory, Operation research, Riemann integration, Perron integration, Theory of probability, Theory of measurement and so on. The concept of mini-mal structure space (brie y m-structure) was introduced by V. Popa and T.Noiri (2000). They also introduced the concepts of m_X -open set and m_X -closed set and characterize those sets using m_X -closure and m_X -interior operators, respectively. C. Boonpok (2010) introduced the concept of biminimal structure space and studied m^1_X m^2_X -open sets and m^1_X m^2_X -closed sets in biminimal structure spaces. R. Gowri and S. Vembu (2015) introduced the concept of Soft minimal and soft biminimal spaces. Also they introduced (2016) boundary set on soft bimimal spaces and Exte-rior sets in soft biminimal spaces. In this paper, we introduce the concept of dense sets in soft biminimal spaces and study some of their simple properties.

II. PRELIMINARIES

Definition 2.1 [11] *Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a nonempty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $e \in A$. $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2 [6] *Let X be an initial universe set, E be the set of parameters and $A \subseteq E$. Let F_A be a nonempty soft set over X and $\tilde{P}(F_A)$ is the soft power set of F_A. A subfamily $\tilde{m}$ of $\tilde{P}(F_A)$ is called a soft minimal set over X if $F_\emptyset \in \tilde{m}$ and $F_A \in \tilde{m}$.*

$(F_A, \tilde{m})$ or $(X, \tilde{m}, E)$ is called a soft minimal space over X. Each member of $\tilde{m}$ is said to be $\tilde{m}$ -soft open set and the complement of an $\tilde{m}$-soft open set is said to be $\tilde{m}$-soft closed set over X.

Example 2.3 [6] *Let $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then*

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$$
\begin{aligned}
&F_{A_1} = \{(x_1, \{u_1\})\},\\
&F_{A_2} = \{(x_1, \{u_2\})\},\\
&F_{A_3} = \{(x_1, \{u_1, u_2\})\},\\
&F_{A_4} = \{(x_2, \{u_1\})\},\\
&F_{A_5} = \{(x_2, \{u_2\})\},\\
&F_{A_6} = \{(x_2, \{u_1, u_2\})\},\\
&F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\},\\
&F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\},\\
&F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\},\\
&F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\},\\
&F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\},\\
&F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\},\\
&F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\},\\
&F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\},\\
&F_{A_{15}} = F_A,\\
&F_{A_{16}} = F_\emptyset \text{ are all soft subsets of } F_A
\end{aligned}
$$

soft minimal $(\tilde{m}) = \{F_A, F_\emptyset, F_{A_1}, F_{A_3}, F_{A_6}, F_{A_{10}}\}$

Definition 2.4 [6] *Let $(F_A, \tilde{m})$ be a soft minimal space over X. For a soft subset F_B of F_A, the $\tilde{m}$-soft closure of F_B and $\tilde{m}$-soft interior of F_B are defined as follows:*

(1) $\tilde{m}Cl(F_B) = \cap\{F_\alpha : F_B \tilde{\subseteq} F_\alpha, F_A - F_\alpha \in \tilde{m}\}$,

(2) $\tilde{m}Int(F_B) = \cup\{F_\beta : F_\beta \tilde{\subseteq} F_B, F_\beta \in \tilde{m}\}$.

Lemma 2.5 [6] *Let $(F_A, \tilde{m})$ be a soft minimal space over X. For a soft subset F_B and F_C of F_A, the following properties hold:*

(1) $\tilde{m}cl(F_A - F_B) = F_A - \tilde{m}Int(F_B)$ and $\tilde{m}Int(F_A - F_B) = F_A - \tilde{m}cl(F_B)$,

(2) If $(F_A - F_B) \in \tilde{m}$, then $\tilde{m}cl(F_B) = F_B$ and if $F_B \in \tilde{m}$, then $\tilde{m}Int(F_B) = F_B$,

(3) $\tilde{m}cl(F_\emptyset) = F_\emptyset$, $\tilde{m}cl(F_A) = F_A$, $\tilde{m}Int(F_\emptyset) = F_\emptyset$ and $\tilde{m}Int(F_A) = F_A$,

(4) If $F_B \tilde{\subseteq} F_C$, then $\tilde{m}cl(F_B) \tilde{\subseteq} \tilde{m}cl(F_C)$ and $\tilde{m}Int(F_B) \tilde{\subseteq} \tilde{m}Int(F_C)$,

(5) $F_B \tilde{\subseteq} \tilde{m}cl(F_B)$ and $\tilde{m}Int(F_B) \tilde{\subseteq} F_B$,

(6) $\tilde{m}cl(\tilde{m}cl(F_B)) = \tilde{m}cl(F_B)$ and $\tilde{m}Int(\tilde{m}Int(F_B)) = \tilde{m}Int(F_B)$.

Lemma 2.6 [6] *Let F_A be a nonempty set and $\tilde{m}$ on X satisfying property B. For a soft subset F_B of F_A, the following properties hold:*

(1) $F_B \in \tilde{m}$ if and only if $\tilde{m}Int(F_B) = F_B$,

(2) If F_B is $\tilde{m}$-closed if and only if $\tilde{m}Cl(F_B) = F_B$,

(3) $\tilde{m}Int(F_B) \in \tilde{m}$ and $\tilde{m}Cl(F_B) \in \tilde{m}$-closed.

Definition 2.7 [6] *Let $(F_A, \tilde{m})$ be a soft minimal space with nonempty set F_A is said to have property B if the union of any family belonging to $\tilde{m}$ belongs to $\tilde{m}$.*

Definition 2.8 [6] *Let X be an initial universe set and E be the set of parameters. Let $(X, \tilde{m_1}, E)$ and $(X, \tilde{m_2}, E)$ be the two different soft minimals over X. Then $(X, \tilde{m_1}, \tilde{m_2}, E)$ or $(F_A, \tilde{m_1}, \tilde{m_2})$ is called a soft biminimal spaces.*

Definition 2.9 [6] *A soft subset F_B of a soft biminimal space $(F_A, \tilde{m_1}, \tilde{m_2})$ is called $\tilde{m_1}\tilde{m_2}$-soft closed if $\tilde{m}cl_1(\tilde{m}cl_2(F_B)) = F_B$. The complement of $\tilde{m_1}\tilde{m_2}$-soft closed set is called $\tilde{m_1}\tilde{m_2}$-soft open.*

Proposition 2.10 [6] *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space over X. Then F_B is a $\tilde{m_1}\tilde{m_2}$-soft open soft subsets of $(F_A, \tilde{m_1}, \tilde{m_2})$ if and only if $F_B = \tilde{m}Int_1(\tilde{m}Int_2(F_B))$.*

Proposition 2.11 [6] *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space.If F_B and F_C are $\tilde{m_1}\tilde{m_2}$-soft closed soft subsets of $(F_A, \tilde{m_1}, \tilde{m_2})$ then $F_B \tilde{\cap} F_C$ is $\tilde{m_1}\tilde{m_2}$-soft closed.*

Proposition 2.12 [6] *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space over X. If F_B and F_C are $\tilde{m_1}\tilde{m_2}$-soft open soft subsets of $(F_A, \tilde{m_1}, \tilde{m_2})$,then $F_B \tilde{\cup} F_C$ is $\tilde{m_1}\tilde{m_2}$ -soft open.*

Definition 2.13 [8] *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space (SBMS), F_B be a soft subset of F_A and $x \in F_A$. Then x is called $\tilde{m_i}\tilde{m_j}$-exterior point of F_B if $x \in \tilde{m_i}Int(\tilde{m_j}Int(F_A \setminus F_B))$. We denote the set of all $\tilde{m_i}\tilde{m_j}$-exterior point of F_B by $\tilde{m}Ext_{ij}(F_B)$ where $i, j = 1, 2$, and $i \neq j$.*
From definition we have $\tilde{m}Ext_{ij}(F_B) = F_A \setminus \tilde{m_i}Cl(\tilde{m_j}Cl(F_B))$.

Definition 2.14 [7] *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space, F_B be a soft subset of F_A and $x \in F_A$. We called x is $(i, j) - \tilde{m}$ boundary point of F_B if $x \in \tilde{m_i}Cl(\tilde{m_j}Cl(F_B)) \cap \tilde{m_i}Cl(\tilde{m_j}Cl(F_A \setminus F_B))$. We denote the set of all $(i, j) - \tilde{m}$-boundary point of F_B by $\tilde{m}Bdr_{ij}(F_B)$ where $i, j = 1, 2$, and $i \neq j$.*
From definition we have $\tilde{m}Bdr_{ij}(F_B) = \tilde{m_i}Cl(\tilde{m_j}Cl(F_B)) \cap \tilde{m_i}Cl(\tilde{m_j}Cl(F_A \setminus F_B))$.

Theorem 2.15 [7] *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space and F_B be a soft subsets of F_A. Then for any $i, j = 1, 2$, and $i \neq j$, we have;*
1. F_B is $\tilde{m_i}\tilde{m_j}$- soft closed if and only if $\tilde{m}Bdr_{ij}(F_B) \tilde{\subseteq} F_B$.
2. F_B is $\tilde{m_i}\tilde{m_j}$- soft open if and only if $\tilde{m}Bdr_{ij}(F_B) \tilde{\subseteq} F_A \setminus F_B$.

III. DENSE SETS IN SOFT BIMINIMAL SPACES

In this section, we introduce the concept of dense sets in soft biminimal spaces and study some of their properties.

Definition 3.1 *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space. A soft subset F_B of F_A is called $\tilde{m_i}\tilde{m_j}$-dense set in F_A if $F_A = \tilde{m_i}Cl(\tilde{m_j}Cl(F_B))$, where $i, j = 1, 2$ and $i \neq j$*

Example 3.2 *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space where $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$.*
Then $\tilde{m_1} = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_7}, F_{A_{11}}, F_{A_{13}}, F_A\}$, and $\tilde{m_2} = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_8}, F_{A_{10}}, F_{A_{12}}, F_A\}$.
Then $\tilde{m_1}Cl(\tilde{m_2}Cl(\{(x_2, \{u_1\})\}))$=$F_A$ and $\tilde{m_2}Cl(\tilde{m_1}Cl(\{(x_2, \{u_1\})\}))$=$F_A$.
$\tilde{m_1}Cl(\tilde{m_2}Cl(\{(x_1, \{u_2\})\}))$= $\{(x_1, \{u_2\})\}$ and $\tilde{m_2}Cl(\tilde{m_1}Cl(\{(x_1, \{u_2\})\}))$=$\{(x_1, \{u_2\})\}$.
Hence $\{(x_2, \{u_1\})\}$ is $\tilde{m_1}\tilde{m_2}$-dense set and $\tilde{m_2}\tilde{m_1}$-dense set in F_A.
But $\{(x_1, \{u_2\})\}$ is not $\tilde{m_i}\tilde{m_j}$-dense set, where $i, j = 1, 2$ and $i \neq j$

Theorem 3.3 *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space and F_B be a soft subset of F_A. F_B is $\tilde{m_i}\tilde{m_j}$-dense set in F_A if and only if $\tilde{m}Ext_{ij}(F_B)$=$F_\emptyset$, where $i, j = 1, 2$ and $i \neq j$*

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Proof: Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space and let F_B be a soft subset of F_A.

Necessity: Suppose that F_B is $\tilde{m}_i\tilde{m}_j$-dense set in F_A. Since F_A=$\tilde{m}_iCl(\tilde{m}_jCl(F_B))$. That implies $F_A \setminus \tilde{m}_iCl(\tilde{m}_jCl(F_B))$= $F_\emptyset \Rightarrow \tilde{m}Ext_{ij}(F_B)$=$F_\emptyset$, [8] where $i, j = 1, 2$ and $i \neq j$.

Sufficiency: Assume that $\tilde{m}Ext_{ij}(F_B)$=$F_\emptyset$. Thus $F_A \setminus \tilde{m}_iCl(\tilde{m}_jCl(F_B))$= $F_\emptyset$, it follows that $\tilde{m}_iCl(\tilde{m}_jCl(F_B))$=$F_A$. Hence F_B is $\tilde{m}_i\tilde{m}_j$-dense set in F_A, where $i, j = 1, 2$ and $i \neq j$ □

Theorem 3.4 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space and F_B be a soft subset of F_A. If F_B is $\tilde{m}_i\tilde{m}_j$-dense set in F_A then for any non-empty soft $\tilde{m}_i\tilde{m}_j$-closed subset G_B of F_A, where $i, j = 1, 2$ and $i \neq j$ such that $F_B \tilde{\subseteq} G_B$, we have G_B=F_A.*

Proof: Assume that F_B is $\tilde{m}_i\tilde{m}_j$-dense set in F_A and G_B is soft $\tilde{m}_i\tilde{m}_j$-closed subset of F_A such that $F_B \tilde{\subseteq} G_B$. Since F_B is $\tilde{m}_i\tilde{m}_j$-dense set in F_A, that implies F_A=$\tilde{m}_iCl(\tilde{m}_jCl(F_B))$. By assumption, G_B is soft $\tilde{m}_i\tilde{m}_j$-closed set and $F_B \tilde{\subseteq} G_B$, it follows that F_A=$\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \tilde{\subseteq} \tilde{m}_iCl(\tilde{m}_jCl(G_B))$= G_B. Consequently, G_B=F_A. □

Note 3.5 *By Theorem 3.4 if F_B is $\tilde{m}_i\tilde{m}_j$-dense set in F_A. Then only F_A is soft $\tilde{m}_i\tilde{m}_j$-closed set in F_A such that containing F_B.*

Remark 3.6 *The Theorem 3.4 is not true if G_B is not soft $\tilde{m}_i\tilde{m}_j$-closed set. The following example supports our claim.*

Example 3.7 *Let $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then $\tilde{m}_1 = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_7}, F_{A_{11}}, F_{A_{13}}, F_A\}$,and $\tilde{m}_2 = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_8}, F_{A_{10}}, F_{A_{12}}, F_A\}$. Then $\tilde{m}_1Cl(\tilde{m}_2Cl(\{(x_2, \{u_1, u_2\})\}))$=$F_A$ and $\tilde{m}_2Cl(\tilde{m}_1Cl(\{(x_2, \{u_1, u_2\})\}))$=$F_A$. Hence $\{(x_2, \{u_1, u_2\})\}$ is $\tilde{m}_i\tilde{m}_j$-dense set in F_A. But $\{(x_2, \{u_1, u_2\})\}$ is not soft $\tilde{m}_i\tilde{m}_j$-closed set in F_A.*

Theorem 3.8 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space and let F_B be a soft subset of F_A. If $\tilde{m}_iInt(\tilde{m}_jInt(F_A \setminus F_B))$=$F_\emptyset$ then for any non-empty soft $\tilde{m}_i\tilde{m}_j$-closed subset G_B of F_A, where $i, j = 1, 2$ and $i \neq j$ such that $F_B \tilde{\subseteq} G_B$, we have $G_B = F_A$*

Proof: Suppose that $\tilde{m}_iInt(m_jInt(F_A \setminus F_B))$=$F_\emptyset$ and F_B be a soft subset of F_A, where $i, j = 1, 2$ and $i \neq j$ such that $F_B \tilde{\subseteq} G_B$. By assumption, we have $F_A \setminus \tilde{m}_iCl(\tilde{m}_jCl(F_B)) = F_\emptyset$ and so F_A=$\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \tilde{\subseteq} \tilde{m}_iCl(\tilde{m}_jCl(G_B))$= G_B. Therefore G_B=F_A. □

Theorem 3.9 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space and let $F_B \tilde{\subseteq} F_A$. If F_B be a soft subset of F_A. If F_B is $\tilde{m}_i\tilde{m}_j$-dense set in F_A. Then $U_B \cap F_B \neq F_\emptyset$ for any non-empty soft $\tilde{m}_i\tilde{m}_j$-open subset U_B of F_A, where $i, j = 1, 2$ and $i \neq j$.*

Proof: Let F_B be a $\tilde{m}_i\tilde{m}_j$-dense set in F_A. Assume that $U_B \cap F_B = F_\emptyset$ for any non-empty soft $\tilde{m}_i\tilde{m}_j$-open subset U_B of F_A. Thus we have $F_B \tilde{\subseteq} U_B{}^C$. It follows that F_A=$\tilde{m}_iCl(\tilde{m}_jCl(F_B)) \tilde{\subseteq} \tilde{m}_iCl(\tilde{m}_jCl(U_B^C)) = F_A \setminus \tilde{m}_iInt(\tilde{m}_jInt(U_B))$. Since U_B is soft$\tilde{m}_i\tilde{m}_j$-open, $G_B{}^C$ is soft $\tilde{m}_i\tilde{m}_j$-closed. By assumption, we have $U_B{}^C$= F_A. That is $F_A \setminus F_B$= F_A. Therefore U_B= $F_\emptyset$. This is contradiction. Hence $U_B \cap F_B \neq F_\emptyset$ for any non-empty soft $\tilde{m}_i\tilde{m}_j$-open subset U_B F_A, where $i, j = 1, 2$ and $i \neq j$. □

Theorem 3.10 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space and let $F_B \tilde{\subseteq} F_A$. If for any non-empty soft $\tilde{m}_i\tilde{m}_j$-closed subset G_B of F_A such that $F_B \tilde{\subseteq} G_B$, then G_B=F_A if and only if $U_B \cap F_B \neq F_\emptyset$ for any non-empty soft $\tilde{m}_i\tilde{m}_j$-open subset U_B of F_A, where $i, j = 1, 2$ and $i \neq j$.*

Proof: Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space and let $F_B \tilde{\subseteq} F_A$.
Necessity: Suppose that if for any non-empty soft $\tilde{m_i}\tilde{m_j}$-closed subset G_B of F_A such that $F_B \tilde{\subseteq} G_B$, then $G_B{=}F_A$. Assume that $U_B \cap F_B = F_\emptyset$ for any non-empty soft $\tilde{m_i}\tilde{m_j}$-open subset U_B of F_A. Thus we have $F_B \tilde{\subseteq} {U_B}^C$. Since U_B is soft $\tilde{m_i}\tilde{m_j}$-open that implies ${U_B}^C$ is soft $\tilde{m_i}\tilde{m_j}$-closed. By assumption, we have ${U_B}^C{=}F_A$ it follows that $U_B{=}F_\emptyset$, this is contradiction. Therefore $U_B \cap F_B \neq F_\emptyset$ for any non-empty soft $\tilde{m_i}\tilde{m_j}$-open subset U_B of F_A, where $i, j = 1, 2$ and $i \neq j$.
Sufficiency: Assume that $U_B \cap F_B \neq F_\emptyset$ for any non-empty soft $\tilde{m_i}\tilde{m_j}$-open subset U_B of F_A and G_B is a non-empty soft $\tilde{m_i}\tilde{m_j}$-closed subset of F_A, such that $F_B \tilde{\subseteq} G_B$. Suppose that $G_B \neq F_A$. Thus ${G_B}^C$ is a non-empty soft $\tilde{m_i}\tilde{m_j}$-open subset of F_A. By assumption, we have ${G_B}^C \cap F_B \neq F_\emptyset$. This is a contradiction with $F_B \tilde{\subseteq} G_B$. Hence $G_B{=}F_A$. □

Theorem 3.11 *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space and let F_B be a soft subset of F_A. If $\tilde{m_i}Int(\tilde{m_j}Int(F_A \setminus F_B)){=}F_\emptyset$ then $U_B \cap F_B \neq F_\emptyset$ for any non-empty soft $\tilde{m_i}\tilde{m_j}$-open subset U_B of F_A, where $i, j = 1, 2$ and $i \neq j$.*

Proof: Assume that $\tilde{m_i}Int(m_jInt(F_A \setminus F_B)){=}F_\emptyset$. Suppose that $U_B \cap F_B{=}F_\emptyset$ for any non-empty soft $\tilde{m_i}\tilde{m_j}$-open subset U_B of F_A such that $F_B \tilde{\subseteq} {U_B}^C$, it follows that $F_A{=}\tilde{m_i}Cl(\tilde{m_j}Cl(F_B)) \tilde{\subseteq} \tilde{m_i}Cl(\tilde{m_j}Cl(U_B)^C){=}{U_B}^C$. Hence $U_B{=}F_\emptyset$. This is contradiction with the property of U_B. Therefore $U_B \cap F_B \neq F_\emptyset$ for any non-empty soft $\tilde{m_i}\tilde{m_j}$-open subset U_B of F_A, where $i, j = 1, 2$ and $i \neq j$. □

Theorem 3.12 *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space and let $F_B \tilde{\subseteq} F_A$. If F_B is $\tilde{m_i}\tilde{m_j}$-dense set in F_A, then $\tilde{m}Bdr_{ij}(F_B){=}\ \tilde{m_i}Cl(\tilde{m_j}Cl(F_A \setminus F_B))$, where $i, j = 1, 2$ and $i \neq j$.*

Proof: Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space and let $F_B \tilde{\subseteq} F_A$.
Assume that F_B is $\tilde{m_i}\tilde{m_j}$-dense set in F_A. Thus we have $\tilde{m}Bdr_{ij}{=}\tilde{m_i}Cl(\tilde{m_j}Cl(F_B)) \cap \tilde{m_i}Cl(\tilde{m_j}Cl(F_A \setminus F_B)){=}\tilde{m_i}Cl(\tilde{m_j}Cl(F_A \setminus F_B))$, where $i, j = 1, 2$ and $i \neq j$. □

Theorem 3.13 *Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space and let F_B be a soft subset of F_A. Then F_B is soft $\tilde{m_i}\tilde{m_j}$-open and $\tilde{m_i}\tilde{m_j}$-dense set in F_A if and only if $\tilde{m}Bdr_{ij}(F_B){=}F_A \setminus F_B$, where $i, j = 1, 2$ and $i \neq j$.*

Proof: Let $(F_A, \tilde{m_1}, \tilde{m_2})$ be a soft biminimal space and let $F_B \tilde{\subseteq} F_A$.
Necessity: Assume that F_B is soft $\tilde{m_i}\tilde{m_j}$-open and $\tilde{m_i}\tilde{m_j}$-dense set in F_A. By Theorem 2.15 and Theorem 3.12, we have $\tilde{m_i}Cl(\tilde{m_j}Cl(F_A \setminus F_B)){=}F_A \setminus F_B$. Hence $\tilde{m}Bdr_{ij}(F_B){=}\tilde{m_i}Cl(\tilde{m_j}Cl(F_A \setminus F_B)){=}F_A \setminus F_B$, where $i, j = 1, 2$ and $i \neq j$.
Sufficiency: Let $\tilde{m}Bdr_{ij}(F_B){=}F_A \setminus F_B$. Assume that F_B is $\tilde{m_i}\tilde{m_j}$-open set in F_A. Then we have $\tilde{m}Bdr_{ij}(F_B){=}\ F_A \setminus F_B{=}\ F_A \setminus \tilde{m_i}Int(\tilde{m_j}Int(F_B)){=}\tilde{m_i}Cl(\tilde{m_j}Cl(F_A \setminus F_B))$. By Theorem 3.12, we have F_B is $\tilde{m_i}\tilde{m_j}$-dense set in F_A, where $i, j = 1, 2$ and $i \neq j$. □

Example 3.14 *Let $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then $\tilde{m_1} = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_7}, F_{A_{13}}, F_A\}$, and $\tilde{m_2} = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_7}, F_{A_{11}}, F_{A_{13}}, F_A\}$. Then $\tilde{m_1}Cl(\tilde{m_2}Cl(\{(x_1, \{u_1\}), (x_2, \{u_1\})\})){=}F_A$ and $\tilde{m_1}Int(\tilde{m_2}Int(\{(x_1, \{u_1\}), (x_2, \{u_1\})\})){=}\{(x_1, \{u_1\}), (x_2, \{u_1\})\}$.
Hence $\{(x_1, \{u_1\}), (x_2, \{u_1\})\}$ is $\tilde{m_i}\tilde{m_j}$-open and $\tilde{m_i}\tilde{m_j}$-dense set in F_A.
Then we have $\tilde{m}Bdr_{ij}(\{(x_1, \{u_1\}), (x_2, \{u_1\})\}){=}\{(x_1, \{u_2\}), (x_2, \{u_2\})\}$.*

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