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Research Paper

Analysis of Occurrence of Digit 0 in Prime Numbers till 1 Trillion

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ABSTRACT : *All prime numbers less than 1 trillion are exhaustively examined for occurrence of digit 0 in them. Multiple occurrences of 0's are investigated. The first and last instances of all possible repetitions of 0's are determined within first 12 blocks of increasing powers of 10.*

Keywords: *Digit 0, all occurrences, prime numbers Mathematics Subject Classification 2010 - 11Y35, 11Y60, 11Y99*

I. INTRODUCTION

Prime numbers have been in limelight for their seemingly irregular pattern of occurrence amongst natural numbers. Theoretically [1], as well as for ranges as huge as till 1 trillion, their distributions have been analyzed [3] to [16]. Due to importance ovzcf 0 [2] in a better number system, this work presents the analysis of occurrence of digit 0 within all prime numbers less than 1 trillion. The required primes in such big range could be generated by choosing most efficient prime generating algorithm chosen by using their exhaustive comparisons [17] to [23].

We have considered prime numbers p, with $1 < p < 10^n$, $1 \leq n \leq 12$. The number 10^n is omitted as it falls in the category of numbers having more than n number of significant digits.

Only significant 0's are considered; those coming after some non-zero digit.

II. OCCURRENCE OF SINGLE SIGNIFICANT DIGIT 0 IN PRIME NUMBERS

0's value in any number system is well-understood; particularly comparison with other number systems not having 0 highlights it more clearly [2]. All prime numbers p in the range $1 \leq p < 10^{12}$ have been critically analyzed for occurrence of digit 0.

Continuous parallel execution of a computer program written in Java Programming Language on many computer systems for long durations could fetch in these results. The need for having very efficient approach for this to curtail the execution time and efforts led us to design of ultimate optimized algorithms. Those are not part of presentation of this paper! But we do give the outcomes of their execution.

Table 1: Number of Prime Numbers in Various Ranges with Single 0 in Their Digits

Sr. No.	Range	Number of Primes with Single 0
1.	$1 - 10^1$	0
2.	$1 - 10^2$	0
3.	$1 - 10^3$	15
4.	$1 - 10^4$	206
5.	$1 - 10^5$	2,223
6.	$1 - 10^6$	22,281
7.	$1 - 10^7$	214,893
8.	$1 - 10^8$	2,022,364
9.	$1 - 10^9$	18,800,595
10.	$1 - 10^{10}$	173,468,013
11.	$1 - 10^{11}$	1,592,608,078
12.	$1 - 10^{12}$	14,567,387,959

The percentage of number of primes containing single 0 in each of aforementioned ranges calculated with base as all integers containing single 0 [24] in the respective ranges is declining.

III. OCCURRENCE OF MULTIPLE SIGNIFICANT DIGITS 0'S IN PRIME NUMBERS

We have comprehensive data about all numbers containing double, triple and higher number of digit 0's in them within the ranges $1\text{-}10^n$, $1 \le n \le 12$ [24]. Here the number of prime numbers in these ranges containing multiple number of digit 0's have been determined and are as follows :

Table 2: Number of Prime Numbers in Various Ranges with Multiple 0's in Their Digits

Sr. No.	Number Range <	Number of Prime Numbers with 2 0's	Number of Prime Numbers with 3 0's	Number of Prime Numbers with 4 0's
1.	10^4	13	0	0
2.	10^5	239	8	0
3.	10^6	3,652	243	9
4.	10^7	46,530	4,985	300
5.	10^8	549,606	79,421	6,436
6.	10^9	6,147,984	1,119,332	122,091
7.	10^{10}	66,366,790	14,515,960	1,982,376
8.	10^{11}	697,616,892	178,273,256	29,292,749
9.	10^{12}	7,188,505,154	2,102,717,194	403,694,095

Table 2: Continued . . .

Sr. No.	Number Range <	Number of Prime Numbers with 5 0's	Number of Prime Numbers with 6 0's	Number of Prime Numbers with 7 0's
1.	10^7	5	0	0
2.	10^8	277	3	0
3.	10^9	7,890	316	8
4.	10^{10}	173,306	9,524	319
5.	10^{11}	3,209,677	234,527	10,762
6.	10^{12}	53,164,085	4,860,847	305,078

Table 2: Continued . . .

Sr. No.	Number Range <	Number of Prime Numbers with 8 0's	Number of Prime Numbers with 9 0's	Number of Prime Numbers with 10 0's
1.	10^{10}	8	0	0
2.	10^{11}	289	2	0
3.	10^{12}	12,537	319	2

This quantum of occurrence of multiple 0's as digits in prime numbers in various ranges of powers of 10 is graphically depicted as follows, where vertical axis in on logarithmic scale.

Their percentage calculated with respect to number of integers with those many 0's in respective ranges fluctuates as follows.

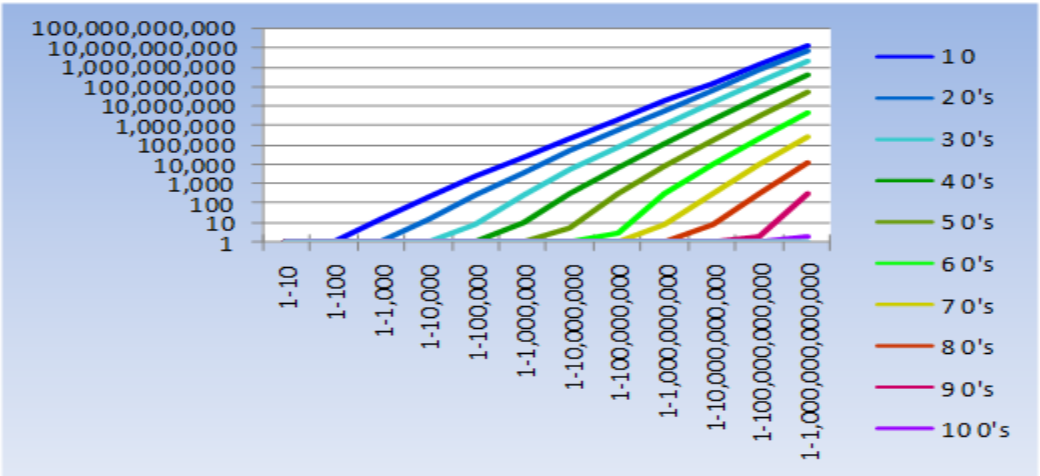

Figure 1: Number of Primes in Various Ranges with Multiple 0's in Their Digits

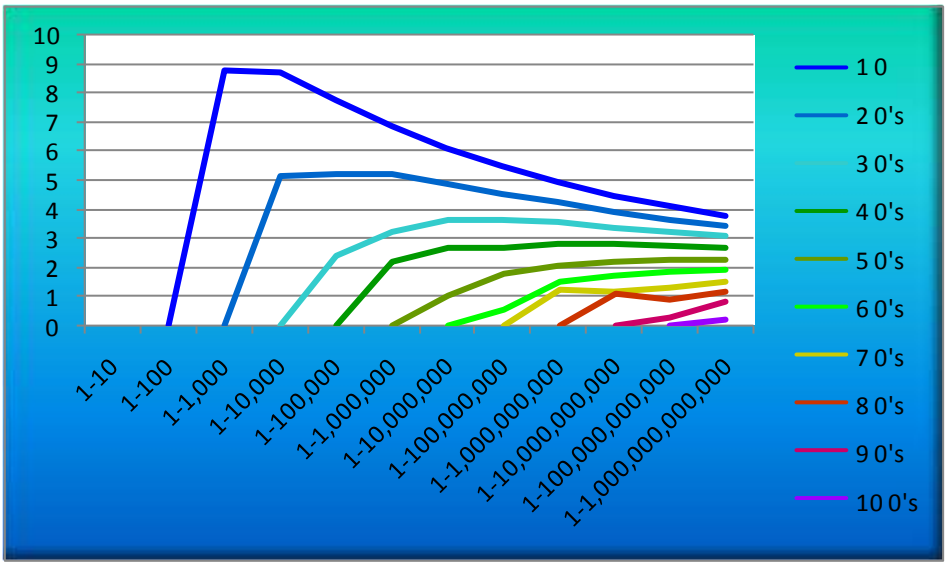

Figure 2: Percentage of Primes in Various Ranges with Multiple 0's in Their Digits With Respect To All Such Integers in Respective Ranges

IV. FIRST OCCURRENCE OF DIGIT 0 IN PRIME NUMBERS

As far as all natural numbers are concerned, the first number containing 0 is clearly 10. For sufficiently large ranges, for 2 0's, the first instance is 100, for 3 it is 1000 and so on. It has been formulated as

Formula 1 [24] : If n and r are natural numbers, then the first occurrence of r zeros in integers in range $1 \le m < 10^n$ is

$$f = \begin{cases} - & , \text{if } r \ge n \\ 10^r & , \text{if } r < n \end{cases}.$$

Now that our focus is prime numbers, and that the prime numbers seem sufficiently randomly distributed, it is worthwhile to search the first occurrences of single and higher number of 0's in them.

Table 3: First Prime Numbers in Various Ranges with Multiple 0's in Their Digits

Sr. No.	Range	First Prime Number in Range with					
		1 0	2 0's	3 0's	4 0's	5 0's	6 0's
1.	$1 - 10^1$	-	-	-	-	-	-
2.	$1 - 10^2$	-	-	-	-	-	-
3.	$1 - 10^3$	101	-	-	-	-	-
4.	$1 - 10^4$	101	1,009	-	-	-	-
5.	$1 - 10^5$	101	1,009	10,007	-	-	-
6.	$1 - 10^6$	101	1,009	10,007	100,003	-	-
7.	$1 - 10^7$	101	1,009	10,007	100,003	1,000,003	-
8.	$1 - 10^8$	101	1,009	10,007	100,003	1,000,003	20,000,003
9.	$1 - 10^9$	101	1,009	10,007	100,003	1,000,003	20,000,003
10.	$1 - 10^{10}$	101	1,009	10,007	100,003	1,000,003	20,000,003
11.	$1 - 10^{11}$	101	1,009	10,007	100,003	1,000,003	20,000,003
12.	$1 - 10^{12}$	101	1,009	10,007	100,003	1,000,003	20,000,003

Table 3: Continued . . .

Sr. No.	Range	First Prime Number in Range with			
		7 0's	8 0's	9 0's	10 0's
1.	$1 - 10^1$	-	-	-	-
2.	$1 - 10^2$	-	-	-	-
3.	$1 - 10^3$	-	-	-	-
4.	$1 - 10^4$	-	-	-	-
5.	$1 - 10^5$	-	-	-	-
6.	$1 - 10^6$	-	-	-	-
7.	$1 - 10^7$	-	-	-	-

Sr. No.	Range	First Prime Number in Range with			
		7 0's	8 0's	9 0's	10 0's
8.	$1 - 10^8$	-	-	-	-
9.	$1 - 10^9$	100,000,007	-	-	-
10.	$1 - 10^{10}$	100,000,007	1,000,000,007	-	-
11.	$1 - 10^{11}$	100,000,007	1,000,000,007	30,000,000,001	-
12.	$1 - 10^{12}$	100,000,007	1,000,000,007	30,000,000,001	100,000,000,003

V. LAST OCCURRENCE OF DIGIT 0 IN PRIME NUMBERS

The formula for last occurrence of r number of 0's in integers in ranges $1 - 10^n$, $1 \le n \le 12$, is available.

Formula 2 [24] : If n and r are natural numbers, then the last occurrence of r zeros in numbers in range $1 \le m < 10^n$ is

$$l = \begin{cases} - & , \text{if } r \ge n \\ 10^n - 10^r & , \text{if } r < n \end{cases}.$$

The last occurrences of prime numbers with r number of 0's in them in these ranges $1 - 10^n$, $1 \le n \le 12$, have been traced out. Those are as follows :

Table 4: Last Prime Numbers in Various Ranges with Multiple 0's in Their Digits

Sr. No.	Number of 0's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1.	1	-	-	907	9,907	99,907	999,907	9,999,907	99,999,703
2.	2	-	-	-	9,007	98,009	999,007	9,997,007	99,994,009
3.	3	-	-	-	-	90,007	990,001	9,970,001	99,990,001
4.	4	-	-	-	-	-	900,007	9,800,009	99,900,001
5.	5	-	-	-	-	-	-	8,000,009	99,000,007
6.	6	-	-	-	-	-	-	-	40,000,003
7.	7	-	-	-	-	-	-	-	-
8.	8	-	-	-	-	-	-	-	-
9.	9	-	-	-	-	-	-	-	-
10.	10	-	-	-	-	-	-	-	-
11.	11	-	-	-	-	-	-	-	-

Table 4: Continued . . .

Sr. No.	Number of 0's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1.	1	999,999,607	9,999,999,707	99,999,999,907
2.	2	999,999,001	9,999,999,001	99,999,998,003
3.	3	999,970,009	9,999,980,009	99,999,970,001
4.	4	999,200,009	9,999,800,003	99,999,400,009
5.	5	997,000,007	9,999,000,001	99,999,000,001
6.	6	970,000,007	9,990,000,001	99,990,000,007
7.	7	700,000,001	9,900,000,007	99,600,000,001
8.	8	-	9,000,000,001	93,000,000,001
9.	9	-	-	40,000,000,003
10.	10	-	-	-
11.	11	-	-	-

Remark : The maximum number of 0's in any prime number in the range $1 - 10^n$, for $n > 2$, is at most $n - 2$.

The numbers in all sections of this work form integer sequences deserving independent treatment in their own right.

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