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Research Paper

Analysis of Occurrence of Digit 4 in Prime Numbers Till 1 Trillion

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ABSTRACT: *Occurrence of 4 in digits of all primes till 10^{12} is analyzed. Multiple occurrences of 4's are also considered. The first and last occurrences of all possible repeated instances of 4's in their digits are determined in blocks of $1 – 10^n$ for $1 \leq n \leq 12$.*
Keywords: *Digit 4, all occurrences, prime numbers*
Mathematics Subject Classification 2010 -11Y35, 11Y60, 11Y99

I. INTRODUCTION

Numbers are quite simple and equally interesting. It is more interesting to have glimpses of past to see how differently these numbers were represented and used by different ancient civilizations [2]. Now we have come quite ahead in mathematics, but some numbers like primes are still maintain their mysterious status due to their hereto unexposed pattern fitting. Although many theoretical properties of primes are being explored; most of them are on asymptotic level [1] and hence explicit inspection of whole ranges of primes becomes necessary [4].

All integers are probed for all types of occurrences of zero [5], [6], [7] and all non-zero [11], [12], [13] digits. This work is about the analysis of occurrence of digit 4 within all primes in ranges of powers of 10 till 1 trillion, i.e., prime numbers p such that $1 < p < 10^n$, $1 \leq n \leq 12$. It is in continuation of earlier such analysis for digits 0 [8], [9], [10], 1 [14], [15], [16], 2[17], [18], [19] and 3 [20], [21], [22].

II. OCCURRENCE OF SINGLE DIGIT 4 IN PRIME NUMBERS

4 is first composite number. The way in which digit 4 occurs in all natural numbers is inferred from work of [11] applicable to all non-zero digits. Here instead of all positive integers, prime numbers p are considered in the range $1 < p < 10^{12}$ for trends of occurrences of digit 4.

Table 1: Number of Prime Numbers in Various Ranges with Single 4 in Their Digits

Sr. No.	Range	Number of Primes with Single 4
1.	1 – 101	0
2.	1 – 102	3
3.	1 – 103	30
4.	1 – 104	294
5.	1 – 105	2,725
6.	1 – 106	25,602
7.	1 – 107	234,745
8.	1 – 108	2,142,049
9.	1 – 109	19,446,059
10.	1 – 1010	176,268,251
11.	1 – 1011	1,595,405,886
12.	1 – 1012	14,425,647,017

III. OCCURRENCE OF MULTIPLE DIGITS 4's IN PRIME NUMBERS

All this work has been done by rigorous executions of selectively chosen algorithms [3] on multiple computer systems simultaneously.

Single, double, triple and all possible multiple occurrences of digit 4 in all positive integers in ranges of $1 – 10^n$ are available [11]. This kind of analysis for prime numbers is done in this work.

Table 2: Number of Prime Numbers in Various Ranges with Multiple 4's in Their Digits

Sr. No.	Number Range <	Number of Prime Numbers with 2 4's	Number of Prime Numbers with 3 4's	Number of Prime Numbers with 4 4's
1.	10^3	2	0	0
2.	10^4	30	2	0
3.	10^5	472	33	1
4.	10^6	5,692	618	39
5.	10^7	65,317	9,526	823
6.	10^8	711,033	131,750	14,483
7.	10^9	7,547,163	1,674,856	232,589
8.	10^{10}	78,211,222	20,253,003	3,371,595
9.	10^{11}	796,687,309	235,800,672	45,798,343
10.	10^{12}	8,006,151,685	2,666,239,163	591,969,174

Table 2: Continued …

Sr. No.	Number Range <	Number of Prime Numbers with 54's	Number of Prime Numbers with 64's	Number of Prime Numbers with 74's
1.	10^6	2	0	0
2.	10^7	45	0	0
3.	10^8	1,000	34	0
4.	10^9	20,587	1,064	45
5.	10^{10}	374,994	27,511	1,386
6.	10^{11}	6,098,052	563,409	36,105
7.	10^{12}	91,990,156	10,207,857	806,904

Table 2: Continued …

Sr. No.	Number Range <	Number of Primes with 8 4's	Number of Primes with 9 4's	Number of Primes with 10 4's	Number of Primes with 11 4's
1.	10^9	1	0	0	0
2.	10^{10}	41	1	0	0
3.	10^{11}	1,498	38	1	0
4.	10^{12}	44,669	1,609	34	1

Graphs of the number of primes with multiples 4's in their digits have following nature when vertical axis is taken on logarithmic scale.

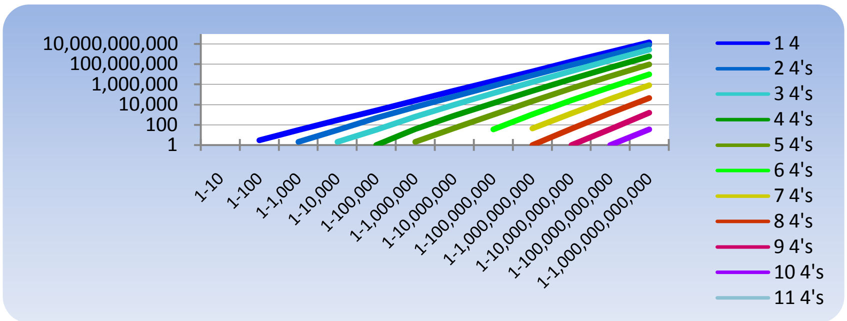
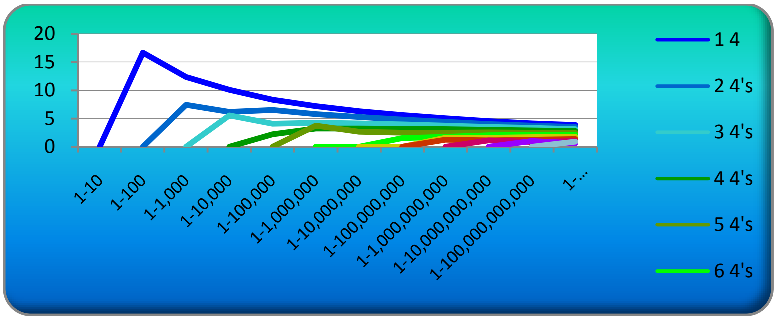


Figure 1: Number of Primes in Various Ranges with Multiple 4's in Their Digits

The percentage of number of primes with multiple 4's in their digits calculated with respect to number of all such positive integers with those many 4's in their digits in respective ranges are plotted graphically.



*Corresponding Author: Neeraj Anant Pande

Figure 2: Percentage of Primes in Various Ranges with Multiple 4's in Their Digits with Respect to All Such Integers in Respective Ranges

The differences of number of multiple occurrences of digits 1, 2 and 3 in primes with those of 4 in them in our ranges are depicted below graphically with compartmentalizing them in two blocks – one with 1 and 3 and the other with 4, as the former ones can occupy units place and the later one doesn't barring one exception. Digit 0 is not considered as it doesn't occupy all places, particularly units and leading n^{th} places in any *n* digit prime number.

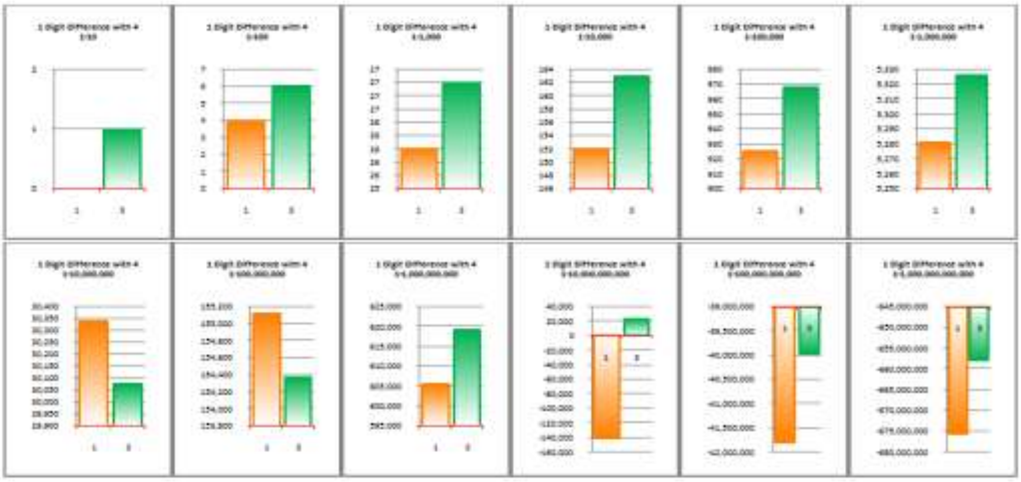


Figure 3: Differences of Number of Primes having One 1 and One3 in their Digits with those having One4 in them in Ranges of $1 – 10^n$.

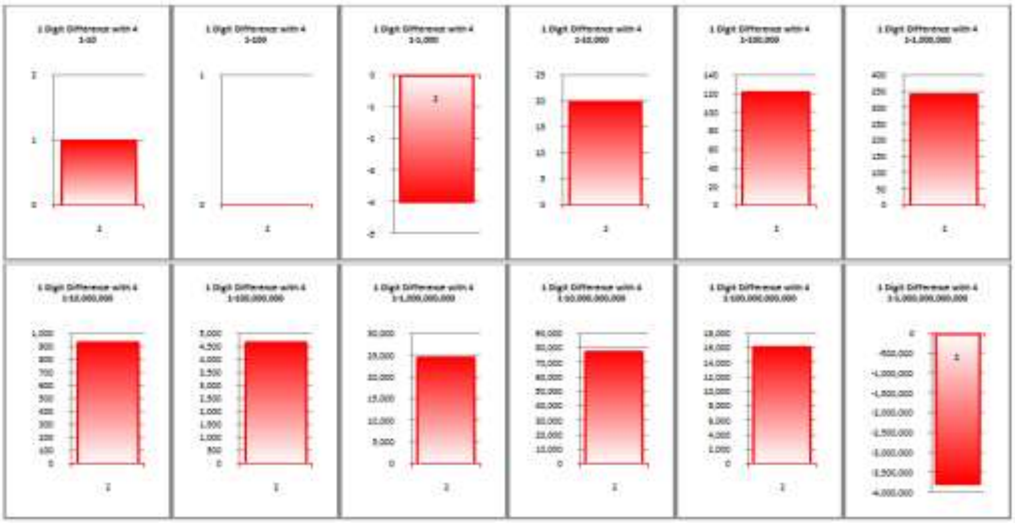


Figure 4: Difference of Number of Primes having One2 in their Digits with those having One4 in them in Ranges of $1 – 10^n$.

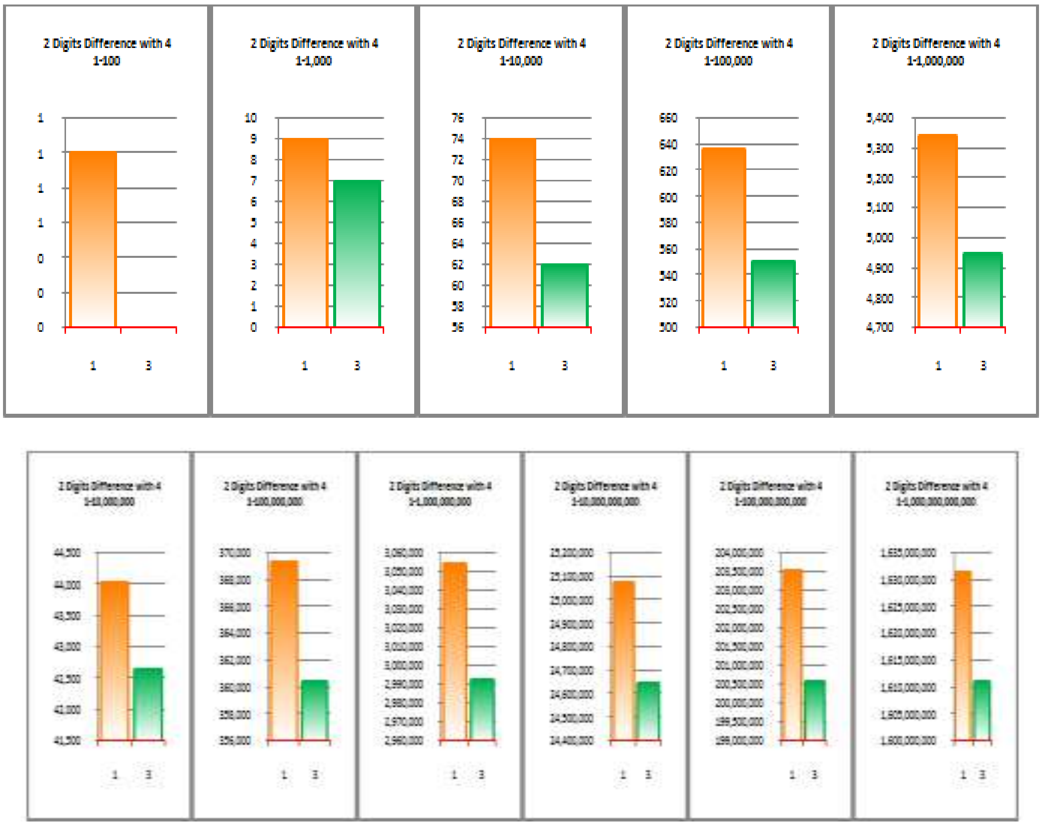


Figure 5: Differences of Number of Primes having Two 1's and Two3's in their Digits with those having Two4's in them in Ranges of $1 - 10^n$.

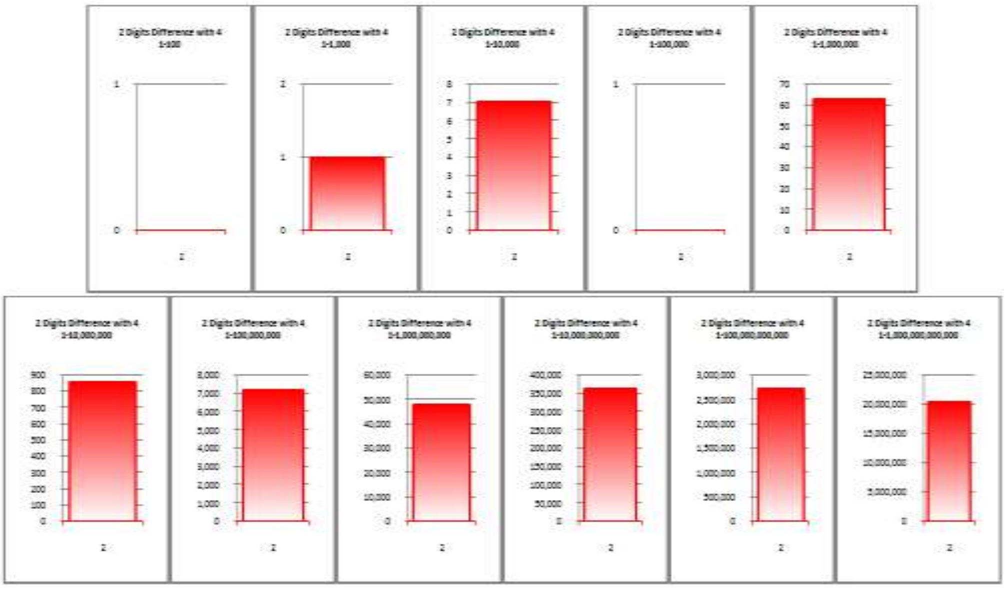


Figure 6: Difference of Number of Primes having Two2'sin their Digits with those having Two4's in them in Ranges of $1 - 10^n$.

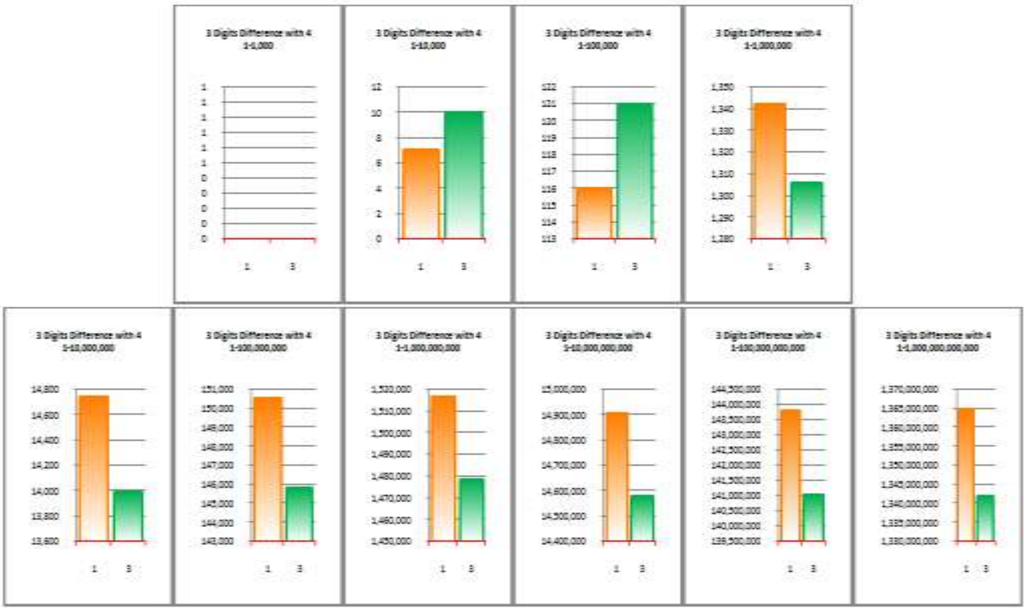


Figure 7: Differences of Number of Primes having Three 1's and Three3's in their Digits with those having Three4's in them in Ranges of $1 - 10^n$.

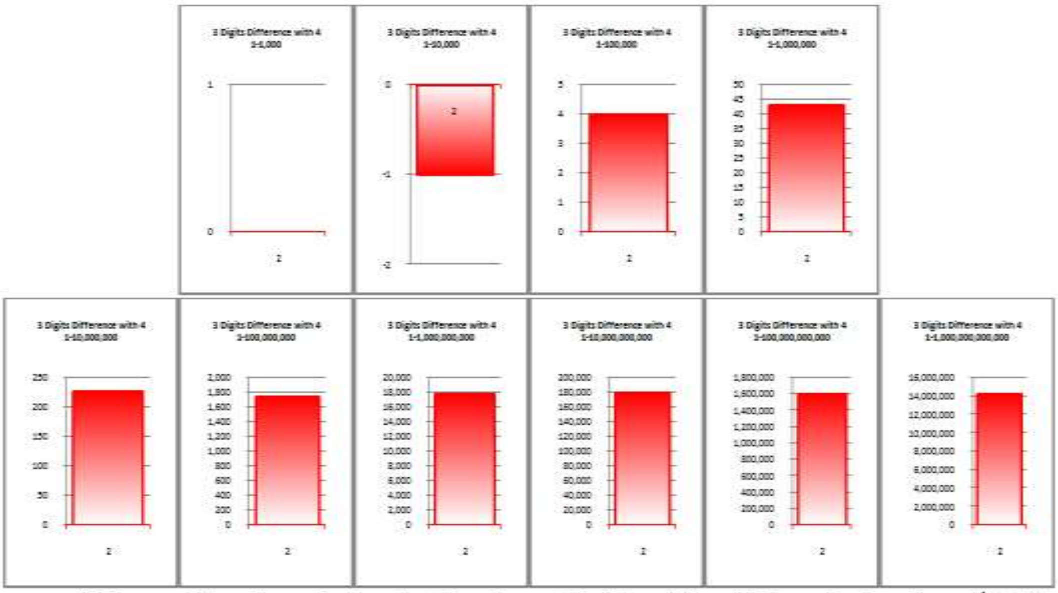


Figure 8: Difference of Number of Primes having Three2'sin their Digits with those having Three4's in them in Ranges of $1 - 10^n$.

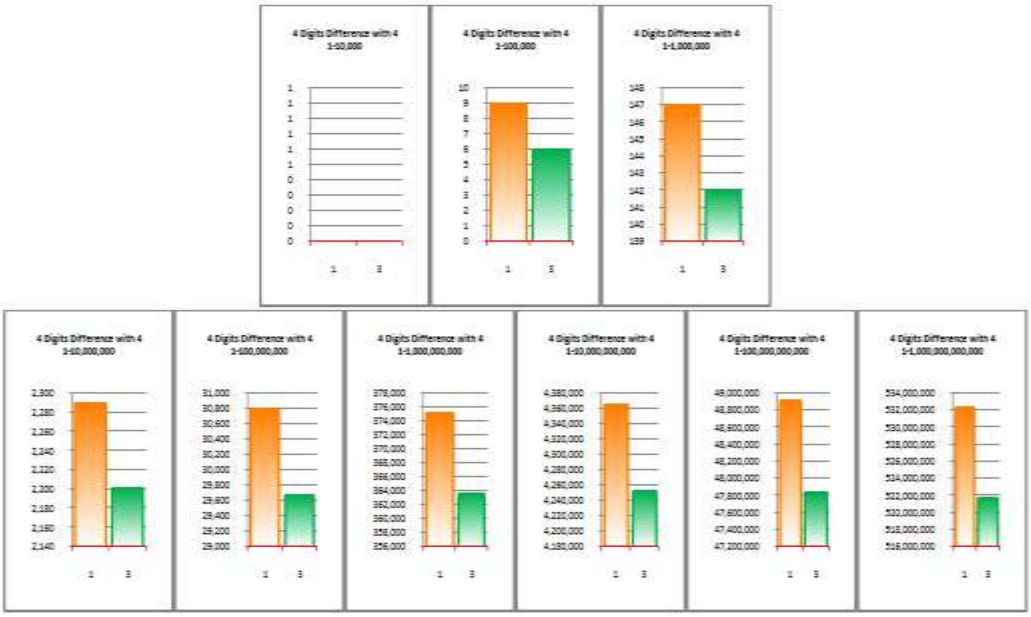


Figure 9: Differences of Number of Primes having Four 1's and Four3's in their Digits with those having Four4's in them in Ranges of 1 – 10^n.

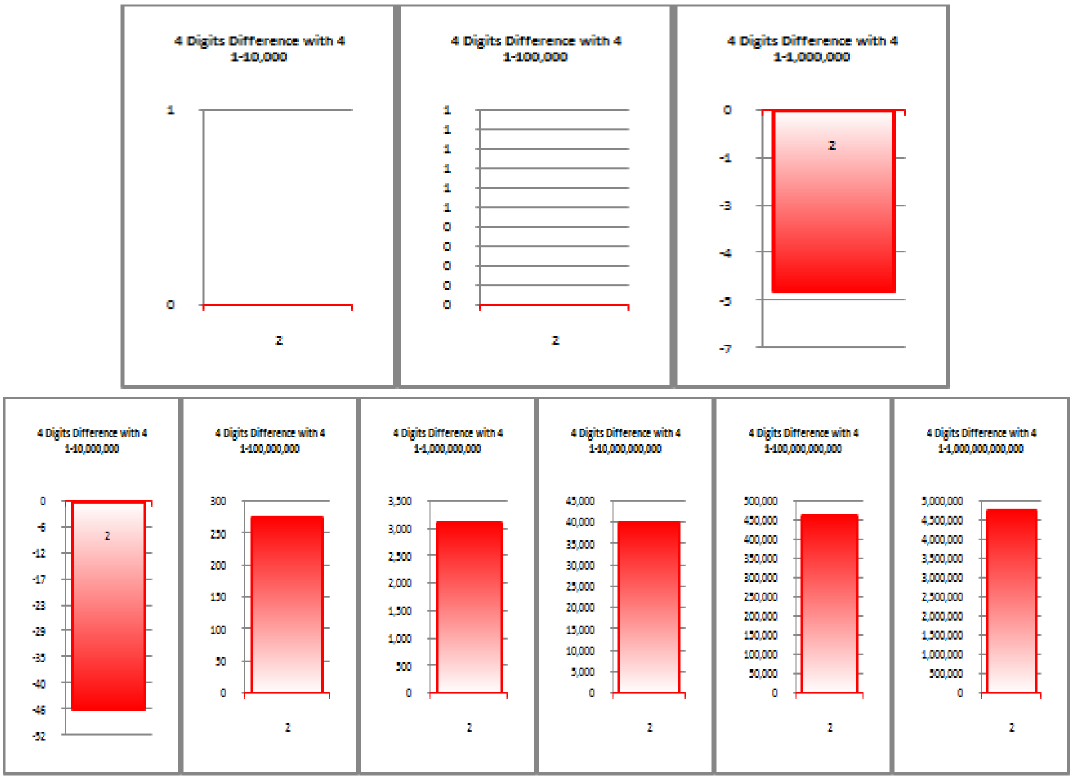


Figure 10: Difference of Number of Primes having Four2'sin their Digits with those having Four4's in them in Ranges of 1 – 10^n.

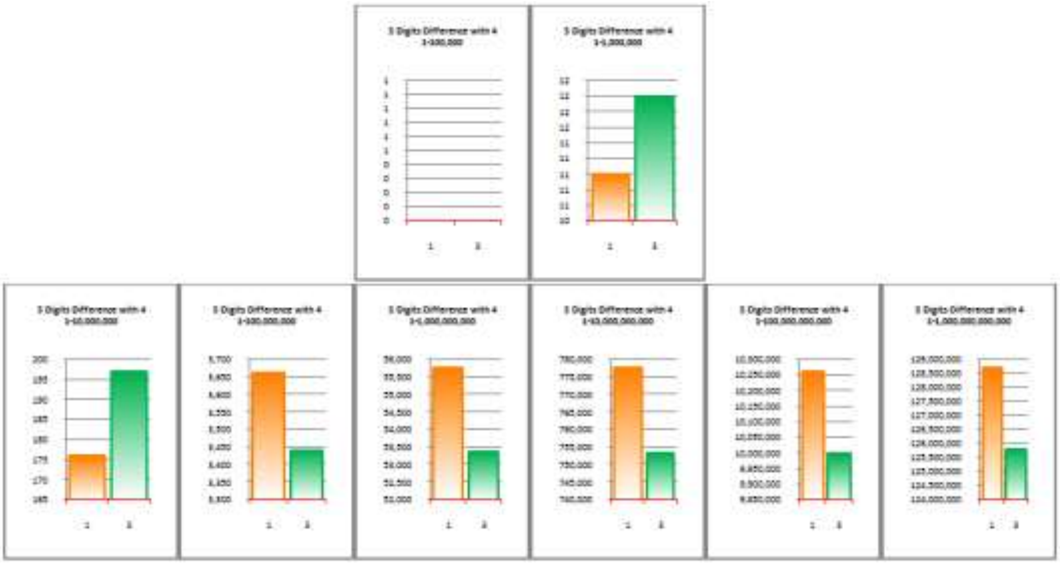


Figure 11: Differences of Number of Primes having Five 1's and Five3's in their Digits with those having Five4's in them in Ranges of $1 - 10^n$.

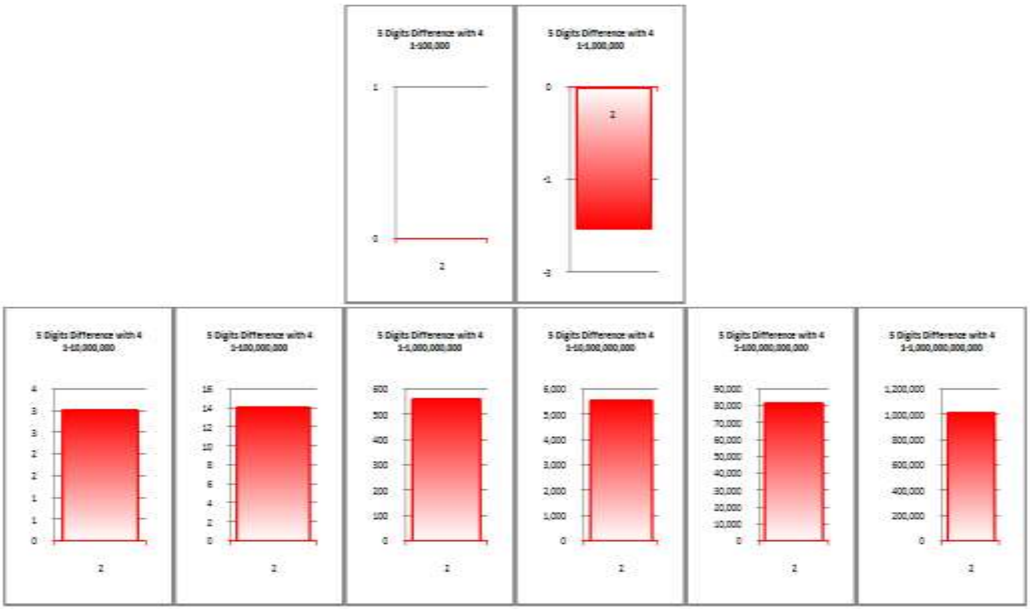


Figure 12: Difference of Number of Primes having Five2'sin their Digits with those having Five4's in them in Ranges of $1 - 10^n$.

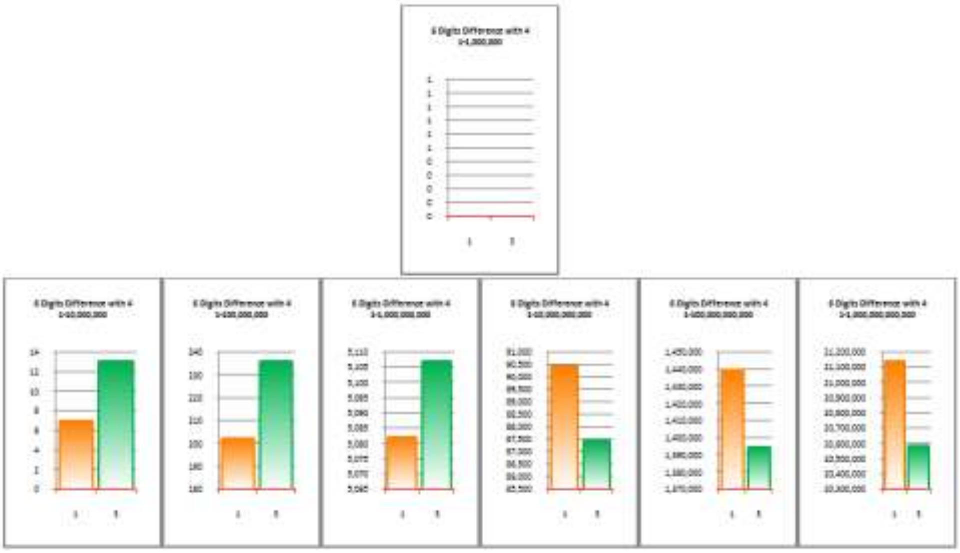


Figure 13: Differences of Number of Primes having Six 1's and Six3's in their Digits with those having Six4's in them in Ranges of $1 - 10^n$.

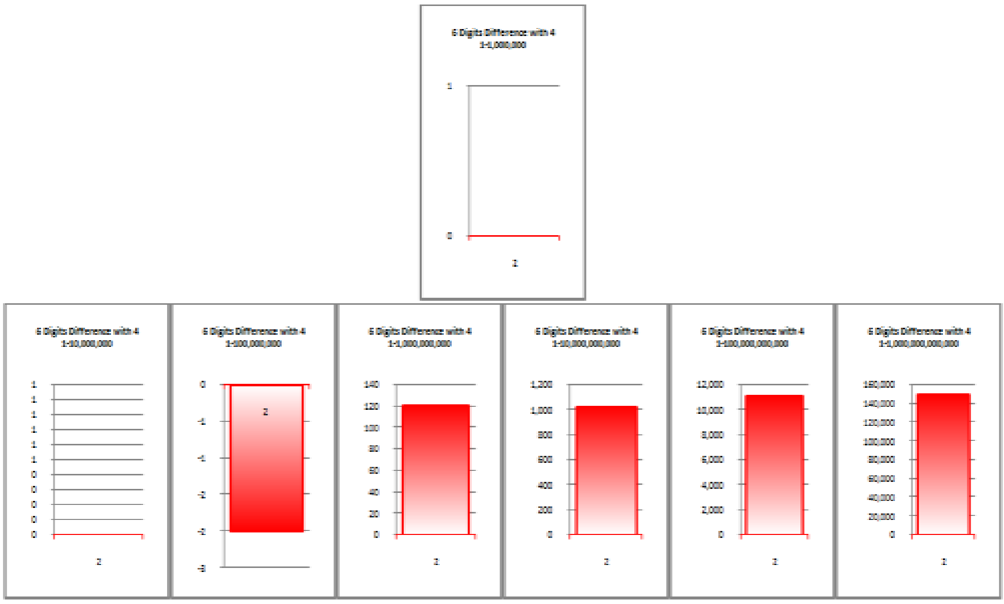


Figure 14: Difference of Number of Primes having Six2'sin their Digits with those having Six4's in them in Ranges of $1 - 10^n$.

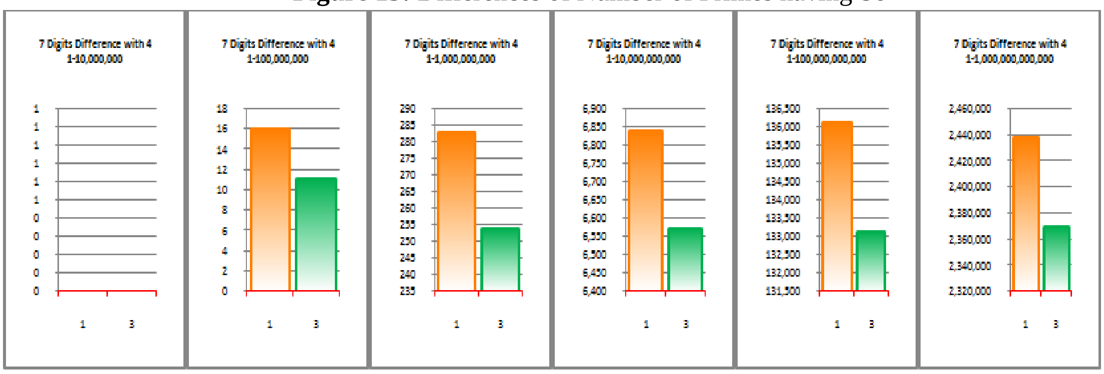


Figure 15: Differences of Number of Primes having Seven 1's and Seven3's in their Digits with those having Seven4's in them in Ranges of $1 - 10^n$.

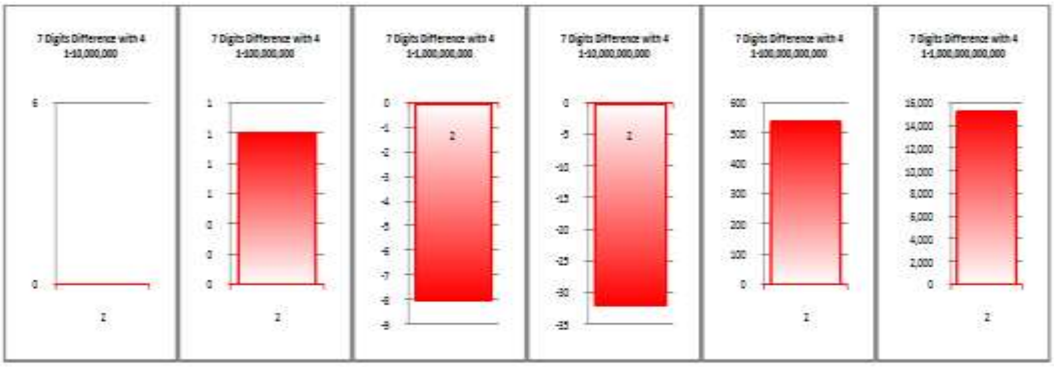


Figure 16: Difference of Number of Primes having Seven2'sin their Digits with those having Seven4's in them in Ranges of 1 – 10^n.

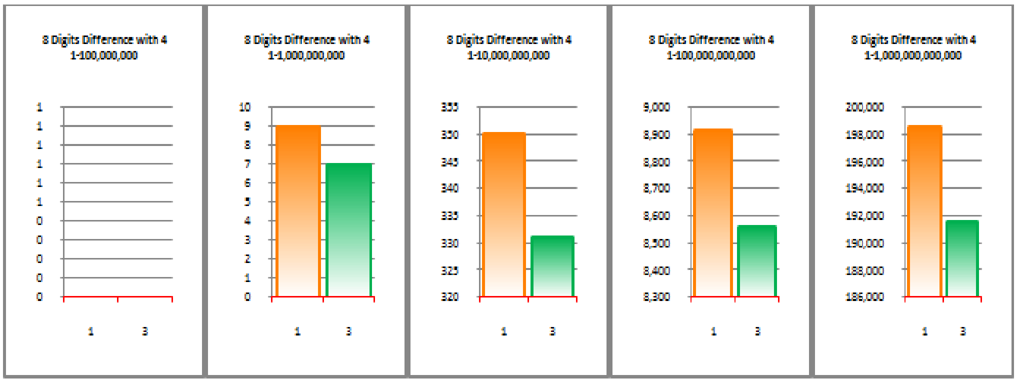


Figure 17: Differences of Number of Primes having Eight 1's and Eight3's in their Digits with those having Eight4's in them in Ranges of 1 – 10^n.

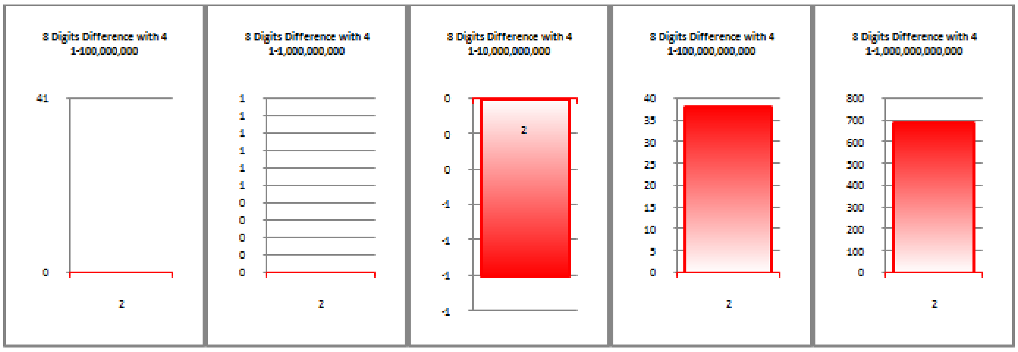


Figure 18: Difference of Number of Primes having Eight2'sin their Digits with those having Eight4's in them in Ranges of 1 – 10^n.

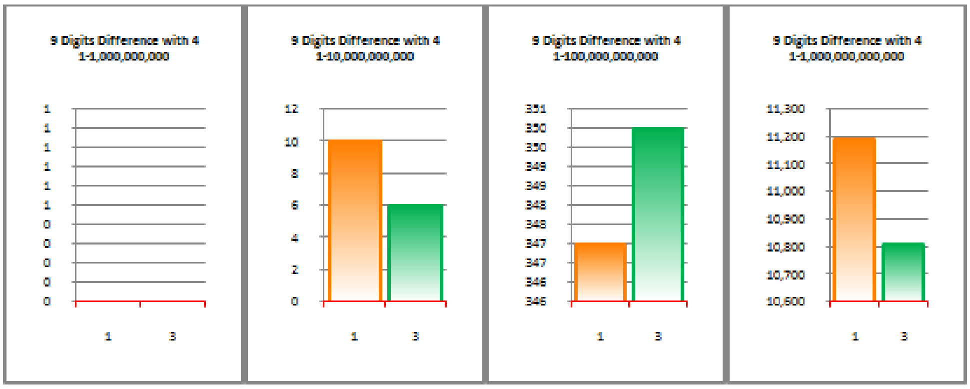


Figure 19: Differences of Number of Primes having Nine 1's and Nine3's in their Digits with those having Nine4's in them in Ranges of 1 – 10^n.

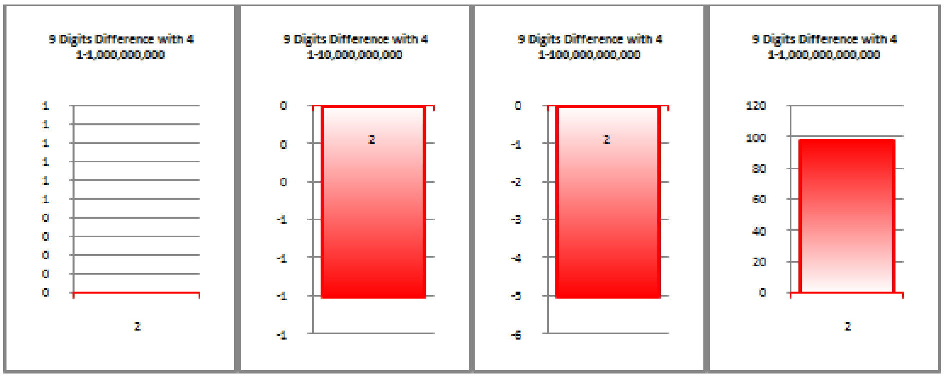


Figure 20: Difference of Number of Primes having Nine2'sin their Digits with those having Nine4's in them in Ranges of $1 – 10^n$.

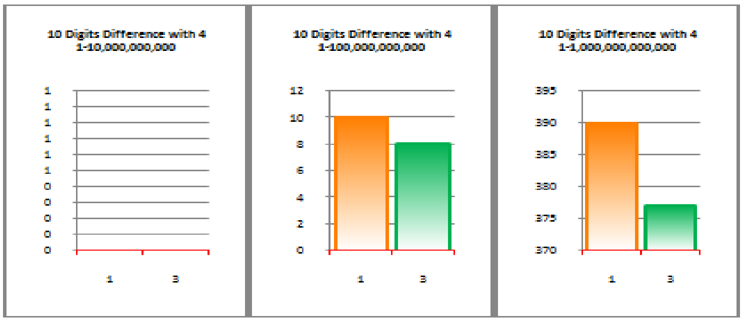


Figure 21: Differences of Number of Primes having Ten 1's and Ten3's in their Digits with those having Ten4's in them in Ranges of $1 – 10^n$.

Figure 22: Difference of Number of Primes having Ten2'sin their Digits with those having Ten4's in them in Ranges of $1 – 10^n$.

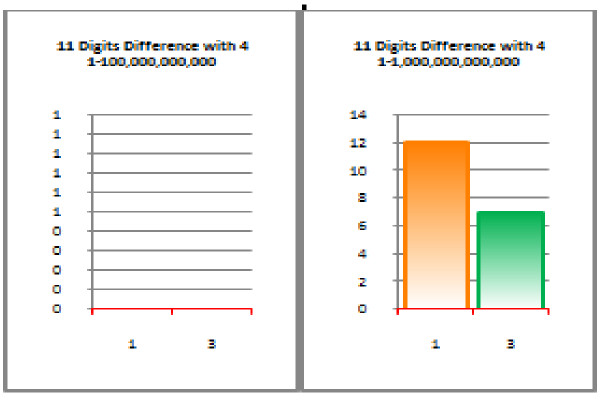


Figure 23: Differences of Number of Primes having Eleven 1's and Eleven3's in their Digits with those having Eleven4's in them in Ranges of $1 – 10^n$.

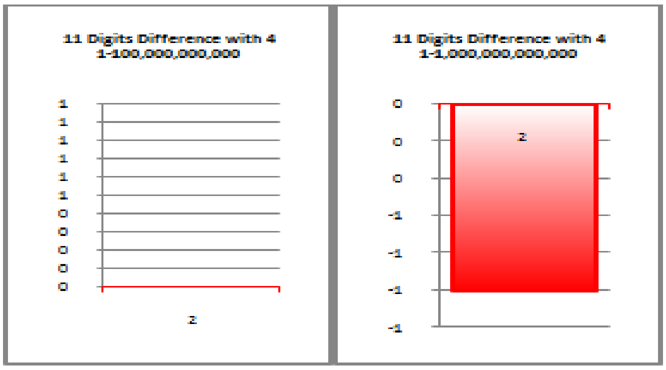


Figure 24: Difference of Number of Primes having Eleven2'sin their Digits with those having Eleven4's in them in Ranges of $1 - 10^n$.

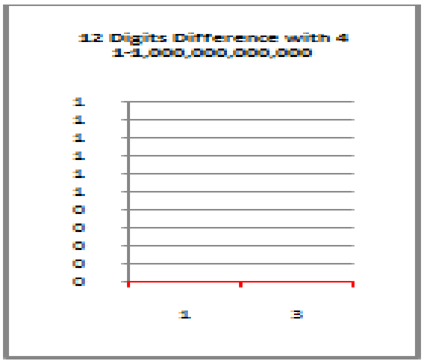


Figure 25: Differences of Number of Primes having Twelve 1's and Twelve3's in their Digits with those having Twelve4's in them in Ranges of $1 - 10^n$.

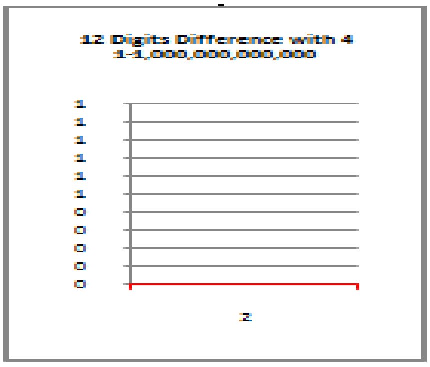


Figure 26: Difference of Number of Primes having Twelve2'sin their Digits with those having Twelve4's in them in Ranges of $1 - 10^n$.

IV. FIRST OCCURRENCE OF DIGIT 4 IN PRIME NUMBERS

The first natural number with single digit 4 is 4 itself! For sufficiently large ranges, first positive integer containing 2 4's 44, 3 4's is 444 and so on. Very simple formulation for this is given in [11].

Formula 1 [12] : If n and r are natural numbers, then the first occurrence of r number of 4's in numbers in range $1 \le m < 10^n$ is

$$f = \begin{cases} - , & \text{if } r > n \\ \sum_{j=0}^{r-1} \left(4 \times 10^j \right), & \text{if } r \le n \end{cases}.$$

But this formula is for all natural numbers. There is no such formula invented yet for the first occurrences of r number of 4's in prime numbers in range $1 \le m < 10^n$. So this has demanded actual determinations which we have come up with.

Table 3: First Prime Numbers in Various Ranges with Multiple 4's in Their Digits

Sr. No.	Range	First Prime Number in Range with						
		1 4	2 4's	3 4's	4 4's	5 4's	6 4's	7 4's
1.	$1 - 10^1$		-	-	-	-	-	-
2.	$1 - 10^2$	41	-	-	-	-	-	-
3.	$1 - 10^3$	41	443	-	-	-	-	-
4.	$1 - 10^4$	41	443	4,441	-	-	-	-
5.	$1 - 10^5$	41	443	4,441	44,449	-	-	-
6.	$1 - 10^6$	41	443	4,441	44,449	444,443	-	-
7.	$1 - 10^7$	41	443	4,441	44,449	444,443	-	-
8.	$1 - 10^8$	41	443	4,441	44,449	444,443	24,444,443	-
9.	$1 - 10^9$	41	443	4,441	44,449	444,443	24,444,443	424,444,441
10.	$1 - 10^{10}$	41	443	4,441	44,449	444,443	24,444,443	424,444,441
11.	$1 - 10^{11}$	41	443	4,441	44,449	444,443	24,444,443	424,444,441
12.	$1 - 10^{12}$	41	443	4,441	44,449	444,443	24,444,443	424,444,441

Table 3: Continued …

Sr. No.	Range	First Prime Number in Range with			
		8 4's	9 4's	10 4's	11 4's
1.	$1 - 10^1$	-	-	-	-
2.	$1 - 10^2$	-	-	-	-
3.	$1 - 10^3$	-	-	-	-
4.	$1 - 10^4$	-	-	-	-
5.	$1 - 10^5$	-	-	-	-
6.	$1 - 10^6$	-	-	-	-
7.	$1 - 10^7$	-	-	-	-
8.	$1 - 10^8$	-	-	-	-
9.	$1 - 10^9$	444,444,443	-	-	-
10.	$1 - 10^{10}$	444,444,443	4,444,444,447	-	-
11.	$1 - 10^{11}$	444,444,443	4,444,444,447	44,444,444,441	-
12.	$1 - 10^{12}$	444,444,443	4,444,444,447	44,444,444,441	444,444,444,443

V. LAST OCCURRENCE OF DIGIT 4 IN PRIME NUMBERS

The last natural number in ranges $1 - 10^n$, $1 \le n \le 12$, with r number of 4's in its digits fits in a formula.
Formula 2 [11] : If n and r are natural numbers, then the last occurrence of r number of 4's in numbers in range $1 \le m < 10^n$ is

$$l = \begin{cases} - & , \text{if } r > n \\ \sum_{j=0}^{r-1}\left(4\times10^j\right) + \begin{cases} 0 & , \text{if } r = n \\ \sum_{j=r}^{n-1}\left(9\times10^j\right), & \text{if } r < n \end{cases} \end{cases}.$$

Again since primes don't fit in any such formula, the last prime numbers in ranges $1 - 10^n$, $1 \le n \le 12$ with r number of 4's in them have been computationally determined.

Table 4: Last Prime Numbers in Various Ranges with Multiple 4's in Their Digits

Sr. No.	Number of 4's	Last Prime Number in Range 1 –							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1.	1	-	47	947	9,949	99,643	999,749	9,999,943	99,999,941
2.	2	-	-	449	8,447	98,443	998,443	9,998,447	99,998,449
3.	3	-	-	-	4,447	94,447	994,447	9,994,441	99,984,449
4.	4	-	-	-	-	44,449	844,447	9,944,449	99,944,447
5.	5	-	-	-	-	-	444,449	7,444,441	98,444,443
6.	6	-	-	-	-	-	-	-	74,444,449
7.	7	-	-	-	-	-	-	-	-
8.	8	-	-	-	-	-	-	-	-
9.	9	-	-	-	-	-	-	-	-
10.	10	-	-	-	-	-	-	-	-
11.	11	-	-	-	-	-	-	-	-

Table 4: Continued …

Sr. No.	Number of 4's	Last Prime Number in Range 1 –		
		10^9	10^{10}	10^{11}
1.	1	999,999,541	9,999,999,943	99,999,999,947
2.	2	999,994,843	9,999,994,409	99,999,996,443
3.	3	999,974,447	9,999,974,449	99,999,994,447
4.	4	999,944,441	9,999,944,447	99,999,644,449
5.	5	999,444,449	9,998,444,441	99,999,444,443
6.	6	994,444,447	9,994,444,441	99,974,444,447
7.	7	944,444,441	9,544,444,447	99,944,444,449
8.	8	444,444,443	5,444,444,443	99,444,444,443
9.	9	-	4,444,444,447	84,444,444,443
10.	10	-	-	44,444,444,441
11.	11	-	-	-

Remark : The maximum number of 4's in digits of any prime in the range 1 – 10^n is at most $n - 1$.

The integers occurring in all sections of this work form new integer sequences meriting their independent analysis.

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