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Research Paper

Designing A Queueing System with A Maximum Expected Waiting Time of 10 Minutes for Medical Centres

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ABSTRACT: *This paper proposes a time-frame approach to arrest the incidence of queues. In this approach, a maximum time frame of ten minutes (10) is given which no patient is expected to exceed accessing medical service in the system. There after a queueing system was designed that accommodates this time frame. The queueing system so designed is given as M/M/5/FCFS/∞/∞. It is assured that no patient spends more than 10 minutes in this system (medical centre). This is confirmed by a queue time of 0.1077minutes with its corresponding queue length of 0.3525 both of which approximate to zero (0) indicating disappearance of queue*
Keywords*: Medical Service, Maximum Time, Queueing System, Queue time, Queue length.*

I. INTRODUCTION

A healthy nation is a wealthy nation. This ideology seems to be the driving force behind the establishment by both the federal and state governments alike, tertiary institutions, manufacturing industries and companies, of medical centers and other health institutions and programs. The sole aim of all these concerns and concerted efforts is to bring health care services to the very door step of their relevant communities. The federal and state governments seek to by this initiative, maintain a healthy citizenry, the academic institutions a healthy academia while the business sector, a healthy work force to enhance productivity. These concerns and efforts by these institutions both government and private underscore the importance of providing adequate health care services to the people. It is the desire of management of these institutions to see that the populace, staff and students access health care services without let or hindrances.

II. STATEMENT OF PROBLEM

It is quite obvious from the facilities on ground in the health sector that both the federal and state governments have put in considerable efforts to the provision of affordable health care services to the citizenry. These efforts however are being undermined by some inherent constraints impeding easy and fast assessing of health care services. One of these constraints is the problem of queues. This situation could lead to disastrous consequences if not checked as human life is involved. Delay in accessing health care services promptly could lead to complications or even death. It is therefore of great necessity to ensure that less time is spent to deliver or access health care services, hence the problem of eliminating queues completely or reducing them to the barest minimum.

III. LITERATURE REVIEW

The problem of queues is a global phenomenon and queues have become a way of life as stated by [1; 2], since various services are required by various customers, i.e anything requiring service [3]. According to [4], Americans spend 37000,000,000 hours per year waiting in queues. Queues as stated by [5] are formed when the current demand for service exceeds the current capacity to provide for the service, while [1] describes the place where customers wait to be served as a queue. The implication of queues in the health sector particularly could be disastrous if not checked as human life is involved. The task of checking the incidence of queues by either complete elimination or reducing them to the barest minimum leads us to queueing theory. According to [6], queueing theory is the development of mathematical models for analysis, forecasting and optimal decision making of systems involving queues. The idea of queueing theory can be traced back to the classical work of A.K Erlang in 1900s, however the work of D.K Kendal in 1951 formed the basis for analytical calculations and the naming convention in queues being used today [7;2]. A schematic representation of the basic queueing process is given below as illustrated by [2].

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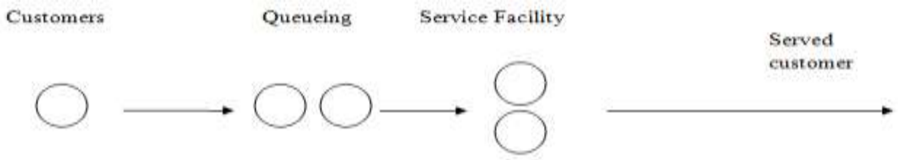

[8] and [2] explain that customers on arriving a queueing system would naturally join the queue. Occasionally, a customer is selected for service by a rule known as queue discipline. They are then attended to by a service facility called channel or server. A queueing system according to [9] involves a number of servers. One with one server is called a single-channel system while one with many servers is called a multiple -channel system. When service is completed the customer leaves the System.

IV. Objectives of the study

The objectives of this study are to:

- Investigate existing structure
- Collect data
- Analyze data and compute values for queueing parameters.

V. Aim

The aim of this work is to design a queueing system that would ensure that no patient spends more than 10 minutes at the health centre.

Significance of the study

The result of the study will contribute immensely to:

The reduction of delays at hospitals.

Ensure prompt health care delivery to patients.

Check avoidable complications or fatalities that may occur due to delays.

VI. METHODOLOGY

The source data was the Out-Patients Department (O.P.D) of the medical centre of Kaduna Polytechnic Kaduna. Data were collected on Arrival time and service time of 350 patients out of those that consulted at the hospital between 8.00am and 10.00am in the morning during the period of this study, with Poisson arrival time and exponential service time. Data were collected for a period of 10 days and were now used to investigate the existing structure. There after a model is structured to accommodate the main characteristics of the queueing system (hospital) to achieve our aim.

The existing structure (m/m/3/fcfs/□ /□)

Though there were about 6 doctors at the health centre, but as at the time of this study, three (3) doctors were seen regularly attending to patients. Being a multi-channel system, the following formulae were used for our calculations.

1. Service intensity which is the rate at which the service facility is utilized by the components of the system is given by

$$\rho = \frac{\lambda}{K\mu} \qquad (1)$$

where K is the number of channels and Kμ is the mean combined service rate for all the service stations.

2. The mean number of patients in the queue (queue length) is given by;

$$Lq = \frac{\mu\lambda\left(\frac{\lambda}{\mu}\right)P_O}{(K-1)!(K\mu-\lambda)^2} \qquad (2)$$

3. The mean number of patients in the system is

$$L = Lq + \frac{\lambda}{\mu} \qquad (3)$$

4. The mean time spent in the queue (queue time) is

$$Wq = \frac{\mu\left(\frac{\lambda}{\mu}\right)P_O}{(K-1)!(K\mu-\lambda)^2} \qquad (4)$$

5. The mean time spent in the system is given by

$$W = \frac{\mu\left(\frac{\lambda}{\mu}\right)P_O}{(K-1)!(K\mu-\lambda)^2} + \frac{1}{\mu} \qquad (5)$$

6. The probability that there is no patient in the hospital is given by

$$P_0 = \left[\sum_{N=0}^{K} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \left(\frac{\lambda}{\mu}\right)^k \frac{1}{k!(1-\rho)} \right]^{-1} \quad (6)$$

The designed queueing system (m/m/5/fcfs/∞/∞)

VII. Results and discussion

After analyzing the data to obtain values for the parameters, calculations and results were as follows:

Total number of patients = 350
Number of channels (doctors) K = 5
Total arrival time = 1146minutes.
Total service time = 2764minutes.

Mean arrival rate $\lambda = \frac{\text{total no.of patients}}{\text{total arrival time}} = \frac{350}{1146} = 0.3054$

Mean service rate $\mu = \frac{\text{total no.of patients}}{\text{total service time}} = \frac{350}{2764} = 0.1266$

The ratio $\frac{\lambda}{\mu} = \frac{0.3054}{0.1266} = 2.4123$

Service intensity $\rho = \frac{\lambda}{k\mu} = \frac{0.3054}{5 \text{ x } 0.1266} = 0.4825$

The probability of no patient in the hospital is

$$P_0 = \left[\sum_{N=0}^{k-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \left(\frac{\lambda}{\mu}\right)^k \frac{1}{k!(1-\rho)} \right]^{-1}$$

$$P_0 = \frac{(2.4123)^0}{0!} + \frac{(2.4123)^1}{1!} + \frac{(2.4123)^2}{2!} + \frac{(2.4123)^3}{3!} + \frac{(2.4123)^4}{4!} + \frac{(2.4123)^5}{5!(1-0.4825)}$$

$$= 1 + 2.4123 + 2.9096 + 2.3396 + 1.4110 + 1.3154$$

$$= (11.3879)^{-1} = 0.0878.$$

Here the mean queue length $L_q = \frac{\mu\lambda(\frac{\lambda}{\mu})^k P_O}{(K-1)!(K\mu-\lambda)^2}$

$L_q = \frac{0.1266\times0.3054\ (2.4123)^5\times0.0878}{4!(5\times0.1266-0.3054)^2} = 0.1077 \approx 0$patient

Mean time spent on queue Wq $= \frac{\mu(\frac{\lambda}{\mu})^5 P_O}{(K-1)!(K\mu-\lambda)^2}$

$W_q = \frac{0.1266\times(2.4123)^5\times0.0878}{4!(5\times0.1266-0.3054)^2} = 0.3525 \approx 0$minute

Mean number of patients in the Hospital

$L = L_q + \frac{\lambda}{\mu} = 0.1077 + 2.4123 = 2.52 \approx 3$ patients

Mean time spent in the Hospital

$W = W_q + \frac{1}{\mu} = 0.3525 + \frac{1}{0.1266} = 8.2514 \approx 8$minutes

VIII. SUMMARY

It is obvious that with this system, the queue disappears since a queue length of 0.1077 implies that there is no body in the queue and the waiting time of 0.3525 implies also that there is no time for queueing in the absence of a queue.

IX. CONCLUSION

There is no doubt that queues have become a part of our daily lives, and that it can adversely affect the quality of life of the people and the economy of the nation. In the health sector its consequences could be disastrous since it has to do with human life. Realization of this fact makes it mandatory that no efforts should be spared in making sure that queues are either eliminated completely or queue time reduced to the barest minimum. The procedure above is one of the measures to curb the incidence of queues. Every channel (doctor, pharmacists etc) in the system being aware of the time frame given for a patient to be in the hospital, puts in their best to maintain it and avoid the consequences of failure from their part.

X. RECOMMENDATIONS

With reference to the results obtained from analytical investigations, it is recommended that:

The Medical Centre adopts the designed system as it has been proven that its structure does not allow any patient to spend more than 10 minutes in the system. More medical doctors should be employed in to the Medical centre in particular and other medical centres or hospitals in the country at large.

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