Quest Journals
Journal of Research in Applied Mathematics
Volume 3 ~ Issue 4 (2017) pp: 13-18
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Tenth-Order Iterative Methods withoutDerivatives forSolving Nonlinear Equations

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Received 14 Jan, 2017; **A**ccepted 07 Feb, 2017

ABSTRACT: *In this paper, we construct new classes of derivative-free of tenth-order iterative methods for solving nonlinear equations. The new methods of tenth-order convergence derived by combining of theSteffensen's method, the Kung and Traub's of optimal fourth-order and the Al-Subaihi's method. Several examples to compare of other existing methods and the results of new iterative methods are given the encouraging results and have definite practical utility.*

Keywords: *nonlinear equations, iterative methods, tenth-order, derivative-free, order of convergence.*

I. INTRODUCTION

The nonlinear equation is one of the most and oldest problems in scientific and engineering applications, so the researchers have beenlooking for optimization of solution for this problem.

For solving the nonlinear equation $f(\gamma) = 0$, where $f: I \subseteq \mathbb{R} \to \mathbb{R}$be a scalar function on an open interval$I$, and$\gamma$is a simple root of $f(x)$.The famous iterative method of optimal quadratic convergence to solve the nonlinear equation is Newton's method (NM).

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \tag{1}$$

The optimal conjectureis2^n, where $n + 1$ is the number of function evaluation [1].We can define the efficiency index as $\delta^{1/\alpha}$, such that δ be the order of the method and α is the total number of functions evaluations per iteration [2,3]. The Newton's method have been modified by Steffensenby approximating$f'(x_n)$ by forward difference, to get the famous Steffensen's method (SM) [4]

$$x_{n+1} = x_n - \frac{f(x_n)^2}{f(x_n + f(x_n)) - f(x_n)}. \tag{2}$$

Which has same optimal order of convergence asNewton's method (NM), the efficiency index of both method (1) and (2) are EI = $2^{1/2} \approx 1.4142$.

In this paper some iterative methods are constructedto develop new derivative-free iterative methods usingtheSteffensen's method, the Kung-Traub [1] and the Al-Subaihi's method [5].

A three- point of eighth-order convergence is constructed by Kung-Traub(KTM) [1].

$$y_n = x_n - \left(\frac{f(x_n)^2}{f(w_n) - f(x_n)}\right),$$

$$z_n = y_n - \left(\frac{f(x_n)f(w_n)}{f(y_n) - f(x_n)}\right)\left(\frac{1}{f[w_n, x_n]} - \frac{1}{f[w_n, y_n]}\right), \tag{3}$$

$$x_{n+1} = z_n - \left(\frac{f(y_n)f(x_n)f(w_n)}{f(z_n) - f(x_n)}\right)\left\{\left(\frac{1}{f(z_n) - f(w_n)}\right)\left[\frac{1}{f[y_n, z_n]} - \frac{1}{f[w_n, y_n]}\right] - \left(\frac{1}{f(y_n) - f(x_n)}\right)\left[\frac{1}{f[w_n, y_n]} - \frac{1}{f[w_n, x_n]}\right]\right\}, \tag{4}$$

where $w_n = x_n + \beta f(x_n)$, $n \in \mathbb{N} and \beta \in \mathbb{R}$.

The conjecture of Kung and Traub's for multipoint iterations, a without memory based which has optimal convergence, and the efficiency index of eighth-order convergence (KTM), (4), is$8^{1/4} \approx 1.6818$.Recently, families of Iterative methods with higher convergence orders for solving nonlinear equations has been devolved by Al-Subaihi [5] as

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(y_n)} - w(t_n)\frac{f\left(z_n - \frac{f(z_n)}{f'(y_n)}\right)}{f'(y_n)}, \tag{5}$$

wherey_n is iterative method of order p, z_n is iterative method of order q, ($q > p$), and $t_n = \frac{f\left(z_n - \frac{f(z_n)}{f'(y_n)}\right)}{f(z_n)}$, where $w(t_n)$ is a weight function satisfying the conditions of $w(0) = 1, w'(0) = 1 and |w''(0)| < \infty$. The families, (5),of order $2q + p$.

II. DESCREPTION OF THE METHODS

In this section a new iterative families of higher-order with derivative-free are described. The construction depend on the Steffensen's, (2), theKung and Traub's, (4), and the Al-Subaihi's methods, (5). Approximation of $f'(x_n)$ by

$$f'(x_n) \approx \frac{f(x_n + f(x_n)) - f(x_n)}{f(x_n)}, \tag{6}$$

and substituting in Newton's method (NM), (1), gives Steffensen's method (SM), (2), which is convergence quadratically and derivative-free.

Let $G_n = x_n + f(x_n)^3$ [6]. UsingtheSteffensen's method of order two and the second step of the Kung and Traub's method of order fourth, (3), and substituting them in Al-Subaihi's method, (5), we have:

$$y_n = x_n - \frac{f(x_n)}{f[G_n, x_n]},$$
$$z_n = y_n - \left(\frac{f(x_n)f(G_n)}{f(y_n) - f(x_n)}\right)\left(\frac{1}{f[G_n, x_n]} - \frac{1}{f[G_n, y_n]}\right),$$
$$x_{n+1} = z_n - \frac{f(z_n)}{f'(y_n)} - w(t_n)\frac{f\left(z_n - \frac{f(z_n)}{f'(y_n)}\right)}{f'(y_n)}. \tag{7}$$

Using the approximation (see for example [7])

$$f'(y_n) = 2f[y_n, x_n] - f[G_n, x_n], \tag{8}$$

where:

$$f[G_n, x_n] = \frac{f(G_n) - f(x_n)}{G_n - x_n}, \tag{9}$$
$$f[G_n, y_n] = \frac{f(G_n) - f(y_n)}{G_n - y_n}, \tag{10}$$
$$f[y_n, x_n] = \frac{f(y_n) - f(x_n)}{y_n - x_n}. \tag{11}$$

Substitute (8) into equation (7), to obtain a new class of methods (YSM)

$$y_n = x_n - \frac{f(x_n)}{f[G_n, x_n]}, \tag{12}$$
$$z_n = y_n - \left(\frac{f(x_n)f(G_n)}{f(y_n) - f(x_n)}\right)\left(\frac{1}{f[G_n, x_n]} - \frac{1}{f[G_n, y_n]}\right), \tag{13}$$
$$x_{n+1} = z_n - \frac{f(z_n)}{2f[y_n, x_n] - f[G_n, x_n]} - w(t_n)\frac{f\left(z_n - \frac{f(z_n)}{2f[y_n, x_n] - f[G_n, x_n]}\right)}{2f[y_n, x_n] - f[G_n, x_n]}, \tag{14}$$

the new iterative method (14) is of order tenth with derivative free, and five functions evaluations, so the efficiency index (EI) = $10^{1/5} \approx 1.5849$.

Theorem1: suppose that $f: I \subseteq \mathbb{R} \to \mathbb{R}$ in an open interval I with simple root $\gamma \in I$. If $x_0$be a sufficiently close to γthen the methods (12–14) has tenth-order of convergence, if $t_n = \frac{f\left(z_n - \frac{f(z_n)}{2f[y_n, x_n] - f[G_n, x_n]}\right)}{f(z_n)}$, and $w(t_n)$ is a function satisfying the conditions of $w(0) = 1, w'(0) = 1 and |w''(0)| < \infty$.

$$e_{n+1} = (24k_2^8 - 44k_2^6k_3 + 30k_2^4k_3^2 - 9k_2^2k_3^3 + k_3^4)k_2e_n^{10} + O(e_n^{11})$$

Proof: let $e_n = x_n - \gamma$, and $k_n = \frac{f^{(n)}(\gamma)}{n! f'(\gamma)}$

By Taylor expansion of$f(x_n)$

$$f(x_n) = f'(\gamma)[k_n + k_2e_n^2 + k_3e_n^3 + k_4e_n^4 + k_5e_n^5 + \cdots + k_8e_n^8 + k_9e_n^9 + k_{10}e_n^{10} + O(e_n^{11})] \tag{15}$$

Dividing (9) by (15)

$$\frac{f(x_n)}{f[G_n, x_n]} = e_n - k_2e_n^2 + (-2k_3 + 2k_2^2)e_n^3 + (-f'(\gamma)^3k_2 - 3k_4 + 7k_2k_3 - 4k_2^3)e_n^4 + (-4k_5 -3f'(\gamma)^3k_3 + \cdots + 10k_2k_4 + 6k_3^2)e_n^5 + \cdots + (396f'(\gamma)^3k_2^2k_3k_4 + 24f'(\gamma)^6k_2^3k_3 -13f'(\gamma)^6k_2^2k_4 - 11f'(\gamma)^6k_2k_3^2 + \cdots - 28f'(\gamma)^3k_8 - 166f'(\gamma)^3k_2k_3k_5 - 9k_{10})e_n^{10} + O(e_n^{11}) \tag{16}$$

Computing (12) by using (16), we have

$$y_n = \gamma + k_2 e_n^2 + (2k_3 - 2k_2^2)e_n^3 + (f'(\gamma)^3 k_2 + 3k_4 - 7k_2k_3 + 4k_2^3) + (3f'(\gamma)^3 k_3 - 8k_2^4 + 20k_2^2k_3 - \cdots - 6k_3^2 + 4k_5)e_n^5 + \cdots + (-396f'(\gamma)^3 k_2^2 k_3 k_4 - 24f'(\gamma)^6 k_2^3 k_3 + \cdots + 166f'(\gamma)^3 k_2 k_3 k_5 + 9k_{10})e_n^{10} + O(e_n^{11}) \quad (17)$$

Use Taylor expansion of (y_n)about γ and simplifying, we obtain

$$f(y_n) = f'(\gamma)[k_2 e_n^2 + (2k_3 - 2k_2^2)e_n^3 + (f'(\gamma)^3 k_2 + 3k_4 - 7k_2k_3 + 5k_2^3)e_n^4 + (3f'(\gamma)^3 k_3 - 12k_2^4 + 24k_2^2k_3 - \cdots - 6k_3^2 + 4k_5)e_n^5 + \cdots + (-614f'(\gamma)^3 k_2^2 k_3 k_4 + 230f'(\gamma)^3 k_2 k_3 k_5 + \cdots - 45k_4k_7 - 49k_5k_6)e_n^{10} + O(e_n^{11})] \quad (18)$$

And

$$\frac{f(x_n)f(G_n)}{f(y_n) - f(x_n)} = f'(\gamma)[-e_n - 2k_2 e_n^2 + (-f'(\gamma)^3 - 3k_3 + k_2^2)e_n^3 + (-7f'(\gamma)^3 k_2 - 4k_4 + 3k_2k_3 - k_2^3)e_n^4 + \cdots + (-155k_2^2k_3k_4 - 86f'(\gamma)^3 k_2k_3k_5 + \cdots + 9k_4k_7 + 9k_5k_6)e_n^{10} + O(e_n^{11})] \quad (19)$$

Substituting (17), (19), (9) and (10) into (13), we obtain

$$z_n = \gamma + (2k_2^3 - k_2k_3)e_n^4 + (-10k_2^4 + \cdots - 2k_2k_4 - 2k_3^2)e_n^5 + \cdots + (-947f'(\gamma)^3 k_2^2k_3k_4 - 72f'(\gamma)^6 k_2^3 k_3 + 46f'(\gamma)^6 k_2^2 k_4 + \cdots + 178f'(\gamma)^3 k_2 k_4^2 + 98f'(\gamma)^3 k_2^2 k_6 + 326f'(\gamma)^3 k_2k_3k_5)e_n^{10} + O(e_n^{11}) \quad (20)$$

Use Taylor expansion of (z_n), we get

$$f(z_n) = f'(\gamma)[(2k_2^3 - k_2k_3)e_n^4 + (-10k_2^4 + \cdots - 2k_2k_4 - 2k_3^2)e_n^5 + \cdots + (-947k_2^2k_3k_4 + 3264f'(\gamma)^3 k_2k_3k_5 + 46f'(\gamma)^6 k_2^2 k_4 - \cdots - 19k_3k_8 - 27k_4k_7 - 31k_5k_6)e_n^{10} + O(e_n^{11})] \quad (21)$$

Expand $2f[y_n, x_n] - f[G_n, x_n]$ *to get*

$$2f[x_n, y_n] - f[x_n, G_n] = f'(\gamma)[1 + (2k_2^2 - k_3)e_n^2 + (-f'(\gamma)^3 k_2 - 4k_2^3 + \cdots - 2k_4)e_n^3 + \cdots + (2f'(\gamma)^9 k_2^4 + 472k_2k_3k_4k_5 + 213k_2^2k_3k_5 - \cdots - 176k_2^4k_5 + 128k_2^2k_3^3)e_n^{10} + O(e_n^{11})] \quad (22)$$

Then we computing $z_n - \frac{f(z_n)}{2f[x_n,y_n] - f[x_n,G_n]}$ *, we have*

$$z_n - \frac{f(z_n)}{2f[x_n, y_n] - f[x_n, G_n]} = \gamma + (4k_2^5 - 4k_2^3k_3 + k_3^2k_2)e_n^6 + (4k_2k_3k_4 - 28k_2^6 + 54k_2^4k_3 - \cdots + f'(\gamma)^3 k_2^2 k_3 - 2f'(\gamma)^3 k_2^4)e_n^7 + \cdots + (-399f'(\gamma)^3 k_2^2k_3k_4 - 44f'(\gamma)^6 k_2^3 k_3 + 5f'(\gamma)^3 k_2^2 k_4 + \cdots + 28f'(\gamma)^3 k_2 k_4^2 + 4f'(\gamma)^3 k_2^2 k_6 + 44f'(\gamma)^3 k_2k_3k_5)e_n^{10} + O(e_n^{11}) \quad (23)$$

Then by Taylor expansion, we have

$$f\left(z_n - \frac{f(z_n)}{2f[x_n, y_n] - f[x_n, G_n]}\right) = f'(\gamma)[(4k_2^5 - 4k_2^3k_3 + k_3^2k_2)e_n^6 + (4k_2k_3k_4 - 28k_2^6 + 54k_2^4k_3 - 8k_2^3k_4 - 24k_2^2k_3^2 + \cdots - 2f'(\gamma)^3 k_2^4)e_n^7 + \cdots + (-399f'(\gamma)^3 k_2^2k_3k_4 + 44f'(\gamma)^3 k_2k_3k_5 + \cdots - 159k_3^3k_4 + 9k_2k_5^2 + 21k_3^2k_6)e_n^{10} + O(e_n^{11})] \quad (24)$$

Substituting (23), (24), (22) and $w(t_n)$ is weight function *into (14), we get*

$$x_{n+1} = \gamma + (24k_2^8 - 44k_2^6k_3 + 30k_2^4k_3^2 - 9k_2^2k_3^3 + k_3^4)k_2 e_n^{10} + O(e_n^{11})$$

Finally, we have

$$e_{n+1} = (24k_2^8 - 44k_2^6k_3 + 30k_2^4k_3^2 - 9k_2^2k_3^3 + k_3^4)k_2 e_n^{10} + O(e_n^{11}) \quad .■$$

And we take some special method of (14), by multiple uses of weight function $w(t_n)$ which satisfies the conditions at Theorem1 as

Method1: choosing

$$w(t_n) = (1 + t_n + \theta {t_n}^2), \theta \in \mathbb{R}$$

A new families of methods can be written as

$$x_{n+1} = z_n - \frac{f(z_n)}{2f[y_n,x_n] - f[G_n,x_n]} - (1 + t_n + \theta {t_n}^2)\frac{f\left(z_n - \frac{f(z_n)}{2f[y_n,x_n] - f[G_n,x_n]}\right)}{2f[y_n,x_n] - f[G_n,x_n]}. \quad (25)$$

Method2: the function

$$w(t_n) = \frac{1}{(1 - t_n + \theta {t_n}^2)}, \theta \in \mathbb{R}$$

Gives another group characterized by the iterative function

$$x_{n+1} = z_n - \frac{f(z_n)}{2f[y_n,x_n]-f[G_n,x_n]} - \left(\frac{1}{(1-t_n+\theta t_n{}^2)}\right)\frac{f\left(z_n - \frac{f(z_n)}{2f[y_n,x_n]-f[G_n,x_n]}\right)}{2f[y_n,x_n]-f[G_n,x_n]}. \quad (26)$$

Method3: the choice

$$w(t_n) = (1+\frac{t_n}{1+\theta t_n}),\ \theta \in \mathbb{R}$$

A new family of methods can be obtained as

$$x_{n+1} = z_n - \frac{f(z_n)}{2f[y_n,x_n]-f[G_n,x_n]} - (1+\frac{t_n}{1+\theta t_n})\frac{f\left(z_n - \frac{f(z_n)}{2f[y_n,x_n]-f[G_n,x_n]}\right)}{2f[y_n,x_n]-f[G_n,x_n]}. \quad (27)$$

Method4: let $w(t_n) = (1+t_n)$, then the new method will become

$$x_{n+1} = z_n - \frac{f(z_n)}{2f[y_n,x_n]-f[G_n,x_n]} - (1+t_n)\frac{f\left(z_n - \frac{f(z_n)}{2f[y_n,x_n]-f[G_n,x_n]}\right)}{2f[y_n,x_n]-f[G_n,x_n]}. \quad (28)$$

The weighted functions (25-28) have been used to give the methods (YSM1 – YSM4), respectively.

III. NUMERICAL RESULT

The present tenth-order method given by (YSM), (14), to compare different methods to solve the nonlinear-equation to verify the effectiveness of the tenth-order methods with derivative-free.
We compare the new families (YSM1 – YSM4) with the Steffensen's method (SM), (2),thefourth –order method with derivative-free of Kung and Traub (KTM), (3),andninth-order without derivative of Malik's, Al-Fahid's and Fayyaz'smethods(FM), [8]

$$y_n = x_n - \frac{\kappa f(x_n)^2}{f(x_n)-f(\eta_n)},$$
$$z_n = y_n - H_1(t_1)\frac{\kappa f(y_n)^2}{f(y_n)-f(\varsigma_n)},$$
$$x_{n+1} = z_n - H_2(t_1,t_2,t_3,t_4)\frac{\kappa f(y_n)f(z_n)}{f(y_n)-f(\varsigma_n)}, \quad (29)$$

whereκ are real parameters,$\eta_n = x_n - \kappa f(x_n)$, $\varsigma_n = y_n - \kappa f(y_n)$,$t_1 = \frac{f(y_n)f(\varsigma_n)}{f(x_n)-f(\eta_n)}$, $t_2 = \frac{f(y_n)f(y_n)}{f(x_n)-f(\eta_n)}$, $t_3 = \frac{f(z_n)}{f(\varsigma_n)}$, $t_4 = \frac{f(z_n)}{f(y_n)}$, and$H_1$, H_2 satisfying the condition $H_1(0) = 1, \frac{\partial H_1}{\partial t_1}(0) = 1, H_2(0,0,0,0) = 1, \frac{\partial H_2}{\partial t_1}(0,0,0,0) = 1, \frac{\partial H_2}{\partial t_2}(0,0,0,0) = 1, \frac{\partial H_2}{\partial t_3}(0,0,0,0) = 1$ and $\frac{\partial H_2}{\partial t_4}(0,0,0,0) = 1$, (Let $H_1 = \frac{1}{1-t_1}$ and $H_2 = \frac{1}{1-t_1-t_2-t_3-t_4}$).

The efficiency index of (FM), (30), is $9^{1/5} \approx 1.5518$, and the eighth –order (KTM1), (4), and the tenth-order with derivative of Hafiz's method (HM), [9]

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$
$$z_n = y_n - \frac{2f(y_n)f'(y_n)}{2[f'(y_n)]^2 - f(y_n)P_f},$$
$$x_{n+1} = z_n - \frac{f(z_n)}{f[z_n,y_n]+(z_n-y_n)f[z_n,y_n,y_n]}, \quad (30)$$

where $P_f = \frac{2}{y_n-x_n}(2f'(y_n)+f'(x_n)-3f[y_n,x_n])$.
The efficiency index of (HM), (29), is $10^{1/5} \approx 1.5849$, which the same as the new families (YSM), (14),

The MATLAB program is used for all the computations using 2000 digit, Table 2 present are the number of iterations to approximate the zero (IT) and containing the value of $f(x_n)\ and\ |x_{n+1}-x_n|$. The convergence order δ of a new iterative methods is over by the equation $\lim_{n\to\infty}\frac{|e_{n+1}|}{|e_n|^\delta} = c \neq 0$, then values of the computational order of convergence (COC) may be approximated as $coc \approx \frac{\ln|\ (x_{n+1}-\gamma)/(x_n-\gamma)|}{ln|(x_n-\gamma)/(x_{n-1}-\gamma)|}$, (see for example [10])
The following criteria

$$|x_{n+1}-x_n| \le 10^{-100},$$

is used to stop the iteration process in all examples.
The following test functions are used to complain methods, with a simple roots in:

Table 1: Test functions and their simple root.

Functions	Roots
$f_1(x) = (x+2)e^x - 1,$	$\gamma = -0.44285440100239$
$f_2(x) = \sin(x)^2 - x^2 + 1,$	$\gamma = 1.40449164821534$
$f_3(x) = (x-1)^3 - 1,$	$\gamma = 2.0$
$f_4(x) = \log(x^4 + x + 1) + xe^x,$	$\gamma = 0.0$
$f_5(x) = \log(x+1) + x^3,$	$\gamma = 0.0$
$f_6(x) = \sin(x)\, e^x + \log(x^2+1),$	$\gamma = 0.0$
$f_7(x) = sin^{-1}(x^2-1) - \frac{x}{2} + 1,$	$\gamma = 0.59481096839837$
$f_8(x) = \cos\left(\frac{\pi x}{2}\right) + x^2 - \pi,$	$\gamma = 2.03472489627913$

Table 2: numerical effects for the results of different functions

Method	IT		$\|x_{n+1} - x_n\|$	COC
				f_1
SM	8	1.08928e-249	1.89983e-125	2
KTM	-	*fails*	-	-
KTM1	-	*fails*	-	-
FM	3	-1.49378e-1136	1.10504e-126	8.99
HM	3	-1.19252e-1393	6.71925e-140	10
YSM1	3	9.61472e-1335	4.7439e-134	10.01
YSM2	3	2.12619e-1283	5.927e-129	10.02
YSM3	3	2.83408e-1259	1.46361e-126	10.02
YSM4	3	1.7715e-1283	5.81981e-129	1
$f_2(x) = \sin(x)^2 - x^2 + 1, x_0 = 1.4$				
SM	9	1.38957e-249	2.19516e-125	2
KTM	4	-4.28939e-605	6.6314e-152	4
KTM1	3	-7.07842e-1192	1.09749e-149	8
FM	3	1.28608e-1421	5.45919e-159	9
HM	3	0	6.49615e-253	10
YSM1	4		8.43293e-528	10
YSM2	4		2.00562e-482	9.72
YSM3	4		5.42247e-421	9.99
YSM4	3	4.04623e-2008	1.98521e-232	
				f_3
SM	8	2.63414e-357	1.48159e-179	2
KTM	4	2.56566e-492	8.46364e-124	4
KTM1	3	1.26822e-967	9.66425e-122	8
FM	3	1.48087e-1272	2.51087e-142	8.99
HM	3	0	3.01795e-203	10
YSM1	3	3.75463e-1916	2.20389e-192	10
YSM2	3	5.35547e-1861	6.5466e-187	10.01
YSM3	3	5.67031e-1836	1.98341e-184	10.01
YSM4	3	5.28414e-1861	6.53783e-187	1
				f_4
SM	9	1.10381e-257	2.7127e-129	2
KTM	5	-9.74336e-947	5.08409e-237	4
KTM1	4	-1.25382e-2009	4.72614e-416	8
FM	-	*div*	-	-
HM	4	8.41923e-2010	3.57641e-916	10
YSM1	4	6.24525e-699	5.08988e-697	10
YSM2	4	-3.13515e-1411	1.94891e-617	10
YSM3	4	-6.25167e-2009	3.61742e-569	10
YSM4	4	1.169e-1510	1.9523e-610	
$f_5(x) = \log(x+1) + x^3, x_0 = 0.25$				
SM	9	-1.40659e-272	1.186e-136	2
KTM	5	6.53964e-524	1.9904e-131	4
KTM1	4	-5.69222e-1923	5.69817e-241	7.99
FM	-	*div*	-	-
HM	4	6.06038e-6026	3.99489e-590	10
YSM1	4	-3.39454e-6029	5.46001e-411	9.99
YSM2	4	-3.23157e-6029	1.95424e-360	10
YSM3	4	-1.99633e-6026	1.54772e-296	10
YSM4	4	2.54739e-6025	4.31825e-346	

Method	IT		$\lvert x_{n+1} - x_n \rvert$	COC
f_6				
SM	10	8.44207e-288	1.45276e-144	2
KTM	5	1.80742e-466	1.85292e-117	4
KTM1	4	3.90546e-1745	3.70115e-219	7.99
HM	4	*-1.02076*e-3888	*6.32121*e-649	10
FM	-	*div*	-	-
YSM1	4	-8.62745e-3728	6.61204e-460	10
YSM2	4	*1.40673*e-3792	*1.16503*e-424	10
YSM3	4	6.64559e-3943	1.15041e-409	9.99
YSM4	4	*1.50312*e-3767	*2.75612*e-427	
f_7				
SM	-	*div*	-	-
KTM	5	*-2.40779*e-900	*3.01354*e-225	4
KTM1	4	*-2.69748*e-2008	*9.45464*e-398	8
FM	3	-2.18467e-1042	2.78079e-115	9.23
HM	4	*-2.69748*e-2008	*5.97482*e-692	10
YSM1	4	*8.02466*e-684	*1.83776*e-669	10
YSM2	4	*-3.35584*e-1597	*1.06178*e-603	10
YSM3	4	0	*1.13456*e-551	10
YSM4	4	*1.06964*e-1703	*2.58104*e-596	
$f_8(x) = \cos\left(\frac{\pi x}{2}\right) + x^2 - \pi, x_0 = 2$				
SM	8	*1.95409*e-255	*1.30323*e-128	2
KTM	4	*1.81305*e-409	*6.10311*e-103	4
KTM1	-	*fails*	-	-
FM	3	9.59404e-930	3.37864e-104	8.98
HM	3	*8.13807*e-1866	*6.36702*e-187	10
YSM1	3	*6.05924*e-1552	*9.27035*e-156	10.05
YSM2	3	*6.85374*e-1423	*6.67285*e-143	10.11
YSM3	3	*9.2786*e-1392	*8.22618*e-140	10.13
YSM4	3	*5.51227*e-1423	*6.52908*e-143	1

IV. CONCLUSION

In this paper, we have presented a new families of tenth-order derivative-free methods for solving nonlinear equations. New scheme of methods containing three steps and five functions, one of them is weighted function.Although the new families are not optimal methods, but it is efficiency index is better than the Steffensen'smethod,Malik's, Al-Fahid's and Fayyaz's methods and equal to the efficiency index of tenth-order of Hafiz's method and no derivative need. Finally, Table2 comparing the other methods using a lot of numerical examples to explain the convergence of the new methods.

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