



Using Mathematical Foundations To Study The Equivalence Between Mass And Energy In Special Relativity

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Abstract: This paper study the equivalence between mass and energy in special relativity, using mathematical methods to connect this work by de-Broglie equation, in this work found the relation between the momentum and energy, It has also been connect the mass and momentum and the speed of light in the energy equation, moreover it has been found that the relative served as an answer to a logical relationship de-Broglie through equivalence relationship between mass and energy.

I. Introduction

Energy plays an important role in the special and general relativity theory, it was also study of energy and its relationship by momentum and mass and speed of light, the general relativity study of Einstein field equation to solve many problem like Horizon, Single and surface. Here we use equations to make relations between the equations of spatial relativity and the equation the de-Broglie hypothesis, also the tensors used as theoretical background.

A. Theoretical background

1-Tensors:

The laws of nature are independent of the coordinates used to describe them mathematically systems if they are acceptable. The study of the results of these requirements led to the subject of tensors analysis used in the study of the theory of general relativity, differential geometry, mechanics, fluid dynamics, flexibility, electromagnetic theory and a number of other fields in science and engineering. [6]

2-Multidimensional spaces:

Any point in space with three dimensions is a set of three numbers known coordinates and sets this point to appoint a special system of coordinates or reference specific framework. For example, $p(x, y, z)$, $z(r, \theta, \phi)$ is a certain point in the coordinates of the rectangle system, cylindrical, spherical coordinates, respectively. Similarly, any point in the multidimensional space is a set of (N) number symbolizes the letters (x_1, x_2, \dots, x_N) .

3-Coordinates transfers:

Let the (x^1, x^2, \dots, x^N) , $(\bar{x}^1, \bar{x}^2, \dots, \bar{x}^N)$ Coordinates of a point in two different systems of coordinates. Let's assume that there is a number (N) of independent coordinate's relations between the two systems have the following formulas:

$$\begin{aligned} x^{-1} &= x^{-1}(x^1, x^2, \dots, x^N) \\ x^{-2} &= x^{-2}(x^1, x^2, \dots, x^N) \\ &\vdots \\ x^{-N} &= x^{-N}(x^1, x^2, \dots, x^N) \end{aligned} \quad (3.1)$$

These relations can be expressed in the following general formula:

$$\bar{x}^{-k} = \bar{x}^{-k}(x^1, x^2, \dots, x^N) ; k = 1, 2, \dots, N \quad (3.2)$$

It is supposed to be included here functions single value, continuous, continuous derivatives. If adversely any group coordinates corresponding set single (x_1, x_2, \dots, x_N) is given as follows:

$$x^k = \bar{x}^k(\bar{x}^1, \bar{x}^2, \dots, \bar{x}^N) \quad k = 1, 2, \dots, N \quad (3.3)$$

Know relations (3.2) and (3.3) diversion of the coordinates of a frame of reference to another.

4 - The term combination:

When you write a formula $S = a_1x^1 + a_2x^2 + \dots + a_Nx^N$ we can use such symbols as follows

$$S = \sum_{j=1}^N a_j x^j$$

We can write this combined process as follows: $S = a_j x^j$ Where we have chosen terminology, which decides when it was the index (below or above) repeater in a certain extent, it combines (1) and even (N) unless specified something else. This is known as a combination. It is clear that can be used as an indicator of any other character instead of (j), for example. Any indication repeats at a certain point, where it applies the term combination, called a silent indicator (dummy).

For the index, which writes once a certain extent, it is called a free index (free index), and can be any number (1, 2, ..., N) such as (k) in equation (2.3) or (3.3), which represents any than the number (N) of the equations.

5-Vector compatible covariance and staggered variation:

If we have a number (N) of the quantities (A_1, A_2, \dots, A_N) in a given system coordinates (x_1, x_2, \dots, x_N) linked to the number (N) of the other quantities in another system by the following coordinates conversion relations:

$$\bar{A}^p = \sum_{q=1}^N \frac{\partial \bar{x}^p}{\partial x^q} A^q ; p= 1, 2, \dots, N \quad (5.1)$$

Which can be simplified by using the term combination as follows:

$$\bar{A}^p = \frac{\partial \bar{x}^p}{\partial x^q} A^q \quad (5.2)$$

These are called relations: vectors vehicles staggered covariance (contra variant), or tensors staggered covariance from Order 1. If there is a number (N) of the quantities (A_1, A_2, \dots, A_N) in the coordinate system (x_1, x_2, \dots, x_N) linked to the number (N) of the other quantities in another system by the following coordinates conversion relations:

$$\bar{A}_p = \sum_{q=1}^N \frac{\partial x^q}{\partial \bar{x}^p} A_q , p = 1, 2, \dots, N \quad (5.3)$$

Or write

$$\bar{A}_p = \frac{\partial x^q}{\partial x^p} A_q \quad (5.4)$$

These are called relations: Vehicles vector compatible covariance (covariant), or tensors compatible covariance from Order 1.

If there is a number (N) Quantities (A_1, A_2, \dots, A_N) In the coordinate system (x_1, x_2, \dots, x_N) Linked to the number (N) of the other quantities ($\bar{A}_1, \bar{A}_2, \dots, \bar{A}_N$) in another system coordinates ($\bar{x}^1, \bar{x}^2, \dots, \bar{x}^N$) by following the conversion relationships:

$$\bar{A}_p = \sum_{q=1}^N \frac{\partial x^q}{\partial x^p} A_q, \quad p=1, 2, \dots, N \quad (5.3)$$

Or write

$$\bar{A}_p = \frac{\partial x^q}{\partial x^p} A_q \quad (5.4)$$

These are called relations: Vehicles vector compatible covariance (covariant), or Metadata compatible covariance from Order 1. Note that the index epitaxial (superscript) is used to denote the covariance alteration vehicles, while the underlying index is used (subscript) to denote the covariance compatible vehicles, rather than talk about tensors their compounds are:

A^p, A_p : Simply going to say that the period is: A^p or A_p .

6- Alteration compatible covariance and staggered heterogeneity and heterogeneous:

If we have a number (N2) of the quantities A^{qs} in a certain system coordinates (x_1, x_2, \dots, x_N) so linked to a similar number of other quantities \bar{A}^{pr} in another system coordinates ($\bar{x}^1, \bar{x}^2, \dots, \bar{x}^N$) by following the conversion relationships:

$$\bar{A}^{pr} = \sum_{q=1}^N \sum_{s=1}^N \frac{\partial \bar{x}^p}{\partial x^q} \frac{\partial \bar{x}^r}{\partial x^s} A^{qs}; \quad p, r = 1, 2, \dots, N \quad (6.1)$$

or write, according to the term combination:

$$\bar{A}^{pr} = \frac{\partial \bar{x}^p}{\partial x^q} \frac{\partial \bar{x}^r}{\partial x^s} A^{qs} \quad (6.2)$$

These are called relations: staggered covariance for tensors from Order 2 vehicles.

Called quantities A^{qs} (N2 and number): AV compatible covariance tensors to from Order 2, if:

$$\bar{A}_{pr} = \frac{\partial x^q}{\partial \bar{x}^p} \frac{\partial x^s}{\partial \bar{x}^r} A_{qs} \quad (6.3)$$

Similarly, the quantities (which number N2) called: tensors vehicles alloy (mixed tensor) from Order 2), if:

$$\bar{A}_r^p = \frac{\partial \bar{x}^p}{\partial x^q} \frac{\partial x^s}{\partial \bar{x}^r} A_s^q \quad (6.4)$$

7- Delta Kronecker:

Known Kronecker delta (δ_k^j) as follows:

$$\delta_k^j = \begin{cases} 0 & ; j \neq k \\ 1 & ; j = k \end{cases} \quad (7.1)$$

It is clear that Kronecker delta (δ_k^j) is an tensors alloy (mixed tensor) from Order 2.[7]

8- Tensor of the larger ranks of 2:

It can be easily defined Tensor of the larger of the two ranks, eg A_{kl}^{qst} alloy is tensors from level 5 (staggered covariance from Order 3, and compatible covariance from Order 2). And carried out the conversion process based on the following relationship:

$$\bar{A}_{ij}^{pmm} = \frac{\partial \bar{x}^p}{\partial x^q} \frac{\partial \bar{x}^r}{\partial x^s} \frac{\partial \bar{x}^m}{\partial x^t} \frac{\partial x^k}{\partial \bar{x}^i} \frac{\partial x^l}{\partial \bar{x}^j} A_{kl}^{qst} \quad (8.1)$$

9-Standard quantities or no invariant:

Let's assume that (ϕ) is a function of the coordinates (x^k), and take ($\bar{\phi}$) to symbolize the value of the deltoid when converting to a new set of coordinates (\bar{x}^k). Called (ϕ) amount of numerical (scalar) tensor (invariant) for converting coordinates if: $\bar{\phi} = \phi$. Called numerical quantity (or invariant) also: tensors from Order 0.

10-Areas tensor (tensor fields):

If we have for any point of the area in the multidimensional space matched by atensors knowledge, we say that the room for tensors has been defined, or room for vectors (if the period from Order 1), or room standard amounts (if they are tensors from Order 0). Note that an tensors, or room for tensors, not just a group of vehicles in a certain system of coordinates, but all possible groups include the conversion of coordinates procedures.

11- Symmetric and asymmetric tensors:

Called tensors: tensors identical (symmetric) for the two indicators are invariant covariance or invariant covariance, if vehicles remained unchanged when switching indicators. If any: $A_{qs}^{mpr} = A_{qs}^{pmr}$, The similar period in the m and p . If similar tensors for any indicators different Covariance or different covariance, it is called: identical (symmetric).
Called tensors: Do not identical (skew symmetric) for the two indicators different or different covariance if the reference compounds change when you switch the indicators or evidence. Thus, if: $A_{qs}^{mpr} = A_{qs}^{pmr}$, the period is not identical in p and m . If not tensors similar for any two indicators different or different covariance, it is called: Do not identical (skew symmetric).[8]

12. Basic operations of tensors:

A / combination: Collect two tensors - or more - of the same grade and type the same process (ie have the same number of covariance different indicators and indicators compatible covariance) is also tensors from the same grade and the same type.

Thus, if: A_q^{mp} and B_q^{mp} are tensors, then:

$$C_q^{mp} = A_q^{mp} + B_q^{mp} \quad (12.1)$$

It is also tensors.

Combining the are process is commutative property (commutative) and synthesis (associative.)

B / subtraction process: the difference between tensors of the same grade and the same type is also tensors from the same grade and the same type.

Thus, if: A_q^{mp} and B_q^{mp} tensors and two, the difference between them D_q^{mp} is also tensors.

$$D_q^{mp} = A_q^{mp} - B_q^{mp} \tag{12.2}$$

C / external battery: The multiplication of tensors is a new tensors arranged by the sum of ranks are Tensors and gives. Know this multiplication process, which includes hit unusually tensors to vehicles, known as the outer beat (outer product). For example, we write the external battery for tensors and the following form:

$$A_q^{pr} B_s^m = C_{qs}^{pmm} \tag{12.3}$$

Note that you can write any tensors terms multiplied e rank infrastructure, and for this reason, the dividing tensors is not always possible.

D / downsizing or shorthand: If we make two indexes, one heterozygous covariance and other compatible covariance, for an tensors equal, the output indicates that the collection process and equal indicators are taken on the basis of the term combination. The combined output of this form is tensors with a rank lower than the original two tensors. This process is known shorthand (contraction).

For example: A_{qs}^{mpr} , in the period of level 5, take: $s = r$, for

$$B_q^{mp} = A_{qr}^{mpr}, \text{ Which it tensors from Order 3.}$$

If we take too: $p = q$, we get: It tensors from Order 1 (vector).

h / beating internal (inner product): conducting external multiplication of tensors then make a reduction of output, we get to know a new tensors an output of the internal battery are tensors, this process is called a process

of internal beatings. For example, if we have are tensors A_q^{mp} and B_{st}^r , The external product $A_q^{mp} B_{st}^r$ Taking:

$q = r$ and $p = s$, we get the internal another product $A_r^{mp} B_{pt}^r$.

The processes of internal and external battery for tensors is commutatively and associative property.

o/ quotient Act (quotient law): Suppose we do not know whether the quantity(x) tensors or not. If possible, conduct an internal multiplication of quantity(x) with tensors arbitrarily chosen so that we get an tensors amount, the amount to be(x) tensors as well, and this is known as the quotient law.

13- matrices:

The matrix A ($m \times n$) is the order of the amounts a_{pq} (called elements) are arranged in a number (m) of (rows) and the number (n) of ((columns, and writes as follows:

$$A = \begin{bmatrix} a_{11} a_{12} \dots a_{1n} \\ a_{21} a_{22} \dots a_{2n} \\ \vdots \\ a_{m1} a_{m2} \dots a_{mn} \end{bmatrix} \tag{13.1}$$

or abbreviated as follows:

$$A = (a_{pq}); p = 1, 2, \dots, m; q = 1, 2, \dots, n \tag{13.2}$$

If: $m = n$, the matrix be square matrix of rank n in m (or: m);. If: $m = 1$, the matrix be descriptive or descriptive vector matrix. If the: $n = 1$, it will be vertical matrix or vector perpendicular.

Qatar called a square matrix: Diameter primary, or main diameter. Called square matrix elements of the country are one and the rest of the elements is zero, called the Matrix unit, symbolized by the letter (1). The matrix is zero (0), they are all matrix elements are equal to zero.

14. matrix algebra:

If they are $A = (a_{pq})$, $B = (b_{pq})$, two matrices have the same rank ($n \times m$), put it:

1. $A = B$, if and only if:.
2. holds reunion (S), and the difference between them (D) are also matrices are Known as follows:

$$S = A + B = (a_{pq} + b_{pq})$$

$$D = A - B = (a_{pq} - b_{pq})$$

2. multiplying matrices $P = AB$ known only when the number of columns (n) in the matrix (A) equal to the number of rows (m) in the matrix (B), and is given as follows:

$$P = AB = (a_{pq})(b_{pq}) = (a_{pr}b_{rq})$$

Where we write method combination:

$$a_{pr}b_{rq} = \sum_{r=1}^n a_{pr}b_{rq}$$

It called arrays that have hit quotient know: matrices harmonious (comfortable).

Overall, the multiplication of matrices are not mutually exclusive $AB \neq BA$, and in any case, the aggregate of the process of multiplication of matrices law be investigators $A(BC) = (AB)C$. Provided that such matrices be consistent. Also be distributive law investigator:

$$A(B + C) = AB + AC$$

$$(A + B)C = AC + BC$$

- 4- symbolizes the specific square matrix $A(a_{pq})$ symbol: $\det A$, or: $\det(a_{pq})$ if: $P = AB$, the: $\det P = \det A \cdot \det B$

- 5- inverse square matrix (A) is the matrix as (A^{-1}) such that $AA^{-1} = I$

The requirement is necessary or sufficient for the existence of the matrix (A^{-1}) is to be: $\det A \neq 0$. If the: $\det A = 0$, called matrix (A) a single matrix (singular).

- 6- multiplied by the number of measurement (λ) in a matrix $A = (a_{pq})$ symbolized to him (λA) , is a matrix (λa_{pq}) where any element which multiplied by (λ)

- 7- Moved matrix A is the matrix (A^T) and are formulated from the matrix A switch ranks and columns. Thus, if: $A = (a_{pq})$, the $A^T = (a_{qp})$. The Moved ((transpose matrix also has the symbol \tilde{A}).

15. Line element and metric tensor:

We write the differential arc length (ds) of the rectangular coordinate system (x, y, z) as follows:

$$ds^2 = dx^2 + dy^2 + dz^2$$

Such a space known as Euclidean space with three dimensions.

Generalizing to a multi-dimensional space (x^1, x^2, \dots, x^N) know-line (ds) element in this space is given by a formula from the second division, called the standard formula (metric form), or standard (metric).

$$ds^2 = \sum_{p=1}^N \sum_{q=1}^N g_{pq} dx^p dx^q$$

Or write, according to the term combination:

$$ds^2 = g_{pq} dx^p dx^q$$

In the case where there is a transfer of the coordinates of the x^j to \bar{x}^k so turned to the standard formula $(d\bar{x}^1)^2 + (d\bar{x}^2)^2 + \dots + (d\bar{x}^N)^2$, the space then called Eqldianmulti-dimensional space. In the general state space called Riemannian space ((Riemannian).

The quantities (g^{pq}) are vehicles for an tensors compatible covariance order 2 is called the tensors standard (metric tensor) or tensors basic (fundamental tensor). This period can be selected to be always identical (symmetric).

16. Escort or inverted tensors:

Let's take $g = \det g^{pq}$ To symbolized the specific elements of (g^{pq}), and assume that $g \neq 0$. We know g^{pq} the following form:

$$g^{pq} = \frac{\text{cofactor of } g_{pq}}{g}$$

So it g^{pq} is an tensors staggered covariance replica of the Order 2, called the tensors accompaniment (conjugate), or an inverted tensors (reciprocal) g^{pq} . You can clarify that:

$$g^{pq} g_{rq} \equiv \delta_r^p$$

$$A_{.q}^p = g^{rp} A_{rq} ; A^{pq} = g^{rp} g^{sq} A_{rs} ; A_{.rs}^p = g_{rq} A_{..s}^{pq}$$

$$A_{..n}^{qm.tk} = g^{pk} g_{sn} g^{rm} .A_{.r..p}^{q.st}$$

This becomes clear if we interpret as g^{rp} meaning tensors multiplication. Take $p = r$ (or $r = p$) in whichever continued and the highest this indicator. Similarly we interpret as g_{rq} meaning in the process of beating. Take $q = r$ (or $q = r$) At whichever is lower, and continued this indicator. tensors all we got from the tensors given to conduct internal multiplication standard Companions and

Companions called tensors associated or shared Associated)) for an tensors given. For example, A^m and A_m two tensorsCompanions, first: staggered covariance, and the second: Compatible covariance. It gives the mark between tensors as follows:

$$A_p = g_{pq} A^q$$

or

$$A^p = g^{pq} A_q$$

In order to be g^{pq} rectangular coordinates if $p = q$. And $g_{pq} = 0$ whether $p \neq q$, therefore $A_p = A^p$. In this case there is no discrimination between aredifferent covariance vehicles and vehicles compatible variation vector.

18- Length of the vector and angle between two vectors:

The amount $A^p B_p$ an inner product the heading A^p and B_p the process, which is a record amount, such as a standard multiplication in rectangular coordinates. They know the length (L) of the vector A^p or A_p the following relationship:

$$L^2 = A^p A_p = g^{pq} A_p A_q = g_{pq} A^p A^q$$

It can be defined A^p and B_p between the angle θ and the following form:

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$$\cos\theta = \frac{A^p B_p}{\sqrt{(A^p A_p)(B^p B_p)}}$$

19- Physical vehicles:

Physical vehicle A^p or A_p vector (denoted as: A_u, A_v, A_w) is a vector falls on tangents of the curves representing the coordinates. The coordinates given in the perpendicular (orthogonal) the following relationships:

$$A_u = \sqrt{g_{11}} \quad A^1 = \frac{A^1}{\sqrt{g_{11}}}$$

$$A_v = \sqrt{g_{22}} \quad A^2 = \frac{A^2}{\sqrt{g_{22}}}$$

$$A_w = \sqrt{g_{33}} \quad A^3 = \frac{A^3}{\sqrt{g_{33}}}$$

Similarly, the physical vehicles for tensors A^{pq} or A_{pq} given to the following relationships:

$$A_{uu} = g_{11} \quad A^{11} = \frac{A^{11}}{g_{11}}$$

$$A_{uv} = \sqrt{g_{11}g_{22}} \quad A^{12} = \frac{A_{12}}{\sqrt{g_{11}g_{22}}}$$

$$A_{uw} = \sqrt{g_{11}g_{33}} \quad A^{13} = \frac{A_{13}}{\sqrt{g_{11}g_{33}}} \dots \text{etc.}$$

20- Christoffel symbols:

The symbols

$$[pq, r] = \frac{1}{2} \left[\frac{\partial g_{pr}}{\partial x^q} + \frac{\partial g_{qr}}{\partial x^p} - \frac{\partial g_{pq}}{\partial x^r} \right]$$

$$\left\{ \begin{matrix} s \\ pq \end{matrix} \right\} = g^{sr} [pq, s]$$

Known as Christoffel symbols of first and second order type. There are other symbols are used instead

$$\left\{ \begin{matrix} s \\ pq \end{matrix} \right\} \text{ of } \{pq, s\} \text{ and } \Gamma_{pq}^s.$$

The symbol Γ_{pq}^s looks like a form of tensors, but generally not the case.

21- Conversion Rules Christoffel symbols:

In the case of the coordinate system we write \bar{x}^k :

$$[\overline{jk}, m] = [pq, r] = \frac{\partial x^p}{\partial \bar{x}^j} \frac{\partial x^q}{\partial \bar{x}^k} \frac{\partial x^r}{\partial \bar{x}^m} + g_{pq} \frac{\partial x^p}{\partial \bar{x}^m} \frac{\partial^2 x^q}{\partial \bar{x}^j \partial \bar{x}^k}$$

$$\left\{ \begin{matrix} n \\ jk \end{matrix} \right\} = \left\{ \begin{matrix} s \\ pq \end{matrix} \right\} \frac{\partial \bar{x}^n}{\partial x^s} \frac{\partial x^p}{\partial \bar{x}^j} \frac{\partial x^q}{\partial \bar{x}^k} + \frac{\partial \bar{x}^p}{\partial x^q} \frac{\partial^2 x^q}{\partial \bar{x}^j \partial \bar{x}^k}$$

They conversion laws to Christoffel symbols with surrounding it is not tensors what were not the second border in the right-hand side equal to zero.

22- Geodesics:

It gives the distance (S) between two points (t_1) and (t_2) the curve $x^r = x^r(t)$: in the space of (Riemanian) the following relationship:

$$S = \int_{t_1}^{t_2} \sqrt{g_{pq} \frac{dx^p}{dt} \frac{dx^q}{dt}} dt$$

It called a curve in space, which makes the distance between any two points in it as little as possible, so-called geodesic space. Using an account that changes the geodesics exist of the following differential equation:

$$\frac{d^2 x^r}{ds^2} + \left\{ \begin{matrix} r \\ pq \end{matrix} \right\} \frac{dx^p}{ds} \frac{dx^q}{ds} = 0$$

Where (S) is the arc length. As examples geodesics on the surface level is straight lines, while geodesics be a spheroid is a parenthesis.

23- The covariant derivative:

Know compatible derivative of covariance tensors A_p for as x^q follows:

$$A_{p,q} \equiv \frac{\partial A_p}{\partial x^q} - \Gamma_{pq}^s A_s$$

is compatible tensors covariance P 2.

And know-derived compatible variation of tensors for as follows:

$$A_{,q}^p = \frac{\partial A^p}{\partial x^q} + \Gamma_{qs}^p A^s$$

It tensors its mixture of Order 2.

For systems rectangular coordinates, the Christoffel symbols equal to zero and be compatible derivatives covariance an ordinary partial derivatives.

The result can be generalized to the above derivatives compatible variation of tensors from the upper ranks.

Thus, we write:

$$A_{r_1 \dots r_n, q}^{p_1 \dots p_m} = \frac{\partial A_{r_1 \dots r_n}^{p_1 \dots p_m}}{\partial x^q} - \Gamma_{r_1 q}^s A_{sr_1 \dots r_n}^{p_1 \dots p_m} - \Gamma_{r_2 q}^s A_{r_1 sr_3 \dots r_n}^{p_1 \dots p_m} \dots + \Gamma_{qs}^{p_1} A_{r_1 \dots r_n}^{sp_2 \dots p_m} + \Gamma_{qs}^{p_2} A_{r_1 \dots r_n}^{p_2 sp_3 \dots p_m} + \dots + \Gamma_{qs}^{p_m} A_{r_1 \dots r_n}^{p_1 \dots p_m, s}$$

It is derived compatible variation of tensors $A_{r_1 \dots r_n}^{p_1 \dots p_m}$ for a variable x^q .

The differentiation rules compatible variation of the total crops or crops producttensors are the same like a normal differentiation.

When you make a differentiation processes, perhaps tensors deal $g_{pq} \cdot g^{pq}$ and δ_q^p as long as the parameters that derivatives compliant covariance equal to zero. As the derivatives compatible covariance reflect the rates of change of independent natural amounts at any frames of reference, therefore, it is of great importance in the expression of the laws of nature.

24- Permutations symbols and tensors:

Know e_{pqr} the following relationships:

$$e_{123} = e_{231} = e_{312} = +1 \text{ Periodical}$$

$$e_{213} = e_{132} = e_{321} = -1 \text{ Non-periodical}$$

$$e_{pqq} = e_{ppq} = e_{prr} = \dots = 0$$

If two or more equal indicators. And you know e^{pqr} in the same way. Icons e_{pqr} and e^{pqr} symbols known permutations in space with three dimensions. Moreover, to know the following tensors:

$$\epsilon_{pqr} = \frac{1}{\sqrt{g}} e_{pqr}$$

$$\epsilon^{pqr} = \sqrt{g} e^{pqr}$$

It is compatible tensors covariance and covariance staggered in order, and you know tensors reciprocity in the space of a three- dimensional circulating that the multiple dimensions.

25- Divergent tensor formula:

1. Gradient: if it ϕ is a record amount (or not variable), the regression ϕ is known as follows:

$$\nabla \phi = grad \phi = \phi_{,p} = \frac{\partial \phi}{\partial x^p}$$

Where ϕ_p it is derived compatible variation of the amount ϕ for the variable x^p .

2- Divergence: The divergence A^p is to reduce the Compliant compatible covariance for the variable x^q .

Any reduction for A^p_q . So we write:

$$div A^p = A^p_{,p} = \frac{1}{\sqrt{g}} \frac{\partial(\sqrt{g} A^k)}{\partial x^k}$$

3- curl: The curl A_p is:

$$A_{p,q} - A_{q,p} = \frac{\partial A_p}{\partial x^q} - \frac{\partial A_q}{\partial x^p}$$

It is a tensor of Order 2.
curl also known as follows:

$$- \epsilon^{pqr} A_{p,q}$$

4- Laplace coefficient Laplacian: The Laplace operator function ϕ is curl to divergent.

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$$\nabla^2 \phi = \text{div } \phi_{,p} = \frac{1}{\sqrt{g}} \frac{\partial(\sqrt{g} g^{ik} \frac{\partial \phi}{\partial x^k})}{\partial x^j}$$

In the case of: $g < 0$, the \sqrt{g} extent must be replaced $\sqrt{-g}$. Either way $g > 0$, and $g < 0$ you can have $\sqrt{|g|}$ rather than instead of \sqrt{g} .

26- Self-derivative or derivative absolute:

Derivative self or the absolute amount A_p of the derivative along the curve $x^q = x^q(t)$

$\frac{\delta A_p}{\delta t}$ [symbolized by the laboratories δt] known as a blow to internal derivative compatible variation of the amount

$\frac{dx^q}{dt}$ and A_f Which $A_{p,q} \frac{dx^q}{dt}$ and it is given as follows:

$$\frac{\delta A_p}{\delta t} \equiv \frac{dA_p}{dt} - \Gamma_{pq}^r A_r \frac{dx^q}{dt}$$

Similarly you know:

$$\frac{\delta A^p}{\delta t} \equiv \frac{dA^p}{dt} + \Gamma_{qr}^p A^r \frac{dx^q}{dt}$$

It is said that two vectors A_p and A^p in parallel Moves along the curve if the derivatives self along the curve is zero, respectively. Similarly, the definition can be self-derivatives to tensors from the upper ranks.

27- Tensors relative and absolute tensor:

Called for an tensors $A_{r_1 \dots r_n}^{p_1 \dots p_m}$ period of relative weight (ω), if the vehicles shift based on the following equation:

$$\bar{A}_{s_1 \dots s_n}^{q_1 \dots q_m} = \left| \frac{\partial x}{\partial \bar{x}} \right|^\omega A_{r_1 \dots r_n}^{p_1 \dots p_m} \frac{\partial \bar{x}^{q_1}}{\partial x^{p_1}} \frac{\partial x^{r_1}}{\partial \bar{x}^{s_1}} \dots \frac{\partial x^{r_n}}{\partial \bar{x}^{s_n}}$$

$$J = \left| \frac{\partial x}{\partial \bar{x}} \right|$$

Where J is the conversion coefficient of Jacob.

If: $\omega = 0$, the tensors period called absolute, which is kind of tensors who care about him.

If: $\omega = 1$, the relative period known as massively tensors. The combination, beatings, ... etc. Relative to tensors are like those conducted for absolute tensors.

B. The de Broglie hypothesis

Says de Broglie “the wave length behavior does not shorten on the photons but extends includes the density of material particles such as electrons and protons, neutrons[2] and here in this paper we will write the conclusion the relationship between the wavelength with the amount of movement as hypothetical de-Broglie We start from the atomic physics:

$$L = p \cdot r = n\hbar = \frac{n\hbar}{2\pi}$$

but $p = mv$

$$mvr = n\hbar = \frac{n\hbar}{2\pi}$$

$$pr = \frac{n\hbar}{2\pi}$$

$$2\pi pr = n\hbar$$

$$2\pi r = \frac{n\hbar}{p}$$

From Bohr–Sommerfeld quantization [1] the path of the electrons around the nucleus to be stable if it is equal to an integer of the wavelength

$$n\pi r = n\lambda$$

$$n\lambda = \frac{n\hbar}{p}$$

$$\lambda = \frac{h}{p}$$

The equivalence between mass and energy in special relativity:

We find that the relative served an logically answer for de Berogli relationship through the equivalence relationship between mass and energy

There for We write

$$E^2 = p^2c^2 + m_0^2c^2$$

$$m_0 = 0$$

$$\text{then } E^2 = p^2c^2$$

$$E = PC$$

$$\text{Then } p = \frac{E}{c}$$

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda}$$

$$\text{then } \lambda = \frac{h}{p}$$

Also we can write :

$$E^2 = p^2c^2 + m_0^2c^2$$

Take [3,4, 5];

$$m_0 = 0$$

Then

$$E^2 = p^2c^2$$

$$E = PC$$

$$p = \frac{E}{c}$$

but $E = mc^2$

$$p = \frac{mc^2}{c} = mc$$

This relation clear theequivalence between mass and momentum which agrees with de Brogli hypothesis. The equivalence between mass and energy:

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