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Research Paper

Effect of an Inclined Magnetic Field on Peristaltic Flow of Williamson Fluid in an Inclined Channel

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ABSTRACT: *This paper deals with the influence ofinclined magnetic field on peristaltic flow of an incompressible Williamson fluid in an inclined channel with heat and mass transfer. Viscous dissipation and Joule heating are taken into consideration.Channel walls have compliant properties. Analysis has been carried out through long wavelength and low Reynolds number approach. Resulting problems are solved for small Weissenberg number. Impacts of variables reflecting the salient features of wall properties, concentration and heat transfer coefficient are pointed out. Trapping phenomenon is also analyzed.*
Keywords*: Williamson fluid,Reynolds number, compliant walls .*

I. INTRODUCTION

Peristalsis is an important mechanism generated by the propagation of waves along the walls of a channel or tube. Itoccurs in the gastrointestinal, urinary, reproductive tracts and many other glandular ducts in a living body.Most of the studies on the topic have been carried out for the Newtonian fluid for physiological peristalsis includingthe flow of blood in arterioles. But such a model cannot be suitable for blood flow unless the non-Newtonian natureof the fluid is included in it. The non-Newtonian peristaltic flow using a constitutive equation for a second order

fluid has been investigated by Siddiqui et al. [7] for a planar channel and by Siddiqui and Schwarz [9] for anaxisymmetric tube. They have performed a perturbation analysis with a wave number, including curvature andinertia effects and have determined range of validity of their perturbation solutions. The effects of third order fluidon peristaltic transport in a planar channel were studied by Siddiqui et al. [8] and the corresponding axisymmetrictube results were obtained by Hayat et al. [2]. Haroun [1] studied peristaltic transport of third order fluid in anasymmetric channel. Subba Reddy et al. [11] have studied the peristaltic flow of a power-law fluid in an asymmetricchannel. Peristaltic motion of a Williamson fluid in an asymmetric channel was studied by Nadeem and Akram [6].It is now well known that blood behaves like a magnetohydrodynamic (MHD) fluid (Stud et al. [10]). Blood is asuspension of cells in plasma. It is a biomagnetic fluid, due to the complex integration of the intercellular protein,cell membrane and the hemoglobin, a form of iron oxide, which is present at a uniquely high concentration in themature red cells, while its magnetic property is influenced by factors such as the state of oxygenation. Theconsideration of blood as a MHD fluid helps in controlling blood pressure and has potential for therapeutic use ithe diseases of heart and blood vessels (Mekheimer [5]). Peristaltic transport to a MHD third order fluid in a circularcylindrical tube was investigated by Hayat and Ali [3]. Hayat et al. [4] have investigated peristaltic transport of athird order fluid under the effect of a magnetic field. Recently, Subba Reddy et al. [12] have studied the peristaltictransport of Williamson fluid in a channel under the effect of a magnetic field.In view of these, we modeled the MHD peristaltic flow of a Williamson fluid in an inclined planar channel, underthe assumptions of long wavelength. The flow is investigated in a wave frame of reference moving with velocity ofthe wave.

(1) Mathematical formulation

Conseder an incompressible magnetohydrodynamic (MHD) flow of Williamson fluid in a symmetric channel of width $2d_1$. Both the magnetic field and channel are inclined at angle $\varphi$and α. Here x-axis is taken along the length of channel and y-axis transvers to it (see Fig 1). The induced magnetic field is neglected by assuming a very small magnetic Reynolds number .Also the electric field is taken absent .the flow is generate by sinusoidal waves propagating along the compliant walls of channel:

$\bar{y} = \pm\bar{\eta}(\bar{x},\bar{t}) = \pm[d + \bar{m}\bar{x} + a\ sin\frac{2\pi}{\lambda}(\bar{x} - c\bar{t})]$ (1)
Where a is the wave amplitude ,λ the wavelength , c the wave

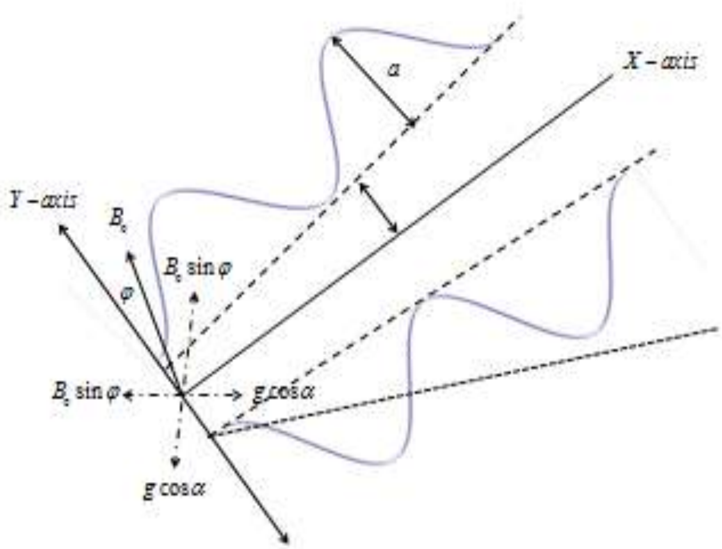


Figure 1 Diagrammatic of the problem

In wave frame, the equations which govern the flow are given by:
$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0$ (2)
$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial}{\partial \bar{x}}\left[2\mu_0(1+\Gamma\bar{\dot{\gamma}})\frac{\partial \bar{u}}{\partial \bar{x}}\right] + \frac{\partial}{\partial \bar{y}}\left[\mu_0(1+\Gamma\bar{\dot{\gamma}})\left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}\right)\right] + \sigma {B_0}^2\cos(\varphi)\,(\bar{u}\cos(\varphi) - v\sin\varphi + \rho g sin(\alpha) - \mu k u$ (3)
$\rho\left(\bar{v}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial}{\partial \bar{x}}\left[\mu_0(1+\Gamma\bar{\dot{\gamma}})\left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)\right] + \frac{\partial}{\partial \bar{y}}\left[2\mu_0(1+\Gamma\bar{\dot{\gamma}})\frac{\partial \bar{v}}{\partial \bar{y}}\right] + \sigma {B_0}^2\sin(\varphi)\,(\bar{u}\cos(\varphi) - v\sin\varphi - \rho g cos(\alpha) - \mu k v$ (4)
$\rho c_p\frac{dT}{d\bar{t}} = k\left[\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right] + \mu_0(1+\Gamma\bar{\dot{\gamma}})[(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}})^2 + 2(\frac{\partial \bar{u}}{\partial \bar{x}})^2 + 2\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2\Big] + \sigma {B_0}^2(\bar{u}^2 cos^2(\varphi) + \bar{v}^2 sin^2(\varphi) - 2uv\sin\varphi\cos(\varphi)$ (5)

where ρ is the density, B_0 is the strength of the magnetic field, $\bar{p}$ is the pressure , $(\bar{u},\bar{v})$ are the components of the velocity vector$\vec{V}$ in the$\bar{x}$ and $\bar{y}$ directions respectively and σ the electrical conductivity . The corresponding boundary condition are given by
$\bar{u} = \pm\bar{\beta}\frac{\partial \bar{u}}{\partial \bar{y}} at \bar{y} = \pm\bar{\eta}$(6)
$$\left[-\tau\frac{\partial^3}{\partial \bar{x}^3} + m\frac{\partial^3}{\partial \bar{x}\partial \bar{t}^2} + \tilde{d}\frac{\partial^2}{\partial \bar{x}\partial \bar{t}} B\frac{\partial^5}{\partial \bar{x}^5} + H\frac{\partial}{\partial \bar{x}}\right](\bar{\eta})$$
$= \frac{\partial}{\partial \bar{x}}2\mu_0(1+\Gamma\bar{\dot{\gamma}})\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial}{\partial \bar{y}}\left[\mu_0(1+\Gamma\bar{\dot{\gamma}})\left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)\right] - \sigma {B_0}^2\cos(\varphi)\,(\bar{u}\cos(\varphi) - \bar{v}\sin(\varphi)) + \rho g sin(\alpha) - \rho\frac{d\bar{u}}{d\bar{t}} at \bar{y} = \pm\bar{\eta}$(6)
$T = T_0 at \bar{y} = \pm\bar{\eta}$(7)
In above equations τ is elastic tension in the membrane ,m the mass per unit area , $\tilde{d}$ the coefficient of viscous damping and H the spring stiffness.

(2) Dimensionless analysis

Defining velocity component u and $v$in terms of stream function , introduce the dimensionless variables $x = \frac{\bar{d}}{\lambda}$,
$y = \frac{\bar{y}}{d}, \eta = \frac{\bar{\eta}}{d}$, $\epsilon = \frac{a}{d}$, $u = \frac{\bar{u}}{c}, v = \frac{\bar{v}}{c\delta}$, $\mathrm{Re} = \frac{cd\rho}{\mu_0}$,

$p = \frac{d^2\bar{p}}{c\lambda\mu_0}$, $t = \frac{c\bar{t}}{\lambda}$, $We = \Gamma\frac{c}{a}$,$\beta = \frac{\bar{\beta}}{d}$,$E_4 = \frac{Bd^3}{c\lambda^3\mu_0}$,$m = \frac{\lambda\bar{m}}{d}$,
$Br = \Pr Ec$,$Pr = \frac{\mu_0 C_P}{k}$, $Ec = \frac{c^2}{T_0 C_P}$, $E_5 = \frac{Hd^3}{c\lambda\mu_0}$, $\theta = \frac{T-T_0}{T_0}$, (8)
$E_1 = -\frac{\tau d^3}{c^3\mu_0 C}$,$E_2 = \frac{mcd^3}{\lambda^3\mu_0}$, $E_3 = \frac{d^3\tilde{d}}{\lambda^3\mu_0}$,$\bar{\dot{\gamma}} = \dot{\gamma}\frac{d}{c}$,$M = \sqrt{\frac{\sigma}{\mu_0}}\beta_0 d$,
And use long wave length assumption ($\delta \ll 1$) the notion equation and temperature equation can be written as
$\frac{\partial^2}{\partial y^2}\left[(1+We\dot{\gamma})\left(\frac{\partial^2\psi}{\partial y^2}\right)\right] - M^2cos^2(\varphi)\frac{\partial\psi}{\partial y} - \frac{1}{k}\frac{\partial\psi}{\partial y} = 0$(9)
$\frac{\partial^2\theta}{\partial y^2} + Br(1+We\dot{\gamma})(\frac{\partial^2\psi}{\partial y^2})^2 + Br\,M^2cos^2(\varphi)(\frac{\partial\psi}{\partial y})^2 = 0$(10)
$\frac{\partial\psi}{\partial y} = \pm\beta\frac{\partial^2\psi}{\partial y^2}\,at\ y = \pm\eta$ (11)
$\left[E_1\frac{\partial^3}{\partial x^3} + E_2\frac{\partial^3}{\partial x\partial t^2} + E_3\frac{\partial^2}{\partial x\partial t} + E_4\frac{\partial^5}{\partial x^5} + E_5\frac{\partial}{\partial x} + \right]\eta =$
$\frac{\partial}{\partial y}\left[(1+We\dot{\gamma})\left(\frac{\partial^2\psi}{\partial y^2}\right)\right] - M^2cos^2(\varphi)\frac{\partial\psi}{\partial y} + \frac{Re\sin(\alpha)}{Fr}at\ y = \pm\eta$(12)
$\theta = 0 \quad aty = \pm\eta$ (13)
$\dot{\gamma} = \frac{\partial^2\psi}{\partial y^2}$(14)
Here$\eta = 1 + \varepsilon\sin[2\pi(x-t)]$

(3)Solution procedure

It seems difficult to solve Eqs. (9) to (13) in closed form . Thus we aim to find series solutions for small Weissenberg number and write
$\psi = \psi_0 + We\,\psi_1$ (15)
$\theta = \theta_0 + We\,\theta_1$(16)
Here we have considered the zeroth and first order systems only due to the fact that the perturbation is taken small ($We \ll 1$) so that contrubation of higher order terms are negligible due to order analysis .

Zeroth-order system with its boundary conditions

Substitute (15) and (16) in to equations (9) to (14) and equating the coefficients of equal power of Weissenberg number we get zero and first order system and they are as :
$\frac{\partial^4\psi_0}{\partial y^4} - A^2\frac{\partial^2\psi_0}{\partial y^2} = 0$ (17)
$\left[E_1\frac{\partial^3}{\partial x^3} + E_2\frac{\partial^3}{\partial x\partial t^2} + E_3\frac{\partial^2}{\partial x\partial t} + E_4\frac{\partial^5}{\partial x^5} + E_5\frac{\partial}{\partial x} + \right]\eta =$
$\frac{\partial}{\partial y}\left[\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\right] - M^2cos^2(\varphi)\frac{\partial\psi_0}{\partial y} + \frac{Re\ sin\ \alpha}{Fr}at\ y = \pm\eta$(18)
$\frac{\partial\psi_0}{\partial y} = \pm\beta\frac{\partial^2\psi_0}{\partial y^2}at\ y = \pm\eta$(19)
$\frac{\partial^2\theta_0}{\partial y^2} + Br(\frac{\partial^2\psi_0}{\partial y^2})^2 + Br\,M^2cos^2(\varphi)(\frac{\partial\psi_0}{\partial y})^2 = 0(20)\theta_0 = 0 aty = \pm\eta$
$Here\ A^2 = M^2cos^2(\varphi) + \frac{1}{k}and\eta = 1 + \varepsilon\sin[2\pi(x-t)]$.

The first-order system with its boundary conditions

$\frac{\partial^4\psi_1}{\partial y^4} - A^2\frac{\partial^2\psi_1}{\partial y^2} + 2(\frac{\partial^3\psi_0}{\partial y^3})^2 + 2\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^4\psi_0}{\partial y^4} = 0$ (21)
$\frac{\partial\psi_1}{\partial y} = \pm\beta\frac{\partial^2\psi_1}{\partial y^2}at\ y = \pm\eta$(22)
$0 = [\frac{\partial^3\psi_1}{\partial y^3} - M^2cos^2(\varphi)\frac{\partial\psi_1}{\partial y} + 2\left(\frac{\partial^2\psi_0}{\partial y^2}\right)\left(\frac{\partial^3\psi_0}{\partial y^3}\right)]at\ y = \pm\eta$(23)
$\frac{\partial^2\theta_1}{\partial y^2} + Br\frac{\partial^2\psi_0}{\partial y^2}(2\frac{\partial^2\psi_1}{\partial y^2} + \left(\frac{\partial^2\psi_0}{\partial y^2}\right)^2) + 2Br\,M^2cos^2(\varphi)(\frac{\partial\psi_0}{\partial y})(\frac{\partial\psi_1}{\partial y}) = 0$
$\theta_1 = 0 \ aty = \pm\eta$(24)
$Here\ A^2 = M^2cos^2(\varphi) + \frac{1}{k}and\eta = 1 + \varepsilon\sin[2\pi(x-t)]$.
Solve the zero and first order systems it is found them the stream function and temperature are given by

$$\boldsymbol{\psi = \frac{e^{Ay}C_1 + e^{-Ay}C_2}{A^2} + C_3 + yC_4 +}$$
$$\boldsymbol{We(C_7 - \frac{e^{-2Ay}({C_2}^2 + {C_1}^2e^{4Ay} - 3e^{Ay}(e^{2Ay}C_5 + C_6))}{3A^2} + yC_8)(25)}$$

$$
\begin{aligned}
\boldsymbol{\theta} = {} & \boldsymbol{b_1 + y b_2 - \frac{1}{8A^4}}\mathbf{Br}e^{-2Ay}\Big(\boldsymbol{8AC_4}e^{Ay}(-\boldsymbol{C_2} + \boldsymbol{C_1}e^{2Ay})\boldsymbol{M^2} + (\boldsymbol{C_2}^2 + \boldsymbol{C_1}^2 e^{4Ay})\boldsymbol{M^2} \\
& + \boldsymbol{2A^4}e^{2Ay}(\boldsymbol{4C_1C_2} + \boldsymbol{C_4}^2\boldsymbol{M^2})\boldsymbol{y^2} + \boldsymbol{2A^2}(\boldsymbol{C_2}^2 + \boldsymbol{C_1}^2 e^{4Ay} - \boldsymbol{2C_1C_2}e^{2Ay}\boldsymbol{M^2y^2}) \\
& + \boldsymbol{M^2}\left(\boldsymbol{C_2}^2 + e^{2Ay}(\boldsymbol{8AC_1C_4}e^{Ay} + \boldsymbol{C_1}^2 e^{2Ay} + \boldsymbol{2A^4C_4}^2\boldsymbol{y^2})\right. \\
& \left. - \boldsymbol{4AC_2}e^{Ay}(\boldsymbol{2C_4} + \boldsymbol{AC_1}e^{Ay}\boldsymbol{y^2})\right)\mathbf{Cos}[\boldsymbol{2\varphi}]\Big) \\
& + \boldsymbol{We}\left(\frac{\begin{pmatrix}\mathbf{Br}e^{-3Ay}(\boldsymbol{10C_2}^3 - \boldsymbol{18C_1C_2}^2 e^{2Ay} \\ + \boldsymbol{C_1}e^{5Ay}(-\boldsymbol{27C_5} + \boldsymbol{10C_1}^2 e^{Ay}) \\ -\boldsymbol{9C_2}e^{Ay}(\boldsymbol{3C_6} + \boldsymbol{2C_1}^2 e^{3Ay}))\end{pmatrix}}{(\boldsymbol{54A^2})} - \atop \boldsymbol{Br(C_2C_5 + C_1C_6)y^2 + b_3 + yb_4}\right) \quad (\mathbf{26})
\end{aligned}
$$

Here the constants $\boldsymbol{C_1, C_2, C_3, C_4, C_4, C_6, C_7, C_8, b_1, b_2, b_3}$ ***and*** $\boldsymbol{b_4}$ can be evaluated using MATHEMATICA .

$(\boldsymbol{a})(\boldsymbol{b})(\boldsymbol{c})$

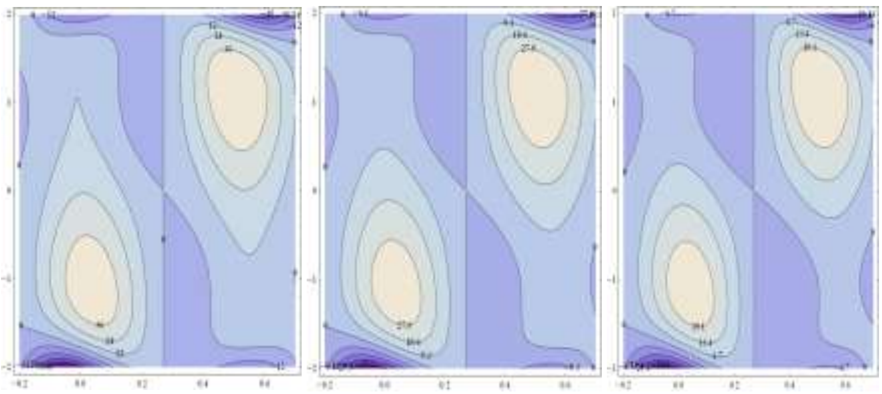


Figure 2- Stream line for different values of $E_1 (a) E_1 = 0.5$, $(b) E_1 = 0.7$, $(c) E_1 = 0.8$ and the other parameters $E_2 = 0.2$, $E_3 = 0.1$, $E_4 = 0.05$, $E_5 = 0.3$, $We = 0.03$, $\varepsilon = 0.2$, $t = 0.02$, $M = 1$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

(a) (b) (c)

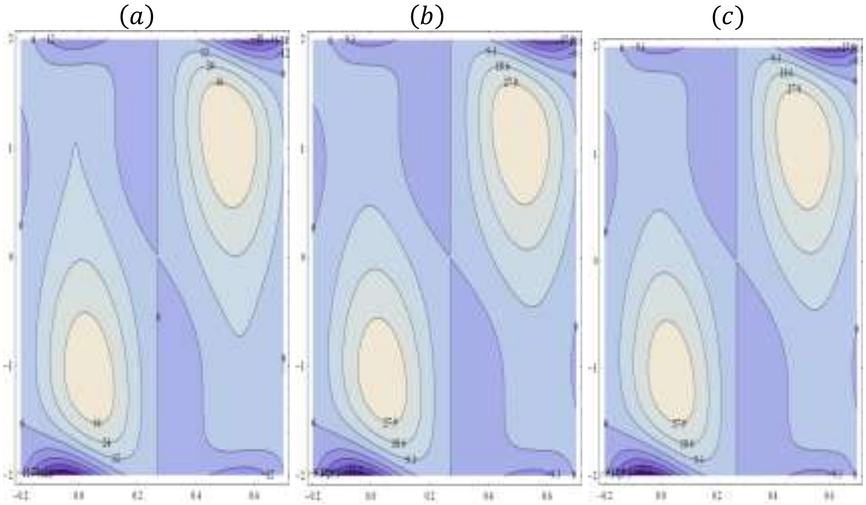


Figure 3- Stream line for different values of $E_2 (a) E_2 = 0.2$, $(b) E_2 = 0.4$, $(c) E_2 = 0.6$ and the other parameters $E_1 = 0.5$, $E_3 = 0.1$, $E_4 = 0.05$, $E_5 = 0.3$, $We = 0.03$, $\varepsilon = 0.2$, $t = 0.02$, $M = 1$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

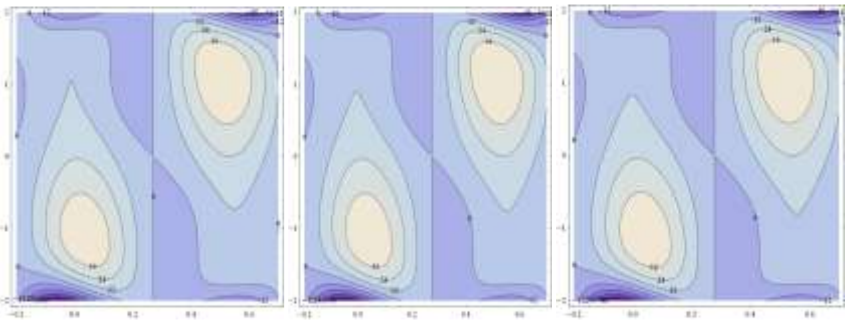


Figure 4- Stream line for different values of $E_3 (a) E_3 = 0.1$, $(b) E_3 = 0.2$, $(c) E_3 = 0.3$ and the other parameters $E_1 = 0.5$, $E_2 = 0.2$, $E_4 = 0.05$, $E_5 = 0.3$, $We = 0.03$, $\varepsilon = 0.2$, $t = 0.02$, $M = 1$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

(a) (b) (c)

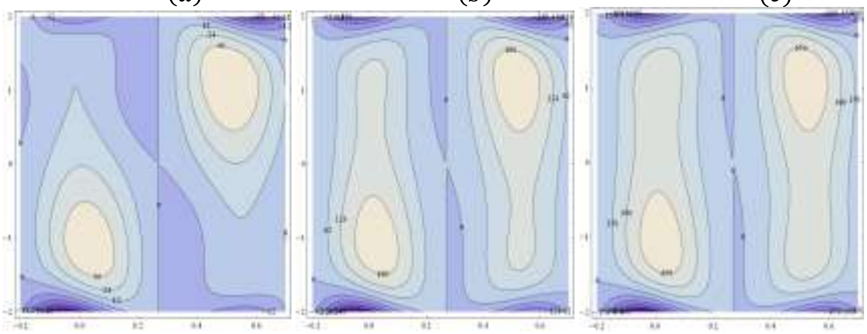


Figure 5- Stream line for different values of $E_4 (a) E_4 = 0.05$, $(b) E_4 = 0.10$, $(c) E_4 = 0.15$ and the other parameters $E_1 = 0.5$, $E_2 = 0.2$, $E_3 = 0.1$, $E_5 = 0.3$, $We = 0.03$, $\varepsilon = 0.2$, $t = 0.02$, $M = 1$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

(a) (b) (c)

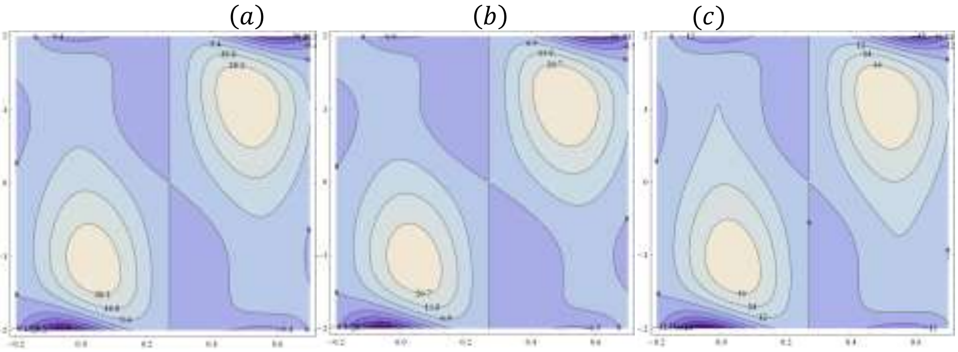


Figure 6- Stream line for different values of $E_5 (a) E_5 = 0.3$, $(b) E_5 = 0.6$, $(c) E_5 = 0.9$ and the other parameters $E_1 = 0.5$, $E_2 = 0.2$, $E_3 = 0.1$, $E_4 = 0.05$, $We = 0.03$, $\varepsilon = 0.2$, $t = 0.02$, $M = 1$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

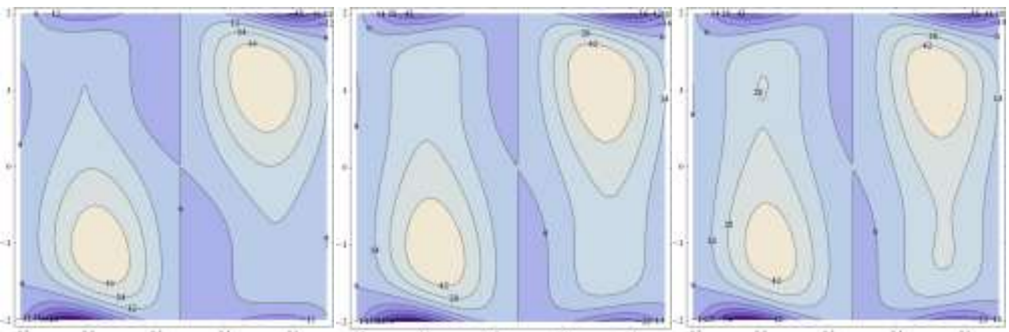


Figure 7- Stream line for different values of $We (a)\ We = 0.03$, $(b)\ We = 0.09$, $(c)\ We = 0.18$ and the other parameters $E_1 = 0.5$, $E_2 = 0.2$, $E_3 = 0.1$, $E_4 = 0.05$, $E_5 = 0.3$, $\varepsilon = 0.2$, $t = 0.02$, $M = 1$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

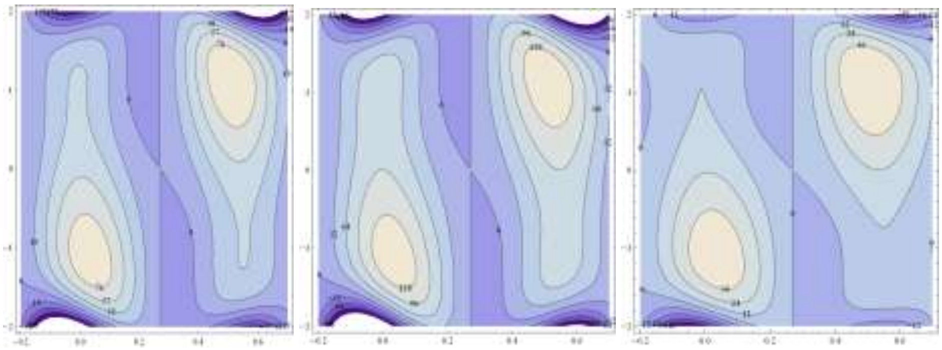


Figure 8- Stream line for different values of $\varepsilon (a)\ \varepsilon = 0.2$, $(b)\ \varepsilon = 0.4$, $(c)\ \varepsilon = 0.8$ and the other parameters $E_1 = 0.5$, $E_2 = 0.2$, $E_3 = 0.1$, $E_4 = 0.05$, $E_5 = 0.3$, $We = 0.03$, $t = 0.02$, $M = 1$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

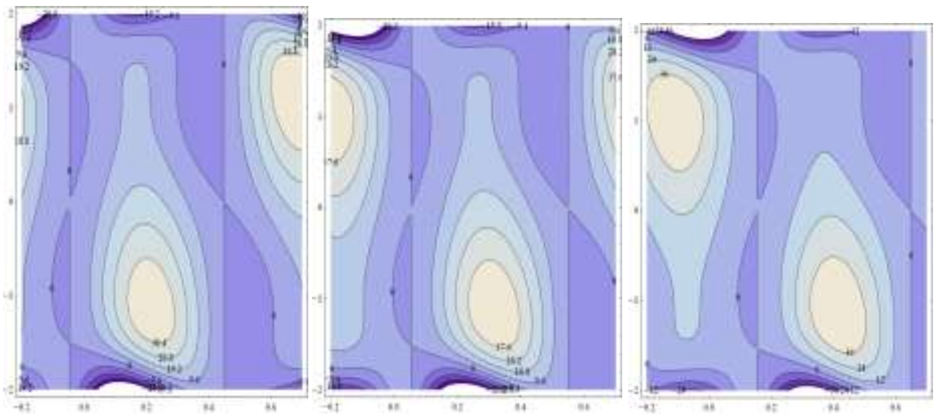


Figure 9- Stream line for different values of $t (a)\ t = 0.02$, $(b)\ t = 0.04$, $(c)\ t = 0.06$ and the other parameters $E_1 = 0.5$, $E_2 = 0.2$, $E_3 = 0.1$, $E_4 = 0.05$, $E_5 = 0.3$, $We = 0.03$, $\varepsilon = 0.02$, $M = 1$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

(a) (b) (c)

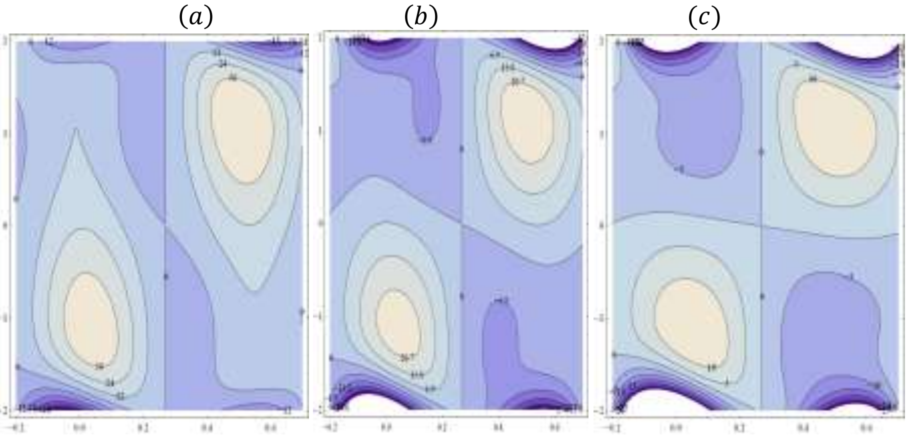


Figure 10- Stream line for different values of $M (a)\ M = 1$, $(b)\ M = 2$, $(c)\ M = 3$ and the other parameters $E_1 = 0.5$, $E_2 = 0.2$, $E_3 = 0.1$, $E_4 = 0.05$, $E_5 = 0.3$, $We = 0.03$, $\varepsilon = 0.2$, $t = 0.02$, $m = 0.01$, $\beta = 0.01$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$.

(a) (b) (c)

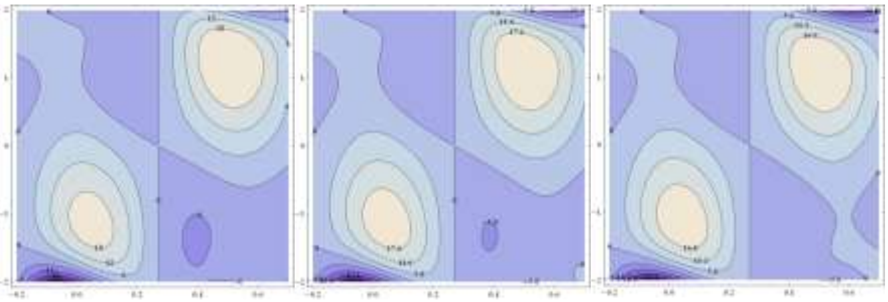


Figure 11- Stream line for different values of $m (a)\ m = 0.01\ , (b)\ m = 002\ , (c)\ m = 0.03$ and the other parameters $E_1 = 0.5\ ,\ E_2 = 0.2\ ,\ E_3 = 0.1\ ,\ E_4 = 0.05\ ,\ E_5 = 0.3\ ,\ We = 0.03\ ,\ \varepsilon = 0.2\ ,\ t = 0.02\ ,\ M = 1\ ,\ Fr = 0.5\ , Re = 0.5\ , \varphi = \frac{\pi}{3}\ , \beta = 0.01$.

(a) (c) (b)

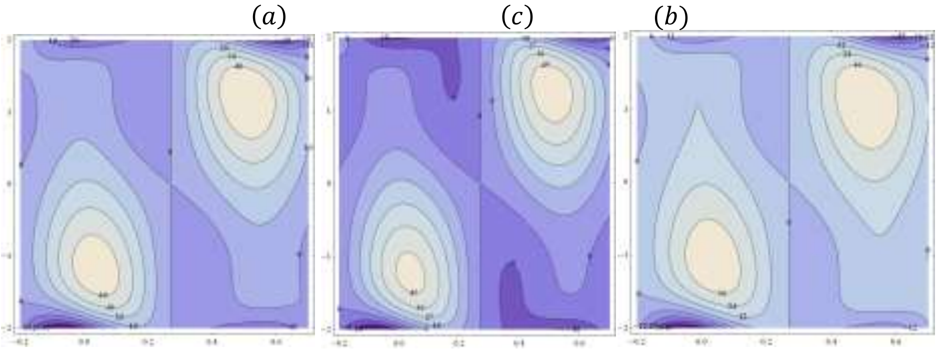


Figure 12- Stream line for different values of $\beta (a)\ \beta = 0.01\ , (b)\ \beta = 002\ , (c)\ \beta = 0.03$ and the other parameters $E_1 = 0.5\ ,\ E_2 = 0.2\ ,\ E_3 = 0.1\ ,\ E_4 = 0.05\ ,\ E_5 = 0.3\ ,\ We = 0.03\ ,\ \varepsilon = 0.2\ ,\ t = 0.02\ ,\ M = 1\ ,\ Fr = 0.5\ , Re = 0.5\ , \varphi = \frac{\pi}{3}\ ,\ m = 0.01$.

(a) $(c\ )$ (b)

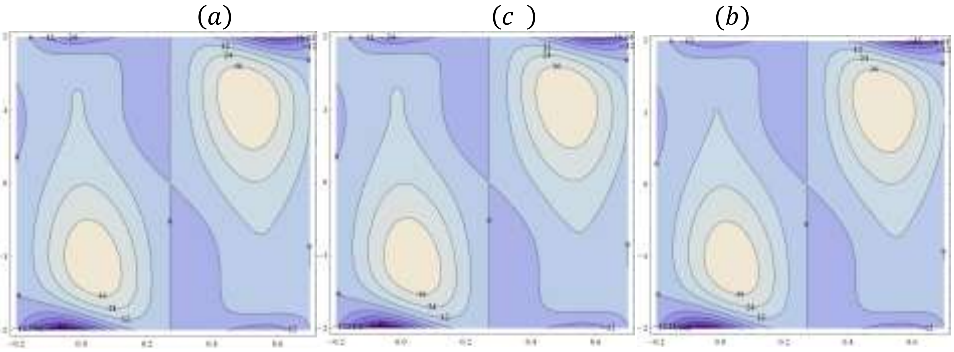


Figure 13- Stream line for different values of $Fr (a)\ Fr = 0.5\ , (b)\ Fr = 1\ , (c)\ Fr = 1.5$ and the other parameters $E_1 = 0.5\ ,\ E_2 = 0.2\ ,\ E_3 = 0.1\ ,\ E_4 = 0.05\ ,\ E_5 = 0.3\ ,\ We = 0.03\ ,\ \varepsilon = 0.2\ ,\ t = 0.02\ ,\ M = 1\ ,\ m = 0.01\ , \beta = 0.1\ , \varphi = \frac{\pi}{3}\ , Re = 0.5$.

(a) (c) (b)

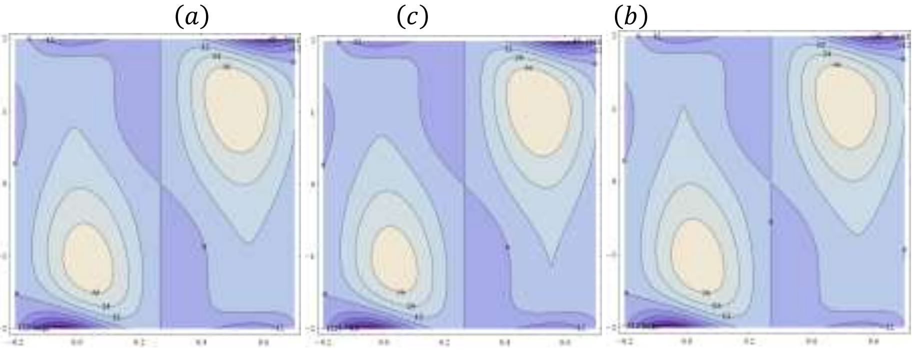


Figure 14- Stream line for different values of $Re(a)\ Re = 0.5\ ,(b)\ Re = 1\ ,(c)\ Re = 1.5$ and the other parameters $E_1 = 0.5\ ,\ E_2 = 0.2\ ,\ E_3 = 0.1\ ,\ E_4 = 0.05\ ,\ E_5 = 0.3\ ,\ We = 0.03\ ,\ \varepsilon = 0.2\ ,\ t = 0.02\ ,\ M = 1\ ,\ m = 0.01\ ,\ \beta = 0.1\ ,\varphi = \frac{\pi}{3}\ ,Fr = 0.5$.

(a) (c) (b)

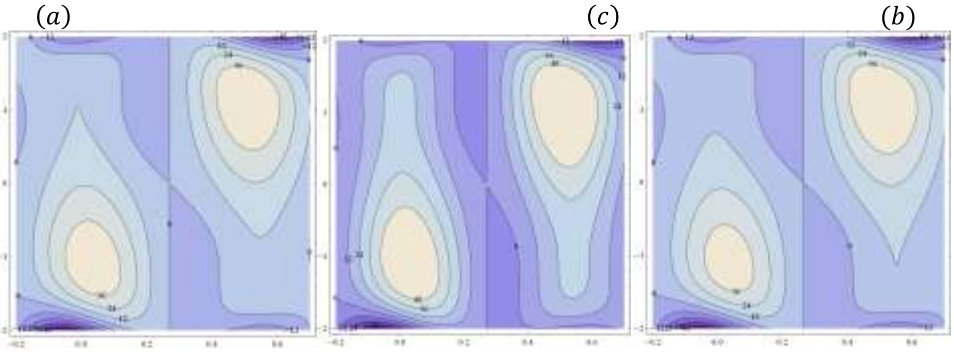


Figure 15- Stream line for different values of $\varphi(a)\varphi = \frac{\pi}{4}\ ,(b)\varphi = \frac{\pi}{3}\ ,(c)\varphi = \frac{\pi}{2}$ and the other parameters $E_1 = 0.5\ ,\ E_2 = 0.2\ ,\ E_3 = 0.1\ ,\ E_4 = 0.05\ ,\ E_5 = 0.3\ ,\ We = 0.03\ ,\ \varepsilon = 0.2\ ,\ t = 0.02\ ,\ M = 1\ ,\ m = 0.01\ ,\ \beta = 0.1\ ,\ Fr = 0.5\ ,Re = 0.5$.

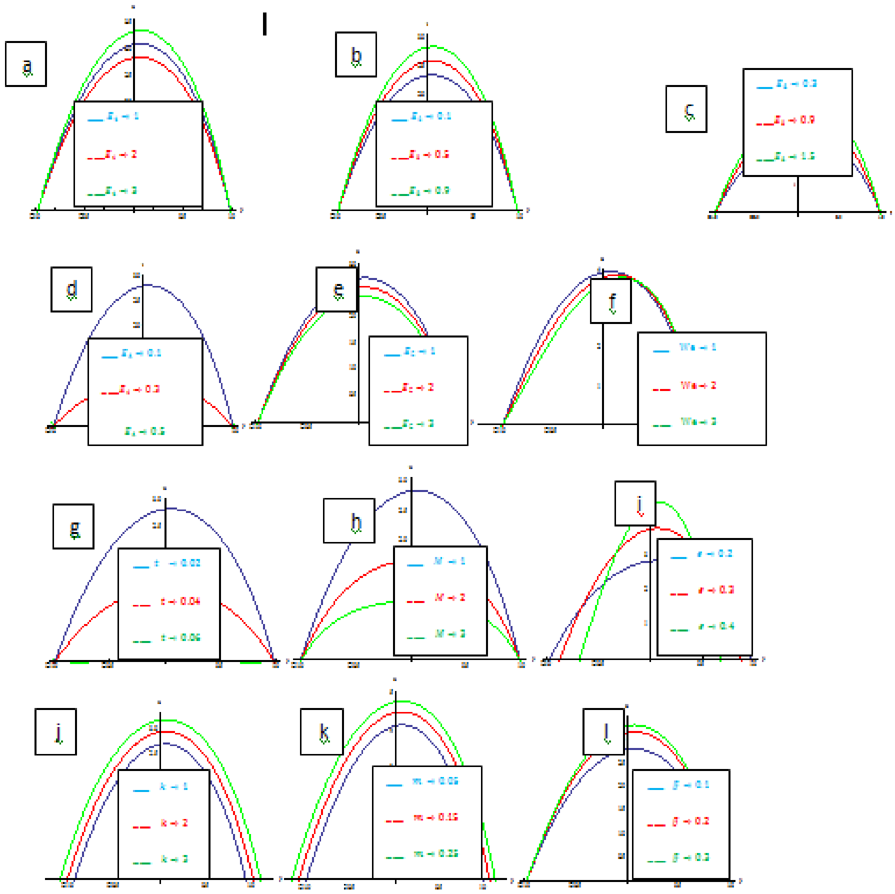
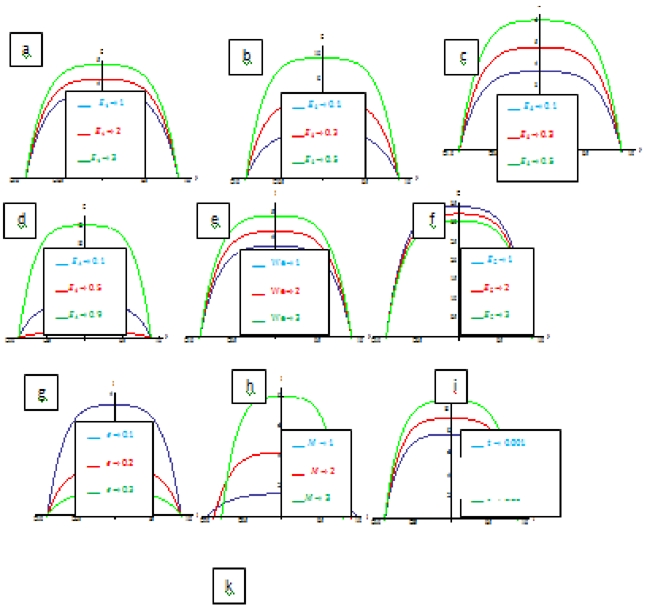


Figure 16-(a) Variation of u with y for different values of E_1at $E_2 = 0.3$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (b)- Variation of u with y for different values of E_2at $E_1 = 1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (c)- Variation of u with y for different values of E_2at $E_1 = 1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (d)- Variation of u with y for different values of E_4at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (e)- Variation of u with y for different values of E_5at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (f)- Variation of u with y for different values of Weat $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$ $E_5 = 0.01$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (g)- Variation of u with y for different values of $t$at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $M = 1$, $k = 1$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (h)- Variation of u with y for different values of M at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (i)- Variation of u with y for different values of ϵat $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $M = 1$, $t = 0.02$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$, $x = 0.8$. (j)- Variation of u with y for different values of $k$at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $\beta = 0.1$, $x = 0.8$. (k)- Variation of u with y for different values of $m$at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $\beta = 0.1$, $x = 0.8$. (l)- Variation of u with y for different values of βat $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $x = 0.8$. (m)- Variation of u with y for different values of $x$at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\theta = \frac{\pi}{3}$, $m = 0.1$, $\beta = 0.1$.(n) –Variation of u with y for different values of Fr at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $\beta = 0.1$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $x = 0.8$.(o)- Variation of u with y for different values of Re at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $\beta = 0.1$, $\varphi = \frac{\pi}{3}$, $m = 0.1$, $x = 0.8$.(p)- Variation of u with y for different values of φ at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\beta = 0.1$, $m = 0.1$, $x = 0.8$



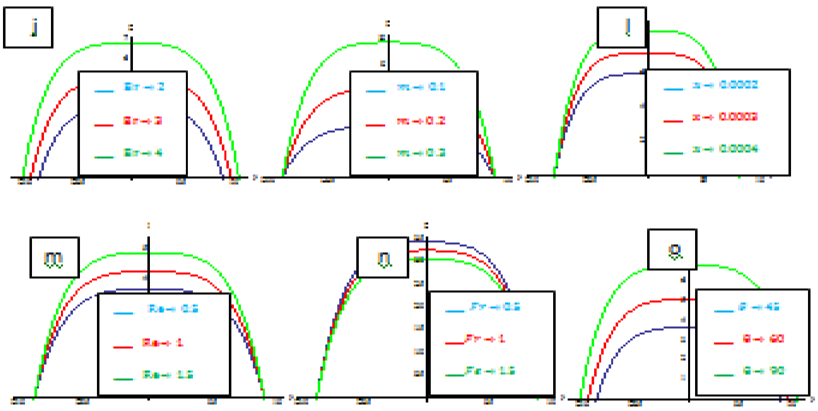


Figure 17-(a) Variation of $\theta$with y for different values of E_1at $E_2 = 0.3$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (b)- Variation of $\theta$with y for different valuesofE_2at$E_1 = 1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (c)- Variation of $\theta$with y for different values of E_2at $E_1 = 1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (d)- Variation of $\theta$with y for different values of E_4at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (e)- Variation of $\theta$with y for different values of Weat $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (f)- Variation of $\theta$with y for different values ofE_5at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$ $we = 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (g)- Variation of $\theta$with y for different values ofϵat $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $t = 0.02$, $M = 1$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (h)- Variation of$\theta$with y for different values of Mat $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (i)- Variation of$\theta$with y for different values of $t$at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $M = 1$, $Br = 2$, $m = 0.1$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (j)- Variation of $\theta$with y for different values of Br at$E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $m = 0.1Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$. (k)- Variation of $\theta$with y for different values of $m$at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $x = 0.8$.(l)- Variation of $\theta$with y for different values of x at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $Fr = 0.5$, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $m = 0.1$. (m) $-$Variation of $\theta$with y for different values of Re at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $Br = 2$, $k = 1Fr = 0.5$, $\varphi = \frac{\pi}{3}$, $\beta = 0.1$, $\theta = \frac{\pi}{3}$, $m = 0.1$, $x = 0.8$.(n)- Variation of $\theta$with y for different values of Fr at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, We $= 0.03$, $Br = 2$, $\epsilon = 0.2$,, $Re = 0.5$, $\varphi = \frac{\pi}{3}$, $t = 0.02$, $M = 1$, $k = 1$, $\beta = 0.1$, $m = 0.1$, $x = 0.8$.(o)- Variation of $\theta$with y for different values of φ at $E_1 = 1$, $E_2 = 0.1$, $E_3 = 0.3$, $E_4 = 0.01$, $E_5 = 0.01$, $Br = 2$, We $= 0.03$, $\epsilon = 0.2$, $t = 0.02$, $M = 1$, $k = 1$, $Fr = 0.5$, $Re = 0.5$, $\beta = 0.1$, $m = 0.1$. $x = 0.8$.

(4)Graphical results and discussion

In this chapter , we have presented a set of figures to observe the behavior of sundry parameters involved in the expressions of longitudinal velocity ($u = \psi_{0y} + We\,\psi_{1y}$) , temperature $\theta$and stream functionψ. Figs **16(a)** to **16(m**)display the effects of various physical parameters on the velocity profile (y) . Figs **16(a)** and **16(b)** depicts that the velocity increases when $E_1\ and\ E_2$ enhanced . It due to the fact that less resistance is offered to the flow because of the wall elastance and thus velocity increases . However reverse effect is observed for $E_3, E_4$and E_5 . As E_3represent the damping which is resistive force so velocity decreases when E_3 is increased . Similar behavior is observed for the velocity in case of rigidity and stiffness due to presence of damping force . Fig **16(h)**shows that the velocity decreases by increasing Hartman number . It is due to the fact that magnetic field applied in the trasverse direction shows damping effect on the flow . Fig **16(f)** illustrates that velocity profile decreases for We in the region [-1,0] whereas it has opposite behavior in the region [0,1] . Figs **16(j)**,**16(k)** and **16(l)**depicts that the velocity increases when $m\,, k\ and\ \beta$ enhanced.Effects of pertinent parameters on temperature profile can be visualized throughFigs **17(a)** to **17(l)**. The variation of compliant wall parameters is studied in Figs **16(a) to 17(e)**. As temperature is the average kinetic energy of the particales and kinetic energy depends upon the velocity .Therefore increase in velocity by $E_1\ and\ E_2$ leads to temperature enhancement .Similarly decrease in velocity by E_3 , E_4 and E_5 shows decay in temperature . Fig**17(h)**reveals that the temperature profile θdecreases when Hartman number M is increased .Effect of Br on temperature can be observed through Fig**17(j)** . The Brinkman number Br is the product of the Prandtl number Pr and the Eckert number Ec. Here Eckert number occurs due to the viscous dissipation effects and the temperatureenhances.Fig**17(k)**depictthattheTemperatureincreaseswhen $m$enhanced.Thefunctionof an internaly circulating bolus of fluid by closed stream lines is shown in Figs **12** . Fig **10** depict that the size of trapped bolus increases when there is an increase in the Hartman number . We have analyzed from Figs**2** and **3** that the size of trapped bolus increases when E_1 *and* E_2 are enhanced . However it decreases when E_3 is increased . Also when we decrease the values of E_4 *and* E_5 the trapped bolus size increases . Fig **7** depict the stream lines pattern for different values of Weissenberg number .

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