



On the Use of the Causal Analysis in Small Type Fit Indices of Adult Mathematics Learners

Adelodun, Olusegun Ayodele

Institute of Education, Obafemi Awolowo University, Ile-Ife, Nigeria

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ABSTRACT: Model evaluation is one of the most important aspects of Structural Equation Modeling (SEM). Many model fit indices have been developed. It is not an exaggeration to say that nearly every publication using the SEM methodology has reported at least one fit index. Fit is the ability of a model to reproduce the data in the variance-covariance matrix form. A good fitting model is one that is reasonably consistent with the data and doesn't require respecification and also its measurement model is required before estimating paths in a covariance structure model. A baseline model of four constructs together with a combination of none, one, two, three or four additional constructs was constructed with latent variables: educational performance, socio-economic label, self concept and parental authority using dichotomous digits 0 or 1 for each additional construct. 16 progressively nested models were considered starting with baseline model using the mathematics adult learners data from the modeling sample and employing some small fit indexes which are commonly used (AIC, CAIC, RMR, SRMR, RMSEA, χ^2 / DF among others) [1] to test the fitness of the model. The measures of model fit based on results from analysis of the covariance structure model are presented.

Keywords: Fit Indices; Structural Equation Modeling; Bernoulli Digits; Latent Constructs; Educational Performance

I. INTRODUCTION

Fit refers to the ability of a model to reproduce the data (i.e., usually the variance-covariance matrix). A good fitting model is one that is reasonably consistent with the data and so does not require respecification and also its measurement model is required before estimating paths in a structural model [2].

[3], [4], and others distinguish between several types of fit indices: *absolute fit indices*, *relative fit indices*, *parsimony fit indices*, and those based on the *noncentrality* parameter. There are several fit indices that fall into the category of *absolute indices*, including the Goodness-of-fit index (GFI), the adjusted goodness of fit index (AGFI), χ^2 / df ratio, Hoelster's CN ("critical N"), Akaike's Information Criterion (AIC), the Bayesian Information Criterion (BIC), the Expected Cross-validation Index (ECVI), the root mean square residual (RMR), and the standardized root mean square residual (SRMR).

Relative fit indices compare a chi-square for the model tested to one from a so-called *null model* (also called a "baseline" model or "independence" model). There are several *relative fit indices*, including Bollen's Incremental Fit Index (IFI), the Tucker-Lewis Index (TLI), Bentler-Bonett Nonnormed Fit Index (BBNFI), and the Bentler-Bonett Normed Fit Index (NFI). A number of *parsimonious fit indices* was developed (which are adjustments of most of the relative fit indices) include PGFI (based on the GFI), PNFI (based on the NFI), PNFI2 (based on Bollen's IFI), PCFI (based on the CFI mentioned below). *Noncentrality*-based indices include the Root Mean Square Error of Approximation (RMSEA), Bentler's Comparative Fit Index (CFI), McDonald and Marsh's Relative Noncentrality Index (RNI), and McDonald's Centrality Index (CI).

Considerable controversy has flared up concerning fit indices recently. Some researchers do not believe that fit indices add anything to the analysis (e.g., [5]) and only the chi square should be interpreted. The worry is that fit indices allow researchers to claim that a mis-specified model is not a bad model. Others (e.g., [6]) argue that cutoffs for a fit index can be misleading and subject to misuse. Most analysts believe in the value of fit indices, but caution against strict reliance on cutoffs. Also problematic is the "cherry picking" a fit index. That is, computing a many fit indices and picking the one index that allows you to make the point that you want

to make. If you decide not to report a popular index (e.g., the TLI or the RMSEA), you need to give a good reason why you are not.

[7] has also argued that fit indices should not even be computed for small degrees of freedom models. Rather for these models, the researcher should locate the source of specification error. SEM scholars distinguish two classes of fit indices: those that reflect “absolute” fit, and those that reflect a model’s “incremental” fit, or the fit of one model relative to another. Absolute indicators of model fit include χ^2 and SRMR, among others. Incremental fit statistics include CFI, among others. However, [8] distinguish two classes of fit indices into large fit indices (*NFI, NNFI, CFI, GFI, PGFI, AGFI, PNFI* and *IFI*) and in this paper, we shall consider the fit indices such as *AIC, CAIC, RMR, SRMR, RMSEA* and χ^2/df with small values considered indicators of good fit to educational performance model with adult mathematics learners as our subjects. Here are their definitions and basic behavioral properties.

Table 1: Equations of some fit indices and their authors

Fit Indices	Equations	Authors
Root Mean Square Residual (RMR)	$RMR = \sqrt{\frac{\sum_{i=1}^p \sum_{j=1}^i (s_{ij} - \hat{s}_{ij})^2}{k(k+1)/2}}$	Browne <i>et al.</i> , 2001
Standardized Root Mean Square Residual (SRMR)	$SRMR = \sqrt{\frac{\sum_{i=1}^p \sum_{j=1}^i [(s_{ij} - \hat{s}_{ij}) / (s_{ii} s_{jj})]^2}{k(k+1)/2}}$	Hu and Bentler, 1999
Chi Square (χ^2)	$\chi^2 = [N \{ \text{tr}(\Sigma \hat{\Sigma}^{-1}) + \log \Sigma - \log S - (p+q) \}]$	Gerbing and Anderson, 1992
Root Mean Square Error of Approximation (RMSEA)	$RMSEA = \sqrt{(\chi^2 - df) / df(N - 1)}$	Kenny and McCoach, 2003

II. MODELS

Let *pqrs denotes a baseline model of four constructs together with a combination of none, one, two, three or four additional constructs; where * indicates the latent variables: educational performance, socio-economic label, self concept and parental authority. The variables p, q, r, s denote Bernoulli or dichotomous digits 0 (if excluded) or 1 (if included) for each additional construct, that is

$$\begin{aligned}
 p &= \begin{cases} 1, & \text{if latent variable CIRCUM is included} \\ 0, & \text{otherwise} \end{cases} ; \\
 q &= \begin{cases} 1, & \text{if latent variable TRAINENV is included} \\ 0, & \text{otherwise} \end{cases} ; \\
 r &= \begin{cases} 1, & \text{if latent variable HEALT is included} \\ 0, & \text{otherwise} \end{cases} ; \quad \text{and} \\
 s &= \begin{cases} 1, & \text{if latent variable SEC is included} \\ 0, & \text{otherwise} \end{cases} .
 \end{aligned}$$

Note that: CIRCUM represents circumstances;
 TRAINENV represents training environment;
 HEALT represents health characteristic; and
 SEC represents socio-economic characteristic.

We shall consider some 16 progressively nested models using the data from model sample as enumerated in Table 2. It varies from the baseline model *0000 to the ultimate model *1111.

Table 2: Coding for Models by included Latent Constructs

Code Name	Latent Constructs
*0000	educational performance, socio-economic label, self concept and parental authority
*1000	educational performance, socio-economic label, self concept, parental authority and circumstances
*0100	educational performance, socio-economic label, self concept, parental authority and training environment
*0010	educational performance, socio-economic label, self concept, parental authority and health characteristic.
*0001	educational performance, socio-economic label, self concept, parental authority and socio-economic characteristic.
*1100	educational performance, socio-economic label, self concept, parental authority, circumstances and training environment
*1010	educational performance, socio-economic label, self concept, parental authority, circumstances and health characteristic.
*1001	educational performance, socio-economic label, self concept, parental authority, circumstances and socio-economic characteristic.
*0110	educational performance, socio-economic label, self concept, parental authority, training environment and health characteristic.
*0101	educational performance, socio-economic label, self concept, parental authority, training environment and socio-economic characteristic.
*0011	educational performance, socio-economic label, self concept, parental authority, health characteristic and socio-economic characteristic.
*1110	educational performance, socio-economic label, self concept, parental authority, circumstances, training environment and health characteristics.
*1101	educational performance, socio-economic label, self concept, parental authority, circumstances, training environment, and socio-economic characteristic.
*1011	educational performance, socio-economic label, self concept, parental authority circumstances, health characteristic and socio-economic characteristic.
*0111	educational performance, socio-economic label, self concept, parental authority, training environment, health characteristic and socio-economic characteristic.
*1111	educational performance, socio-economic label, self concept, parental authority, circumstances, training environment, health characteristic and socio-economic characteristic.

III. GOODNESS-OF-FIT STATISTICS ON MODELING SAMPLE

Having considered some 16 progressively nested models starting with model *0000 using the data from the modeling sample, we shall now employ some fit indexes which are commonly used in the literature (such as χ^2 / df , *GFI*, *AGFI*, *NNFI*, *CFI*, *RMSR*, *RMSEA*, among others) to test the fitness of the model.

As the values in Table 3 reveal, the fit indexes of the models are included in the values which are acknowledged in the literature [1]. The commonly used measures of model fit, based on results from analysis of the structural model, are summarized in Table 3. In practice, model AIC, sat. AIC, model CAIC, RMR and χ^2 are indicative of small values, and SRMR has less than 0.1, RMSEA has less than 0.06 or 0.08 and Chi-square/degree of freedom has less than 3.00 for good fit.

From Table 3, models *0100 (with value 262.89), *0110 (with value 376.02) and *0111 (with value 633.63) have smaller values compared with other competing models for model AIC. Models *0100 (with value 156), *0010 (with value 156), *0110 (with value 210) and *0111 (with value 342) have smaller values compared with other competing models for saturated AIC. Moreso, models *0100 (with value 458.98), *0110 (with value 607.76) and *0111 (with value 924.80) have smaller values compared with other competing models for model CAIC. Models *0100 (with value 619.49), *0010 (with value 619.49), *0110 (with value 833.93) and *0111 (with value 1358.11) have smaller values compared with other competing models for saturated CAIC. Furthermore, models *1000 (with value 1.82), *1001 (with value 1.47), *1101 (with value 1.34) and *1011 (with value 1.34) have smaller values compared with other competing models for RMR. Models *0100 (with value 0.045), *1100 (with value 0.046) and *1101 (with value 0.050) have smaller values less 0.1 compared with other competing models for SRMR. In addition, models *1000 (with value 0.055), *1100 (with value 0.050) and *1101 (with value 0.049) have smaller values less than 0.06 compared with other competing models for RMSEA. Models *0100 (with value 196.89), *0110 (with value 298.02) and *0111 (with value 535.63) have smaller values for χ^2 compared with other competing models. Finally, models *1000 (with value 4.16), *1100 (with value 3.59) and *1101 (with value 3.51) have smaller values compared with other competing models for χ^2 / Df .

Table 3: Summary Statistics of Small Type Fit Indices on Modeling Sample

Fit Index	Model AIC	Sat. AIC	Model CAIC	Sat. CAIC	RMR	SRMR	RMSEA	χ^2	χ^2 / DF
Ideal Value	Small Value	Small Value	Small Value	Small Value	Small value	≤ 0.10	≤ 0.08	Small Value	≤ 3.00
Model *0000	221.32	110	369.87	436.82	2.82	0.051	0.067	171.32	5.71
Model *1000	477.44	272	721.07	1080.13	1.82 ⁺	0.048	0.055 ⁺	395.44	4.16 ⁺
Model *0100	262.89 ⁺	156 ⁺	458.98 ⁺	619.49 ⁺	2.37	0.045 ⁺	0.057	196.89 ⁺	4.38
Model *0010	299.43	156 ⁺	495.52	619.49 ⁺	2.37	0.051	0.064	233.43	5.19
Model *0001	541.04	210	754.96	833.93	2.08	0.062	0.075	469.04	6.70
Model *1100	537.24	342	822.47	1358.11	1.62	0.046 ⁺	0.050 ⁺	441.24	3.59 ⁺
Model *1010	619.82	342	899.10	1358.11	1.63	0.054	0.056	525.82	4.24
Model *1001	742.58	420	1039.69	1667.85	1.47 ⁺	0.054	0.054	642.58	4.02
Model *0110	376.02 ⁺	210 ⁺	607.76 ⁺	833.93 ⁺	2.04	0.051	0.058	298.02 ⁺	4.52
Model *0101	502.42	272	751.99	1080.13	1.80	0.056	0.058	418.42	4.45
Model *0011	544.46	272	794.03	1080.13	1.80	0.058	0.061	460.46	4.90
Model *1110	724.70	420	1033.69	1667.85	1.50	0.054	0.053	620.70	3.93
Model *1101	801.81	506	1140.51	2009.37	1.34 ⁺	0.050 ⁺	0.049 ⁺	687.81	3.51 ⁺
Model *1011	891.43	506	1230.14	2009.37	1.34 ⁺	0.055	0.054	777.43	3.97
Model *0111	633.63 ⁺	342 ⁺	924.80 ⁺	1358.11 ⁺	1.60	0.056	0.057	535.63 ⁺	4.39
Model *1111	1005.2	600	1385.53	2382.65	1.23	0.055	0.051	877.23	3.72

“+” indication of good fit model with some class of models

where

- Model AIC - Model Akaike Information Criterion
- Model CAIC - Model Consistent Akaike Information Criterion
- Sat. AIC - Saturated Akaike Information Criterion
- Sat. CAIC - Saturated Consistent Akaike Information Criterion
- RMR - Root Mean Square Residual
- SRMR - Standardized Root Mean Square Residual
- RMSEA - Root Mean Square Error of Approximation
- χ^2 - Chi-square
- Chi-square / degree of freedom

IV. CONCLUSION

The study considered some 16 progressively nested models for educational performance on small type fit indices of mathematics adult learners.

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