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Research Paper

Modelling of Sex Preference: A Case of Male Preferring Stopping Rule

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ABSTRACT: *We will find many situations wherein the families end the childbearing with male child (i.e. male preferring stopping rule of childbearing) even now. Whereas the girl-preferring stopping rule of childbearing is less. Even the families ending with girl child, they don't want to take risk of having more girls. When we talk of stopping rule depending on sex (male or female) of the child, the ultimate meaning is we require the last child should be of this sex (male or female). We have observed that sex preference and socio-economic status and other demographic factors are statistically significant at 5% level of significance. We have developed probability model for male preferring stopping rule under population homogeneity and heterogeneity. We assume population homogeneity regarding the probability p of having a boy. That is we assume that a free couple in the population follows the same stopping rule with the boys. Under the population heterogeneity the shape parameter θ is not equal to zero. Therefore we derived probability model under population homogeneity (θ is zero) and heterogeneity (θ is not zero).*

Keywords*: Male preference, birth order, negative binomial, stopping rule and Chi-square test*

I. INTRODUCTION

There were days when the couples never thought of the sex of the issues born. Especially in the rural setup male preferring stopping rule was never in the mind. Whether male or female, the family simply kept on adding the member (new born) to the family. The ignorant and illiterate rural mass had no idea of family size. Even the elder of the family encouraged the younger couples to give birth to more number of issues. A female issue even today in some illiterate families is never a source of joy. In the recent years the economic and social status plays a vital role in deciding the size of the family.

The demographic study of any human population is closely related to the distribution of number of children per family in that particular population. The shape of this distribution is affected, at least in some populations by the sex of the previous child. The frequency of families having more than two children has been reported to be significantly lower when the first two children were of different sex than they were of the same sex (Medina, 1977). A considerable earlier literature Goodman (1961); Repetto (1972); Hatzold (1974); Sheps (1963); McDonald (1973) and Das (1987) analyzed the magnitude of the effect of sex-preference and assessed by how much the ability to control the sex of children to lower the birth rate. Arnold (1992) observed that preference for a balance exists in tandem with a moderate preference for sons in North Africa, Kenya and Sri Lanka. Among women with two children in Morocco, those with one girl and one boy are least likely to want another child (60%), those with two boys are next most likely (71%), and those with two girls are the most likely to want another child (80%). A preference for a balance coexists with a slight son preference in Burundi, Mexico and Zimbabwe (Arnold, 1992).

Hank and Kohler, (2003) observed that childless women tend to have stronger sex preferences (particularly in favour of girls) than their male counterparts and that the sex of the first child is the most influential predictor of parents' preferences for the sex distribution of prospective offspring. The more highly educated families seem to have a higher propensity to favour daughters. In a study of the sex composition of previous offspring and third births in the United States, Pollard and Morgan (2002) suggested that changes in the society's gender system may have led to parental gender indifference, resulting in a decreasing effect of children's sex on parents' fertility decisions (see also Andersson, et al., 2006). The present study is an attempt to empirically verify the presence of sex preference and establish relation between sex preference and various socio-demographic factors. If sex-preference is present then there must be present stopping rule for the desired sex. Therefore we attempt at the development of the probability model for the preferred sex where male preferring rule in practicing.

II. SEX PREFERENCE: SOME EVIDENCES

The following tables (1(a-c)) give the evidence of presence of the sex preferences.

Table 1 (a): Family size and male preference

3 first children	Family stopped	Family increased	Total
Male absent	27 (1.87)	119 (8.25)	146 (10.12)
Male present	373 (25.83)	924 (64.03)	1297 (89.88)
Total	400 (27.72)	1043 (72.28)	1443

Calculated χ^2 = 6.89^{s}, χ_1^2=3.84 at 5% level of significance.

Table 1 (b): Family size and female preference

3 first children	Family stopped	Family increased	Total
Female absent	70 (4.85)	197 (13.65)	267 (18.5)
Female present	330 (22.87)	846 (58.63)	1176 (81.5)
Total	400 (27.72)	1043 (72.28)	1443

Calculated χ^2 = 0.34^{ns}, χ_1^2 =3.84 at 5% level of significance.

Table 1(c): Influence of sex of previous children on family size

Sex	Family stopped	Family increased	Total
Same	142 (8.05)	756 (42.83)	898 (50.88)
Different	180 (10.20)	687 (38.92)	867 (49.12)
Total	322 (18.24)	1443 (81.76)	1765

Calculated χ^2 =7.24^{s}, χ_1^2 =3.84 at 5% level of significance.

Note: ^{s} indicates significant at 5% level. Figures in parenthesis indicate percentage values (Source: Medina, 1977)

Table 1 (a) shows that, the absence of a male among the three first children, increases the probability of having more than three children. This preference for a male among the three first children can influence the sex ratio since it is not compensated by an increased probability of having more children if the first three are males. There are 97 families who stopped childbearing after having three first children of the same sex. Whereas 727 (50.4%) families continued childbearing after having three first children of different sex (Table 1(a) and 1(b)).

Table 2(a): Families who have expressed a preference by place of residence

Residence	Sex preferred		Total
	Male	Female	
Rural	140 (35.81)	65 (16.62)	205 (52.43)
Urban	88 (22.51)	98 (25.06)	186 (47.57)
Total	228 (58.31)	163 (41.69)	391

Calculated χ^2 =17.66^{s}, χ_1^2 =3.84 at 5% level of significance.

Table 2(b): Families who have expressed a preference by their Socio-economic status.

Socio-economic status	Sex preferred		Total
	Male	Female	
High	53 (13.55)	79 (20.20)	132 (33.76)
Middle	81 (20.72)	45 (11.51)	126 (32.23)
Low/very low	94 (24.04)	39 (9.97)	133 (34.02)
Total	228 (58.31)	163 (41.69)	391

Calculated χ^2 =28.1^{s}, χ_2^2 =5.991 at 5% level of significance.

Table 2(c): Families who have expressed a preference by offspring sex

Offspring's sex	Sex preferred		Total
	Male	Female	
Females only	126 (32.23)	3 (0.77)	129 (32.99)
No offspring	19 (4.86)	13 (3.32)	32 (8.18)
Both sex	73 (18.67)	57 (14.58)	130 (33.25)
Males only	10 (2.56)	90 (23.02)	391

Calculated χ^2 =178.5^{s}, χ_3^2 =7.815 at 5% level of significance.

Note: [s] indicates significant at 5% level. Figures in parenthesis indicate percentage values **(**Source: El-Gilany and Shady, 2007)

Table 2(a) shows that sex preference and place of residence are statistically significant at 5% level of significance. More than 35% of the rural families have male preference whereas about 25% of urban families have female preference. Out of 391 families 228 (58.31%) families preferred male and 163 (41.69%) families preferred female child. There are 94 (24.04%) families with low/very low income expressed male preference and only about 10% have expressed female preference. More than 20% of families with middle income expressed male preference and also more than 20% of families with high income have expressed female preference (Table 2(b)). Sex preference and Socio-economic status are statistically significant at 5% level of significance. There is highly statistically significant difference between sex preference and offspring's sex (Table 2(c)). More than 32% families preferred male child but they ended with only female child. Similarly 23% families preferred female child but they ended with only male child. More than 18% of families who preferred only male but got both types of children and more than 15% families who preferred only female child and got both kinds of children. It is also true that the families having more than two children are significantly lower when the first two children are boys when compared to the families that they are female. Therefore, male preference plays an important role in sex ratio. It is also important to know the influence of male preference on the family size. In all the above cases we find that there is some preference for male children.

III. MALE PREFERRING STOPPING RULE AND FAMILY SIZE

Govt of India passed the Pre Natal Diagnostic Techniques (PNDT) Act in 1994 which prohibited the sex determination of the fetus before the birth of the child. This Act was implemented in 1996 in all the states of India except Jammu and Kashmir. The thing is if the PNDT Act has been effective then the couples should either resort to the increased use of contraceptives or accept the gender of his or her child or they should resort to sex preferring differential stopping behaviour (SP-DSB) where they continue to have children until the desired number of sons is born. If son preference is a particular mindset of the people (under the absence of sex selective tests) then they will resort to SP-DSB.

Usually all couples expect particular child (male/female) or a combination of both. They go on child bearing until they get the desired child. Some may get the desired child early and some may get late. Many stop childbearing as soon as they get the desired child irrespective of the number of children they want. Because sex of the child unobservable before its birth, parents may go for child bearing until they get the desired sex. If a couple wants a male child, they will go for child bearing until they get a male child or stop inevitably. It is not like tossing a coin until we get head. The problem is whether all prefer same sex child, (may not be) or the sex preference (male or female) changes with education, religion, living status, geographical area etc., of the parents. We will find many situations wherein the families end the childbearing with male child (i.e. male preferring stopping rule of childbearing) even now. Whereas the girl-preferring stopping rule of childbearing is less. Even the families ending with girl child, then don't want to take risk of having more girls. When we talk of stopping rule depending on sex of the child, the ultimate meaning is we require the last child should be of this sex. For example, if we follow stopping rule by considering male child, we should allow birth to girl children until we get a boy or until the desired number of boys.

Based on the birth order effect one can judge the presence of SP-DSB. Basu (2009) computed two effects called, a) Within family birth order effect for boys (WFBOB): The average birth order of the boys in the family should go up if couples are practicing SP-DSB and b) Within family birth order effect for girls (WFBOG): The average birth orders of the girls in the family should decrease if couples are practicing SP-DSB. The difference between the two, WFBOB - WFBOG is called the birth order effect. Basu (2009) measured this birth order effect and proved that if the difference between WFBOG from the WFBOB is positive then it means that male children are born earlier in the birth cohort and couples are practicing SP-DSB. In order to check the presence of sex preference and male preferring rule we have randomly selected the families of size with more than three children and calculated average within family birth order for males and females. The following table gives practice of sex preference (or male preferring stopping rule) of the families based on the birth order effect.

Table 3. Effect of birth order among the families having more than three children

Family No.	Birth Order	WFBO for males	WFBO for females	Diff	SP-DSB	Male Stopping rule
1	GGB	1.00	1.50	-0.50	Not practicing	No
2	GBBB	3.00	1.00	2.00	Practicing	Yes
3	BBBBG	2.50	5.00	-2.50	Not practicing	No
4	GGBGBB	4.67	2.33	2.33	Practicing	Yes
5	BGB	2.00	2.00	0.00	Practicing	Yes
6	GGB	3.00	1.50	1.50	Practicing	Yes
7	BBB	2.00	0.00	2.00	Can't say	
8	GGB	3.00	1.50	1.50	Practicing	Yes
9	BGB	2.00	2.00	0.00	Can't say	
10	GBB	2.50	1.00	1.50	Practicing	Yes
11	BBB	2.00	0.00	2.00	Can't say	
12	BBB	2.00	0.00	2.00	Can't say	
13	GBB	2.50	1.00	1.50	Practicing	Yes
14	GGBB	3.50	1.50	2.00	Practicing	Yes
15	GBGGG	2.00	3.25	-1.25	Not practicing	No
16	GGGGGB	6.00	3.00	3.00	Practicing	Yes
17	BGB	2.00	2.00	0.00	Can't say	
18	GGB	3.00	1.50	1.50	Practicing	Yes
19	GGB	3.00	1.50	1.50	Practicing	Yes
20	BBB	2.00	0.00	2.00	Can't say	
21	BGB	2.00	2.00	0.00	Can't say	
22	GGGB	4.00	2.00	2.00	Practicing	Yes
23	BGGB	2.50	2.50	0.00	Can't say	
24	BBBGBG	2.75	5.00	-2.25	Not practicing	No
25	BBG	1.50	3.00	-1.50	Not practicing	No
26	BBG	1.50	3.00	-1.50	Not practicing	No
27	BBG	1.50	3.00	-1.50	Not practicing	No
28	GBGBG	3.00	3.33	-0.33	Not practicing	No
29	GBB	2.50	1.00	1.50	Practicing	Yes
30	GGB	3.00	1.50	1.50	Practicing	Yes
31	BGB	2.00	2.00	0.00	Can't say	
32	BBG	1.50	3.00	-1.50	Not practicing	No
33	GGBGG	3.00	3.00	0.00	Can't say	
34	GBG	2.00	2.00	0.00	Can't say	
35	GGGGGBB	6.50	3.00	3.50	Practicing	Yes
36	GBB	2.50	1.00	1.50	Practicing	Yes
37	GGB	3.00	1.50	1.50	Practicing	Yes
38	GBB	2.50	1.00	1.50	Practicing	Yes
39	GGGB	4.00	2.00	2.00	Practicing	Yes
40	GGGGB	5.00	2.50	2.50	Practicing	Yes
41	GGGBB	4.50	2.00	2.50	Practicing	Yes
42	GBB	2.50	1.00	1.50	Practicing	Yes
Family No.	**Birth Order**	**WFBO for males**	**WFBO for females**	**Diff**	**SP-DSB**	**Male Stopping rule**
43	GBB	2.50	1.00	1.50	Practicing	Yes
44	GBBB	3.00	1.00	2.00	Practicing	Yes
45	GBB	2.50	1.00	1.50	Practicing	Yes
46	GGGB	4.00	2.00	2.00	Practicing	Yes
47	BBB	2.00	0.00	2.00	Can't say	
48	GGBB	3.50	1.50	2.00	Practicing	Yes
49	BBB	2.00	0.00	2.00	Can't say	
50	GGG	0.00	2.00	-2.00	Can't say	
51	BBG	1.50	3.00	-1.50	Not practicing	No
52	GGBB	3.50	1.50	2.00	Practicing	Yes
53	GGB	3.00	1.50	1.50	Practicing	Yes
54	BGG	1.00	2.50	-1.50	Not practicing	No
55	BGG	1.00	2.50	-1.50	Not practicing	No
56	GGBG	3.00	2.33	0.67	Practicing	Yes
57	BGGB	2.50	2.50	0.00	Can't say	
58	GGB	3.00	1.50	1.50	Practicing	Yes
59	GBGGG	2.00	3.25	-1.25	Not practicing	No

60	GGB	3.00	1.50	1.50	Practicing	Yes
61	GGBGG	3.00	3.00	0.00	Can't say	
62	GBG	2.00	2.00	0.00	Can't say	
63	GBGGB	3.50	2.67	0.83	Practicing	Yes
64	GGB	3.00	1.50	1.50	Practicing	Yes
65	BBGGBB	3.50	3.50	0.00	Can't say	
66	BBB	2.00	0.00	2.00	Can't say	
67	BBG	1.50	3.00	-1.50	Not practicing	No
68	GBG	2.00	2.00	0.00	Practicing	Yes
69	GBB	2.50	1.00	1.50	Practicing	Yes
70	BGB	2.00	2.00	0.00	Can't say	
71	BBB	2.00	0.00	2.00	Can't say	
72	BGGG	1.00	3.00	-2.00	Not practicing	No
73	BGGB	2.00	2.50	-0.50	Not practicing	No
74	BBB	2.00	0.00	2.00	Can't say	
75	GBGB	2.50	2.00	0.50	Practicing	Yes
76	BGG	1.00	2.50	-1.50	Not practicing	No
77	BBG	1.50	3.00	-1.50	Not practicing	No
78	GGGBGG	4.00	3.40	0.60	Practicing	Yes
79	BGGG	1.00	3.00	-2.00	Not practicing	No
80	BBG	1.50	3.00	-1.50	Not practicing	No
81	GGB	3.00	1.50	1.50	Practicing	Yes
82	GGGGGGB	7.00	3.50	3.50	Practicing	Yes

Family No.	Birth Order	WFBO for males	WFBO for females	Diff	SP-DSB	Male Stopping rule
83	GGBG	3.00	2.33	0.67	Practicing	Yes
84	BGBGG	2.00	3.67	-1.67	Not practicing	No
85	BBG	1.50	3.00	-1.50	Not practicing	No
86	GGB	3.00	1.50	1.50	Practicing	Yes
87	GBG	2.00	2.00	0.00	Practicing	Yes
88	GBGG	2.00	2.67	-0.67	Not practicing	No
89	BBGGBB	3.50	3.50	0.00	Can't say	
90	BGG	1.00	2.50	-1.50	Not practicing	No
91	GBB	2.50	1.00	1.50	Practicing	Yes
92	GGBB	3.50	1.50	2.00	Practicing	Yes
93	GBB	2.50	1.00	1.50	Practicing	Yes
94	BBB	2.00	0.00	2.00	Practicing	Yes
95	GBB	1.67	1.00	0.67	Practicing	Yes
96	GGGB	4.00	2.00	2.00	Practicing	Yes
97	GGB	3.00	1.50	1.50	Practicing	Yes
98	GBG	2.00	2.00	0.00	Can't say	
99	GBG	2.00	2.00	0.00	Can't say	
100	GBGG	2.00	2.67	-0.67	Not practicing	No
101	GGB	3.00	1.50	1.50	Practicing	Yes
102	GGGB	4.00	2.00	2.00	Practicing	Yes
103	GGGGB	5.00	2.50	2.50	Practicing	Yes
104	GBG	2.00	2.00	0.00	Can't say	
105	GGGG	0.00	2.50	-2.50	Can't say	
106	GGG	0.00	2.00	-2.00	Can't say	
107	GBG	2.00	2.00	0.00	Can't say	
108	GGGB	4.00	2.00	2.00	Practicing	Yes
109	GGB	3.00	1.50	1.50	Practicing	Yes

110	GGB	3.00	1.50	1.50	Practicing	Yes
111	BGB	2.00	2.00	0.00	Can't say	
112	GGB	3.00	1.50	1.50	Practicing	Yes
113	GBGB	3.00	2.00	1.00	Practicing	Yes
114	BGGG	1.00	3.00	-2.00	Not practicing	No
115	BBG	1.50	3.00	-1.50	Not practicing	No
116	BGGG	1.00	3.00	-2.00	Not practicing	No
117	GBB	2.50	1.00	1.50	Practicing	Yes
118	GGG	0.00	2.00	-2.00	Can't say	
119	BGG	1.00	2.50	-1.50	Not practicing	No
120	BBG	1.50	3.00	-1.50	Not practicing	No
121	GGBG	3.00	2.33	0.67	Practicing	Yes
122	BBBG	2.00	4.00	-2.00	Not practicing	No

IV. FAMILY BUILDING STRATEGY WITH PARENTAL CONTROL OVER SEX OF CHILDREN

If couples want to have exactly one boy and they continue to have children until a boy arrives will average two children. If parents want at least one boy and one girl and the chance of either one-half on a given birth, they have no choice but to produce at random, and they will average three children (see Table 3). Therefore, the parents cannot influence the proportion of boys by stopping rule (Keyfitz, 1986). This argument is applied to any target size of the family, when the population is homogeneous. When the population is heterogeneous Goodman (1961) proved the effect of heterogeneity on the outcome of male-preferring stopping rule and the relationship between the harmonic mean and arithmetic mean. According to Keyfitz, (1986), under the absence of gender preference the male preferring stopping rule makes the expected proportion of boys larger in a family. The following Table 4 gives the possible combinations of boys and girls and desired family size.

Table 4: Observed Numbers of Males and Females offspring among 470 Families

Number of male offspring	Number of female offspring									Total number of families
	0	1	2	3	4	5	6	7	8	
9				1(100.0)						**0**
8				0 (0.0)						**0**
7			1(12.5)	1(20.0)						**2(0.06)**
6		7(28.0)	1(6.7)	2(25.0)	1(20.0)	1 (50.0)				**12(2.5)**
5	1(1.7)	3(9.1)	4(16.0)	4(26.7)	2(25.0)	2(40.0)				**16(3.4)**
4	4(6.5)	9(15.5)	10(30.3)	5(20.0)	3(20.0)	1(12.5)	0.0			**32(6.8)**
3	8(8.5)	18(29.0)	18 (31.0)	12(36.4)	7(28.0)	4(26.7)	1(12.5)	1(20.0)	1(50.0)	**70(14.9)**
2	25(32.5)	46(48.9)	29(49.8)	19(32.8)	7(21.2)	1(4.0)	1(6.7)	1(12.5)	0.0	**129(27.4)**
1	34 (58.6)	37(48.0)	33 (35.1)	9(14.5)	11(19.0)	1(3.0)	1(4.0)	2(13.3)	0.0	**128(27.2)**
0	32 (100.0)	24(41.4)	15(19.5)	7(7.5)	2(3.2)	0.0	0.0	0.0	0.0	**80(17.0)**
Total number of families	**104(22.1)**	**144(30.6)**	**111(23.6)**	**60(12.8)**	**33(7.0)**	**10(2.1)**	**3(0.006)**	**4(00.8)**	**1(0.002)**	**470(100.0)**

Note: Figures in parenthesis indicate percentage value contribution of each combination to the size of a family (Source : Wali and Talawar, 2005)

V. DEVELOPMENT OF THE MODEL FOR MALE PREFERRING STOPPING RULE

Many developing countries in East, South and South-East Asia and North Africa are characterized by strong son preference. The following authors (Ben-Porath and Welch, 1976; Yamaguchi (1989); Arnold et al., 1998; Clark, 2001; Jensen, 2002; Basu and Jong, 2010), gave the existence in society of a strong preference for male as opposed to female offspring. Furthermore, this strong preference is reflected in son targeting fertility behaviour, also referred to in the literature as differential stopping behaviour (DSB) or male-preferring stopping rules. The main idea behind such stopping rules is that the sex composition of already-existing children determines the subsequent fertility behaviour of families for evidence on DSB (see Arnold et al., 1998 and Larsen et al., 1998). In our model development, we define DSB as follows: couples continue childbearing until they reach their "desired" number, k, of boys or when they hit the ceiling for the maximum number, N, of children that they think to be feasible (given their resource constraints). The theoretical results in the paper are derived on the basis of the assumption that every couple in the population follows this behaviour.

Jensen (2002) arrived at results of the sibling effect, though he used a different stopping rule. In his model, couples want n children and b boys; but if they reach n children with less than b boys, they continue childbearing until they attain b boys or reach some maximum number of children, n+k. This stopping rule is a variant of that used by Seidl (1995) and 3this is also a special case of Basu and Jong (2006) model with the desired number of boys k=b and the maximum number of children N=n+k. However, there are two major differences between Basu and Jong (2006) and Jensen's (2002). First, while Basu and Jong (2006) discussed both the sibling effect and the birth-order effect, Jensen (2002) limits himself only to the former. Second, unlike Jensen (2002) they have used household level data on birth sequences and desired family size to estimate the full model with MLE. Yamaguchi (1989) developed a formal theory for male-preferring stopping rules of childbearing and obtained sex differences in birth order and in the number of siblings. In the present paper we develop a probability model for male preferring stopping rule. We derived results for male preferring stopping rule when families ending childbearing with one male child, second male children etc.

Following the notations of Yamaguchi (1989), we denote X for the number of children in the family if parents stop childbearing with M boys, then the random variable X is assumed to have a negative binomial distribution with the probability function

$$p(k) = \binom{k-1}{k-M}(1-p)^{k-M}p^{M} \qquad if\ k \geq M \qquad (1)$$
$$= 0 \qquad if\ k < M$$

where k gives the total number of children in the family. Clearly different couples can be expected to exhibit variation in the probability of conception per menstrual cycle, and this probability follows some probability distribution. The probabilities have the (0, 1) interval for its range. Hence we assume p has beta distribution with pdf

$$f(p) = \frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha,\beta)} \qquad 0 < p \leq 1;\ \alpha,\beta > 0 \qquad (2)$$

$$where\ B(\alpha,\beta) = \frac{\Gamma\alpha\ \Gamma\beta}{\Gamma(\alpha+\beta)}$$

We write the probability density function of X for a particular value of p as the conditional probability P[X=k|p].

Now we find the unconditional probability p(k) as

$$p(k) = \int_0^1 p(k|p)f(p)dp$$
$$= \int_0^1 \binom{k-1}{k-M}(1-p)^{k-M}p^{M}\frac{p^{\alpha-1}(1-P)^{\beta-1}}{B(\alpha,\beta)}dp$$
$$= \binom{k-1}{k-M}\frac{1}{B(\alpha,\beta)}\int_0^1 p^{(\alpha+M-1)}(1-\mathrm{p})^{(k+\beta-M)-1}dp$$
$$= \binom{k-1}{k-m}\frac{1}{B(\alpha,\beta)}\ B\big((\alpha+M),(\beta+k-M)\big)$$
$$= \frac{\Gamma k}{\Gamma(k-M+1)\Gamma M}\frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\ \Gamma\beta}\frac{\Gamma(M+\alpha)\ \Gamma(\beta+k-M)}{\Gamma(\beta+\alpha+k)} \qquad (3)$$

When M=1, equation (3) reduces to

$$p(k) = \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\ \Gamma\beta}\frac{\Gamma(\alpha+1)\Gamma(\beta+k-1)}{\Gamma(\alpha+\beta+k)}$$
$$= \frac{\alpha\ (\beta+k-2)(\beta+k-3)\ldots\ldots\ldots\ \beta}{(\alpha+\beta+k-1)(\alpha+\beta+k-2)\ldots\ldots\ldots(\alpha+\beta)}$$

Using re-parameterization in terms of the two parameters

$$\mu = \frac{\alpha}{(\alpha+\beta)}\ and\ \theta = \frac{1}{(\alpha+\beta)}$$

$$p(k) = \frac{\mu}{(\alpha+\beta)}\frac{\left(1-\frac{\alpha}{(\alpha+\beta)}\right)\left(1-\frac{\alpha}{(\alpha+\beta)}+\frac{1}{(\alpha+\beta)}\right)\ldots\ldots\ldots\left(1-\frac{\alpha}{(\alpha+\beta)}+\frac{(k-2)}{(\alpha+\beta)}\right)}{\left(1+\frac{1}{(\alpha+\beta)}\right)\left(1+\frac{2}{(\alpha+\beta)}\right)\ldots\ldots\ldots\ \left(1+\frac{(k-1)}{(\alpha+\beta)}\right)}$$
$$= \mu\frac{(1-\mu)(1-\mu+\theta)\ldots\ldots\ldots(1-\mu+(k-2)\theta)}{(1+\theta)(1+2\theta)\ldots\ldots\ldots(1+(k-1)\theta)}$$
$$= \mu\ \frac{\prod_{i=1}^{k-1}(1-\mu+(i-1)\theta)}{\prod_{i=1}^{k}(1+(i-1)\theta)} \qquad (4)$$

Which is Beta-Geometric distribution derived by Weinberg and Gladen (1986).

We assume population homogeneity regarding the probability p of having a boy. That is we assume that a free couple in the population follows the same stopping rule with the boys. In this situation, the parameter θ (the shape parameter) is considered as zero. Then equation (4) reduces to a single parameter distribution as

$$p(k) = \mu (1-\mu)^{k-1} \qquad k = 1, 2, \quad \ldots, \tag{5}$$

The maximum likelihood estimator of μ is

$$\hat{\mu} = \frac{1}{1+\frac{\sum n_k (k-1)}{n}} \tag{6}$$

Under the population heterogeneity the shape parameter θ is not equal to zero. Then equation (4) has mean and variance (see also Weingberg and Gladen, 1986).

$$E(X) = \frac{1-\theta}{\mu-\theta} \; and \; V(X) = \frac{(1-\mu)(1-\theta)}{(\mu-\theta)^2(\mu-2\theta)}$$

From equation (4), the likelihood equation can be written as

$$L = \prod_{k=1}^{\infty}\left[\mu \, \frac{\prod_{i=1}^{k-1}(1-\mu+(i-1)\theta)}{\prod_{i-1}^{k}(1+(i-1)\theta)}\right]^{n_k}$$

$$Log\, L = l = n \log \mu + \sum_{k=1}^{\infty} n_k \left[\sum_{i=1}^{k-1} log\,(1-\mu+(i-1)\theta) - \sum_{i=1}^{k} log(1+(i-1)\theta)\right]$$

Differentiating this with respect to μ and θ we get,

$$\frac{dl}{d\mu} = \frac{n}{\mu} - \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k-1} \frac{1}{1-\mu+(i-1)\theta} \tag{7}$$

$$\frac{dl}{d\theta} = \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k-1} \frac{(i-1)}{1-\mu+(i-1)\theta} - \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k} \frac{(i-1)}{1+(i-1)\theta} \tag{8}$$

Therefore, consider equation (7)

$$g(\mu) = \frac{n}{\mu} - \sum_{i=1}^{k-1} \frac{n_j}{1-\mu+(i-1)\theta} \quad \text{and}$$

$$g^{'}(\mu) = -\frac{n}{\mu^2} - \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k-1} \frac{1}{1-\mu+(i-1)\theta^2}$$

Thus

$$\hat{\mu}_{i+1} = \hat{\mu}_i - \frac{g(\hat{\mu})}{g^{'}(\hat{\mu})}\bigg|_{\hat{\mu}=\hat{\mu}_i}$$

Similarly, $\hat{\theta}(\hat{\mu})$ is derived by using (8),

$$g(\theta) = \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k-1} \frac{(i-1)}{1-\mu+(i-1)\theta} - \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k} \frac{(i-1)}{1+(i-1)\theta}$$

$$g'(\theta) = \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k-1} \frac{(i-1)^2}{(1-\mu+(i-1)\theta)^2} - \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k} \frac{(i-1)^2}{(1+(i-1)\theta)^2}$$

$$\hat{\theta}_{i+1} = \hat{\theta}_i - \frac{g(\hat{\theta})}{g^{'}(\hat{\theta})}\bigg|_{\hat{\theta}=\hat{\theta}_i}$$

The parameters are estimated using Newton-Raphson's method.

If M=2 in (3), we get

$$f(k) = \frac{\Gamma k}{\Gamma(k-1)\Gamma 2} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\,\Gamma\beta} \frac{\Gamma(\alpha+2)\,\Gamma(\beta+k-2)}{\Gamma(\beta+\alpha+k)}$$

$$= \frac{k(\alpha+1)\alpha(\beta+k-3)(\beta+k-4)\ldots\ldots\ldots\,\beta}{(\alpha+\beta+k-1)(\alpha+\beta+k-2)\ldots\ldots\ldots(\alpha+\beta)}$$

$$= \frac{\alpha(\alpha+1)k}{(\alpha+\beta)} \frac{\prod_{i=1}^{k-2}\left(1-\frac{\alpha}{(\alpha+\beta)}+\frac{i-1}{\alpha+\beta}\right)}{\prod_{i=1}^{k-1}\left(1+\frac{i-1}{\alpha+\beta}\right)}$$

Using re-parameterization in terms of the two parameters

$$\mu = \frac{\alpha}{(\alpha+\beta)} \text{ and } \theta = \frac{1}{(\alpha+\beta)}$$

$$= \mu\left(1+\frac{\mu}{\theta}\right)k\frac{\prod_{i=1}^{k-2}(1-\mu+(i-1)\theta)}{\prod_{i=1}^{k-1}(1+(i-1)\theta)} \quad (9)$$

The log-likelihood function is

$$\text{Log L} = l = n\log\mu + \log\left(1+\frac{\mu}{\theta}\right) + \log k + \sum_{k=1}^{\infty} n_k\left[\sum_{i=1}^{k-2}\log(1-\mu+(i-1)\theta) - \sum_{i=1}^{k-1}\log(1+(i-1)\theta)\right]$$

Differentiating this with respect to μ and θ we get,

$$\frac{dl}{d\mu} = \frac{n}{\mu} + \frac{1/\mu}{(1+\frac{\mu}{\theta})} - \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k-2}\frac{1}{1-\mu+(i-1)\theta} \quad (10)$$

$$\frac{dl}{d\theta} = \frac{-\mu/\theta^2}{(1+\frac{\mu}{\theta})} + \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k-2}\frac{(i-1)}{1-\mu+(i-1)\theta} - \sum_{k=1}^{\infty} n_k \sum_{i=1}^{k-1}\frac{(i-1)}{1+(i-1)\theta} \quad (11)$$

The parameters of the model are estimated again by using Newton-Raphson's method.

VI. CONCLUSION

We have observed from Table 3 that the families with BGB, BBB, BGGB, GGBGG, GBG, GGG, GGBGG, BBGGBB, GGGG, GGG (where the birth order effect either zero or negative) are not practicing sex (male or female) preferring stopping rule. According to the birth order effect many of the families are practicing male preferring stopping rule. We have made an attempt to model the male preferring stopping rule for various male birth orders.

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