Quest Journals
Journal of Research in Applied Mathematics
Volume 3 ~ Issue 7 (2017) pp: 19-28
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Truncated Gompertz Distribution and its Optimization of CASP-CUSUM Schemes

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Received 17 October, 2017; **A**ccepted 28 October, 2017 © The author(s) 2017. **P**ublished with open access at **www.questjournals.org**

ABSTRACT: *Acceptance sampling plans was studied mainly to draw valid conclusions with regard to accept or reject the lots of finished products. In this direction many optimal techniques were developed to improve and control the quality of products based on the assumption that the variable with regard to quality characteristic is distributed according certain probability law. In this paper we optimized CASP-CUSUM Schemes based on the assumption that the continuous variable, which is under the consideration follows a Truncated Gompertz distribution used in Quality Control and Reliability analysis. Particularly the distribution is used in estimating the optimal truncated point and probability of acceptance of lot. Thus this paper study Optimization CASP-CUSUM Schemes and critical comparisons are made based on the Obtained numerical results.*
Keywords: *Optimization, CASP-CUSUM Schemes, Quality Characteristics, OC Curve, ARL, Truncated Gompertz Distribution.*

I. INTRODUCTION

It is difficult to define quality, but make sure that the item posses or not posses the required specifications such as serviceability, availability, price, color, taste etc. This assessment of quality is subjective and it can vary from different perception of individuals. A product of good quality can be defined as one that satisfies the consumer's needs at the price they are willing to pay. It is believed that the item is fit for usage when it conform the characteristics.

Quality of the product is one of the most important characteristic that determines the demand of a product and would be a strategy for the economic development of country. The term quality could be defined in multi dimensional. Thus, it is very essential to improve the quality of products, in any manufacturing companies by using some strategic measures say improving a life of a form of the products, make sure that the item being worked properly or the item is conforming the quality specifications required by consumer. In order to improve the reliability or quality of the products one should adopt certain measures such as life testing through various probability models preventive measures, sampling inspection CUSUM Schemes etc. In the process, in improving the quality of products it should be examined whether, the items produced performing their intended duties or not, the items are available up to the warranty time, and how best they satisfy the consumer needs.

Life tests are experiments carried out in order to obtain the life time of an item (i.e. time to its failure or the stops working satisfactory). Sometimes, it might be time consuming to wait until all the products fail in a life test, if the life time of products is high. One can use the truncated life test for saving time and money, because 100% inspection involves more time, more money, man power, material, machinery etc. even the sample finite 100% inspection practically not feasible in case of explosive type materials like crackers, bombs, batteries, bulbs etc. The test can be performed without waiting until all the products fail, and then testing time can be reduced significantly. For the purpose of reduces test time and cost, obliviously truncated life models are good.

Cumulative Sum (CUSUM) quality control schemes are becoming widely used in industry. Because, they are powerful, versatile and easy to use. They Cumulative recent process data to quickly detect out of control situations. They also serve as a powerful diagnostic tool.

Hawkins, D. M. [3] proposed a fast accurate approximation for ARL's of a CUSUM Control Charts. This approximation can be used to evaluate the ARL's for Specific parameter values and the out of control ARL's of location and scale CUSUM Charts .

Kakoty. S., Chakravaborthy A.B. [5] proposed CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal Distribution. Generally truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production engineering items, the sorting procedure eliminates items above or below designated tolerance limits. It is worthwhile to note that any continuous variable be first approximated as an exponential variable.

Vardeman.S, Di-ou Ray [9] was introduced CUSUM control charts under the restriction that the values are regard to quality is exponentially distributed. Further the phenomena under study is the occurrence of rare of rare events and the inter arrival times for a homogenous poison process are identically independently distributed exponential random variables.

Lonnie. C. Vance, [6] consider Average Run Length of cumulative Sum Control Charts for controlling for normal means and to determine the parameters of a CUSUM Chart.To determine the parameters of CUSUM Chart the acceptable and rejectable quality levels along with the desired respective ARL's are consider.

Mohammed Riaz, Nasir Abbas and Ronald J.M.M Does [7] proposed two Runs rules schemes for the CUSUM Charts. The performance of the CUSUM and EWMA Charts are compared with the usual CUSUM and weighted CUSUM, the first initial response CUSUM compared with usual EWMA Schemes. This comparison stated that the proposed schemes perform better for small and moderate shifts.

Mohammed Akhtar. P and Sarma K.L.A.P [1] proposed an optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distribution and evaluate L (0) L' (O) and probability of Acceptance and also Optimized CASP-CUSUM Schemes based on numerical results.

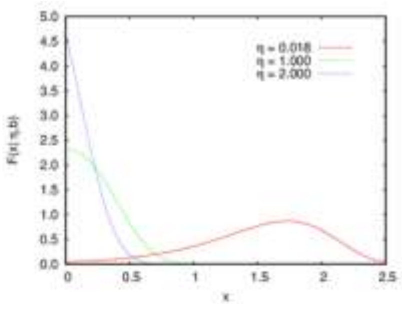
In the present paper it is proposed CASP-CUSUM Chart when the variable under study follows truncated Gompertz Distribution. Thus it is more worthwhile to study some interesting characteristics of this distribution.

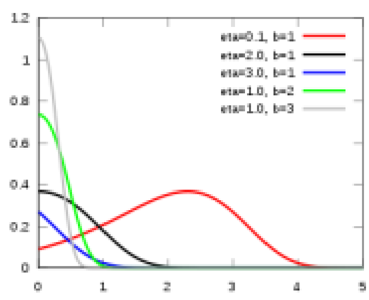
The British Statistician Benziman Gompertz has developed a distribution of adult life spans by demographic and actuaries. The simple formula which he derived describes the exponential rise in death rates between sexual, maturity and old age commonly which is referred as the Gompertz equation. This is a valuable tool in demography, Reliability and other scientific disciplines. The Gompertz is without doubt the best known model, used in a range of disciplines from botany to sociology.It has been widely used, especially in actuarial biological applications, demography and Reliabilty. Gompertz distribution is a special case of a generalized logistic distribution.

A continuous random variable X assuming non-negative values is said to have Gompertz Distribution with parameters c, k > 0, its probability density function is given by:

$$f(x;\eta,b) = b\eta e^{bx} e^{\eta} \exp(-\eta e^{bx}) \qquad \text{…… (1.1)}$$

Where b > 0 is the scale parameter and η > 0 is the shape parameter of the Gompertz distribution. In the actuarial and biological sciences and in demography, the Gompertz distribution is parametrized slightly.





Truncated Gompertz Distribution:-

It is the ratio of probability density function of the Gompertz distribution to the their cumulate distribution function at the point 'B

The random variable X is said to follow a truncated Gompertz Distribution as:

$$f_B(x) = \frac{b\eta e^{bx} e^{\eta} \exp(-\eta e^{bx})}{1-\exp(-\eta(e^{Bx}-1))} \qquad \text{...... (1.2)}$$

Where B is the truncated point of the Gompertz Distribution.

Truncated Gompertz distributions can be used to simplify the asymptotic theory of robust of location and regression.

II. DESCRIPTION OF THE PLAN AND TYPE- C OC CURVE

Battie [2] has suggested the method for constructing the continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum

$S_m = \sum (X_i - k_1) X_i's(i = 1,2,3........)$ are distributed independently and k_1 is the reference value. If the sum lies in the area of normal chart, the product is accepted and if it lies of the return chart, then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., h+h'
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

   When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure in brief is given below.

1. Start plotting the CUSUM at 0.
2. The product is accepted when $S_m = \sum (X_i - k) < h;$ when $S_m < 0$, return cumulative to 0.
3. When h <S_m< h+h' the product is rejected: when S_m crossed h, i.e., when S_m>h+h' and continue rejecting product until S_m>h+h' return cumulative to h+h'

The type-C, OC function, which is defined as the probability of acceptance of an item as function of incoming quality, when sampling rate is same in acceptance and rejection regions. Then the probability of acceptance P (A) is given by

$$P(A) = \frac{L(0)}{L(0)+L'(0)} \qquad \text{...... (2.1)}$$

Where L (0) = Average Run Length in acceptance zone and

L' (0) = Average Run Length in rejection zone.

Page E.S. [8] has introduced the formulae for L (0) and L' (0) as

$$L(0) = \frac{N(0)}{1-P(0)} \qquad \text{...... (2.2)}$$

$$L'(0) = \frac{N'(0)}{1-P'(0)} \qquad \text{...... (2.3)}$$

Where P (0) =Probability for the test starting from zero on the normal chart,

N (0) = ASN for the test starting from zero on the normal chart,

P' (0) = Probability for the test on the return chart and

N' (0) = ASN for the test on the return chart

He further obtained integral equations for the quantities

P (0), N (0), P' (0), N' (0) as follows:

$$P(z) = F(k_1 - z) + \int_0^h P(y)\, f(y + k_1 - z)dy \, , \qquad \text{…… (2.4)}$$

$$N(z) = 1 + \int_0^h N(y)\, f(y + k_1 - z)dy, \qquad \text{…… (2.5)}$$

$$P'(z) = \int_{k_1+z}^{B} f(y)dy + \int_0^h P'(y)\, f(-y + k_1 + z)dy \qquad \text{…… (2.6)}$$

$$N'(z) = 1 + \int_0^h N'(y)\, f(-y + k_1 + z)dy, \qquad \text{…… (2.7)}$$

$$F(x) = 1 + \int_A^h f(x)dx :$$

$$F(k_1 - z) = 1 + \int_A^{k_1 - z} f(y)dy$$

And z is the distance of the starting of the test in the normal chart from zero.

III. METHOD OF SOLUTION

We first express the integral equation (2.4) in the form

$$F(X) = Q(X) + \int_c^d R(x,t)F(t)dt \qquad \text{…… (3.1)}$$

Where

$$F(X) = P(z),$$

$$Q(X) = F(k - z),$$

$$R(X,t) = f(y + k - z)$$

Let the integral $I = \int_c^d f(x)dx$ be transformed to

$$I = \frac{d-c}{2}\int_c^d f(y)dy = \frac{d-c}{2}\sum a_i f(t_i) \qquad \text{…… (3.2)}$$

Where $y = \dfrac{2x - (c-d)}{d-c}$ where a_i's and t_i's respectively the weight factor and abscissa for the Gass-Chibyshev polynomial, given in Jain M.K. and et al [4] using (3.1) and (3.2),(2.4) can be written as

$$F(X) = Q(X)\frac{d-c}{2}\sum a_i R(x,t_i)F(t_i) \qquad \text{…… (3.3)}$$

Since equation (3.3) should be valid for all values of x in the interval (c, d), it must be true for x=t_i , i = 0 (1) n then obtain.

$$F(t_i) = Q(t_i) + \frac{d-c}{2}\sum a_i R(t_j,t_i)F(t_i) \qquad j = 0(1)n \qquad \text{…… (3.4)}$$

Substituting

$F(t_i) = F_i, Q(t_i) = Q_i, i = 0(1)n, in \quad (3.4),$ we get

$$F_0 = Q_0 + \frac{d-c}{2}[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \text{………} a_n R(t_0,t_n)F_n)]$$

$$F_1 = Q_1 + \frac{d-c}{2}[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + \ldots\ldots\ldots a_n R(t_1,t_n)F_n)]$$

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$$F_n = Q_n + \frac{d-c}{2}[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + \ldots\ldots\ldots a_n R(t_n,t_n)F_n)] \qquad \ldots\ldots (3.5)$$

In the system of equations except F_i , i= 0,1,2……………n are known and hence can be solved for F_i, we solved the system of equations by the method of Iteration. For this we write the system (3.5) as

$$[1 - Ta_0 R(t_0,t_0)]F_0 = Q_0 + T[a_0 R(t_0,t_0)F_0 + a_1 R(t_0,t_1)F_1 + \ldots\ldots a_n R(t_0,t_n)F_n)]$$

$$[1 - Ta_1 R(t_1,t_1)]F_1 = Q_1 + T[a_0 R(t_1,t_0)F_0 + a_1 R(t_1,t_1)F_1 + \ldots\ldots a_n R(t_1,t_n)F_n)]$$

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$$[1 - Ta_n R(t_n,t_n)]F_n = Q_n + T[a_0 R(t_n,t_0)F_0 + a_1 R(t_n,t_1)F_1 + \ldots\ldots a_n R(t_n,t_n)F_n)] \qquad \ldots.. (3.6)$$

Where $T = \frac{d-c}{2}$

To start the Iteration process, let us put $F_1 = F_2 = \ldots. = F_n = 0$ in the first equation of (3.6), we then obtain a rough value of F_0. Putting this value of F_0 and $F_1 = F_2 = \ldots. = F_n = 0$ on the second equation, we get the rough value F_1 and so on. This gives the first set of values F_i i= 0,1,2,...,n which are just the refined values of F_i i= 0,1,2,…,n. The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions P' (0), N (0), N' (0) can be obtained.

IV. COMPUTATION OF ARL's AND P (A)

We developed computer programs to solve these equations and we get the following results given in the Tables (4.1) to (4.16).

TABLE-4.1
Values of ARL's AND TYPE-C OC CURVES when
b=0.9, η =0.9, k=2, h=0.07, h'=0.07

B	L(0)	L'(0)	P(A)
2.8	153.95929	1.082930	0.9930152
2.7	155.20174	1.082936	0.9930707
2.6	158.11087	1.082948	0.9931973
2.5	164.62766	1.082975	0.9934646
2.4	179.13860	1.083027	0.9939906
2.3	213.87343	1.083122	0.9949612
2.2	322.48798	1.083289	0.9966521
2.1	1972.30798	1.083563	0.9994509

TABLE-4.2
Values of ARL's AND TYPE-C OC CURVES when
b=0.9, η =0.9, k=1.8, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
2.6	54.02376	1.1230685	0.9796349
2.5	54.73786	1.1239082	0.9798946
2.4	56.19780	1.1231891	0.9804053
2.3	59.10182	1.1233367	0.9813477
2.2	64.93006	1.1235960	0.9829897
2.1	77.51970	1.1240157	0.9857075
2.0	111.45309	1.1246805	0.9900097
1.9	326.60300	1.1256839	0.9965652

TABLE-4.3
Values of ARL's AND TYPE-C OC CURVES when
b=0.92, η =0.92, k=2, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
2.9	438.22754	1.127123	0.9974346
2.8	439.94510	1.127125	0.9974446
2.7	444.74644	1.127129	0.9974721
2.6	457.20480	1.127139	0.9975408
2.5	488.55444	1.127164	0.9976982
2.4	571.35962	1.127215	0.9980310
2.3	856.67487	1.127316	0.9986858
2.2	10489.86719	1.127501	0.9998925

TABLE-4.4
Values of ARL's AND TYPE-C OC CURVES when
b=0.92, η =0.92, k=1.8, h=0.09, h'=0.09

B	L(0)	L'(0)	P(A)
2.6	69.16707	1.112920	0.9841645
2.5	69.84242	1.112941	0.9843149
2.4	71.31420	1.112986	0.9846330
2.3	74.39430	1.113074	0.9852587
2.2	80.81352	1.113237	0.9864118
2.1	95.01366	1.113519	0.9884162
2.0	133.51714	1.113981	0.9917257
1.9	364.63031	1.114707	0.9969522

TABLE-4.5
Values of ARL's AND TYPE-C OC CURVES when
b=0.94, η =0.94, k=2, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
2.8	307.83606	1.061434	0.9965638
2.7	308.93326	1.061435	0.9965760
2.6	312.00513	1.061437	0.9966096
2.5	319.97650	1.061443	0.9966937
2.4	339.83118	1.061457	0.9968863
2.3	390.96716	1.061486	0.9972923
2.2	554.17291	1.061543	0.9980881
2.1	2346.08545	1.061647	0.9995477

TABLE-4.6
Values of ARL's AND TYPE-C OC CURVES when
b=0.94, η =0.94, k=1.8, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
2.6	100.74538	1.13136	0.9888948
2.5	101.52316	1.13137	0.9889798
2.4	103.34875	1.13140	0.9891711
2.3	107.42040	1.13147	0.9895766
2.2	116.39317	1.13160	0.9903713
2.1	137.38971	1.13184	0.9918292
2.0	199.30188	1.13225	0.9943510
1.9	752.08203	1.13292	0.9984959

TABLE-4.7
Values of ARL's AND TYPE-C OC CURVES when
b=0.96, η =0.96, k=2, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
2.8	527.92407	1.063339	0.9979898
2.7	529.27972	1.063339	0.9979950
2.6	533.55121	1.06340	0.9980110
2.5	545.66559	1.063343	0.9980551
2.4	578.37878	1.063352	0.9981649
2.3	669.78583	1.063370	0.9984149
2.2	1000.85693	1.063409	0.9989386
2.1	24996.84180	1.063484	0.9999574

TABLE-4.8
Values of ARL's AND TYPE-C OC CURVES when
b=0.96, η =0.96, k=1.8, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
2.6	144.28017	1.135738	0.9921897
2.5	145.11156	1.135746	0.9922341
2.4	147.22308	1.135764	0.9923445
2.3	152.26682	1.135807	0.9925959
2.2	164.02798	1.135896	0.9931226
2.1	192.97304	1.136068	0.9941472
2.0	283.86282	1.136381	0.9960127
1.9	1449.16248	1.136916	0.9992161

TABLE-4.9
Values of ARL's AND TYPE-C OC CURVES when
b=0.98, η = 0.98, k=2, h=0.06, h'=0.06

B	L(0)	L'(0)	P(A)
2.9	166246362	1.0794643	0.9993511
2.8	1663.87097	1.0794646	0.9993517
2.7	1669.20862	1.0794647	0.9993537
2.6	1687.84082	1.0794653	0.9993609
2.5	1748.04541	1.0794673	0.9993829
2.4	1937.15381	1.0794730	0.9994431
2.3	2644.17407	1.0794870	0.9995919
2.2	14842.68945	1.0795182	0.9999273

TABLE-4.10
Values of ARL's AND TYPE-C OC CURVES when
b=0.94, η =0.94, k=2, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
3.0	1438.24817	1.131352	0.9992140
2.9	1440.39648	1.131352	0.9992152
2.8	1447.90918	1.131353	0.9992192
2.7	1471.51538	1.131355	0.9992318
2.6	1541.07385	1.131360	0.999664
2.5	1747.92615	1.131374	0.9993532
2.4	2521.86011	1.131405	0.9995515
2.3	37860.58594	1.131472	0.9999710

TABLE-4.11
Values of ARL's AND TYPE-C OC CURVES when
b=0.98, η = 0.98, k=1.8, h=0.10, h'=0.10

B	L(0)	L'(0)	P(A)
2.6	215.82964	1.140280	0.9947445
2.5	216.74687	1.140284	0.9947667
2.4	219.29948	1.140294	0.9948272
2.3	225.87407	1.140320	0.9949769
2.2	242.23000	1.140379	0.9953142
2.1	284.97427	1.140500	0.9960138
2.0	431.01505	1.140733	0.9973603
1.9	5345.51318	1.141151	0.9997866

TABLE-4.12
Values of ARL's AND TYPE-C OC CURVES when
b=0.99, η=0.99, k=2, h=0.06, h'=0.06

B	L(0)	L'(0)	P(A)
3.0	3477.76465	1.080710	0.9996893
2.9	3478.44800	1.080710	0.9996894
2.8	3481.18481	1.080710	0.9996896
2.7	3495.62427	1.080710	0.9996909
2.6	3549.60889	1.080710	0.9996957
2.5	3736.19727	1.080712	0.9997109
2.4	4400.39941	1.080716	0.9997545
2.3	8140.99072	1.080727	0.9998673

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TABLE-4.13
Values of ARL's AND TYPE-C OC CURVES when
b=0.99, η =0.99, k=1.8, h=0.09, h'=0.09

B	L(0)	L'(0)	P(A)
2.6	239.97432	1.126449	0.9953279
2.5	240.76172	1.126451	0.9953431
2.4	243.05774	1.126458	0.9953868
2.3	249.19177	1.126476	0.9954998
2.2	264.83987	1.126516	0.9957644
2.1	306.03323	1.126604	0.9963322
2.0	1442.10120	1.126778	0.9974578
1.9	2440.83838	1.127099	0.9995384

TABLE-4.14
Values of ARL's AND TYPE-C OC CURVES when
b=0.99, η =0.99, k=2, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
2.9	1596.56934	1.066317	0.9993325
2.8	1597.29651	1.066317	0.9993328
2.7	1600.35779	1.066317	0.9993342
2.6	1611.68298	1.066318	0.9993388
2.5	1649.27429	1.066319	0.9993539
2.4	1768.17578	1.066322	0.9993973
2.3	2172.55396	1.066331	0.9995094
2.2	4676.54980	1.066351	0.9997720

TABLE-4.15
Values of ARL's AND TYPE-C OC CURVES when
b=0.98, η =0.98, k=2, h=0.05, h'=0.05

B	L(0)	L'(0)	P(A)
2.9	1035.83496	1.065308	0.9989726
2.8	1036.38599	1.065308	0.9989731
2.7	1038.47339	1.065308	0.9989752
2.6	1045.72107	1.065309	0.9989823
2.5	1068.73511	1.065310	0.9990042
2.4	1137.15881	1.065315	0.9990640
2.3	1350.87781	1.065326	0.9992120
2.2	2342.59106	1.065352	0.9995455

TABLE-4.16
Values of ARL's AND TYPE-C OC CURVES when
b=0.96, η =0.96, k=2, h=0.09, h'=0.09

B	L(0)	L'(0)	P(A)
2.9	959.49017	1.091105	0.9988641
2.8	960.73035	1.091105	0.9988656
2.7	965.20062	1.091106	0.9988708
2.6	979.24066	1.091108	0.9988870
2.5	1020.11755	1.091113	0.9989315
2.4	1138.68774	1.091125	0.9990427
2.3	157.91260	1.091152	0.9992956
2.2	6249.32764	1.091209	0.9998254

V. NUMERICAL RESULTS AND CONCLUSIONS

At the hypothetical values of the parameters c, k, k_1, h and h' are given at the top of each table, we determine optimum truncated point B at which P (A) the probability of accepting an item is maximum and also obtained ARL's values which represents the acceptance zone L(0) and rejection zone L'(0) values. The values of truncated point B of random variable X, L(0), L'(0) and the values for Type-C Curve, i.e. P (A) are given in columns I, II, III, and IV respectively.

From the above tables 4.1 to 4.16 we made the following conclusions

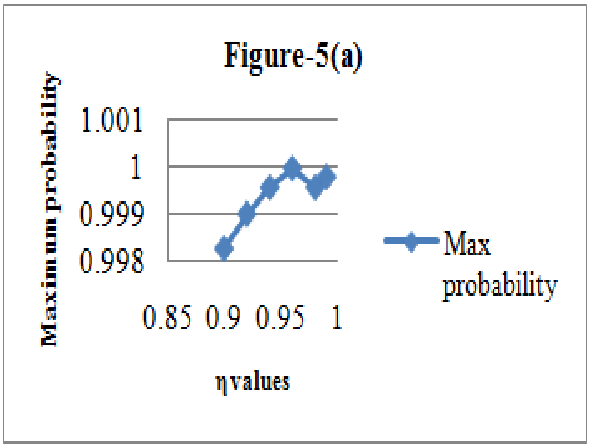
1. From the table 4.1 to 4.16, it is observed that the values of P (A) is increased as the value of truncated point decreases thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.

2. And also we observe that it can be minimized the truncated point B by increasing value of k.
3. From table 4.1 to 4.16, it is observed that at the maximum level of probability of acceptance P (A) the truncated point B from 3.0 to 1.1 as the value of h changes from 0.05 to 0.10.
4. From the table 4.1 to 4.16, it was observed that the value of L (0) and P (A) are increased as the value of truncated point decreases thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.
5. From the table 4.1 to 4.16, it was observed that the truncated point B changes from 3.0 to1 .1 and P (A) is as $h \to 0.05$ maximum **i.e. 0.999970.** Thus truncated point B and h are inversely related and h and P (A) are positively related.
6. From Table 4.1 to 4.16 it is observed that the optimal truncated point changes from 2.1 to 2.5 as $h \to 0.25$.
7. From Table 4.1 to 4.16 it can be observed that the values of P (A) increased as the increases values of **b** and η.
8. It is observed that the Table 5(a) values of Maximum Probabilities increased as the values of η as shown below the figure.

Table-5 (a)

k=2 h=0.05 h'=0.05 B=2.1

η	Max probability
0.90	0.9982447
0.92	0.9989838
0.94	0.9995477
0.96	0.9999574
0.98	0.9995455
0.99	0.9997720

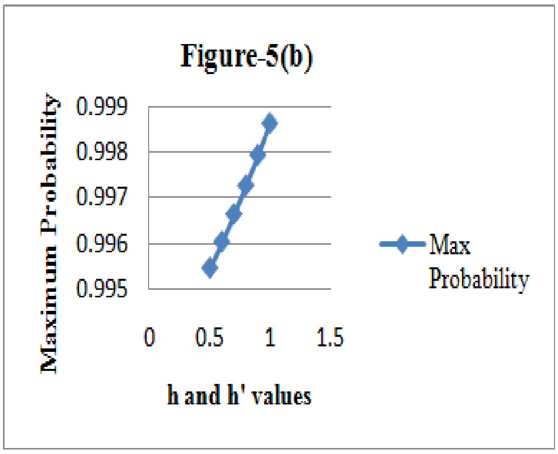


9. It is observed that the Table- 5(b) values of Maximum Probabilities are increased as the increases the values of h and h' as shown below the figure.

TABLE-5(b)

η=0.90 B=2.1 k=2 b=0.90

h and h'	Max probability
0.05	0.9954773
0.06	0.9960494
0.07	0.9966521
0.08	0.9972867
0.09	0.9979548
0.10	0.9986581



10. It is observed that the Table 5(c) values of Maximum Probabilities are increased as the increases values of **k** as shown below the figure.

Table-5(c)

b= 0.90 η=0.90 h=0.05 h'=0.05

k	Max probability
2	0.9982447
1.8	0.9927901
1.5	0.9355075
1	0.7902284

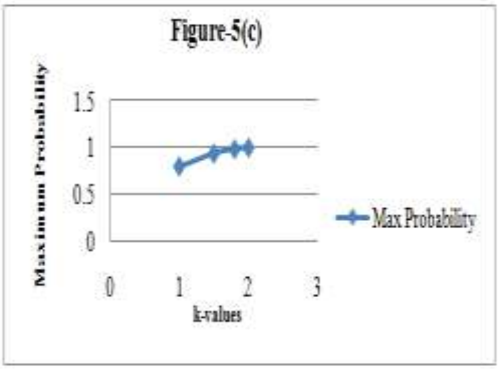


Figure-5(c)

11. The various relations exhibited among the ARL's and Type-C OC Curves with the parameters of the CASP-CUSUM based on the above table 4.1 to 4.16are observed from the following Table

Table – 4.17 Consolidated Table

B	b	□	h	h'	K	P(A)
2.1	0.9	0.9	0.07	0.07	2	0.9994509
1.9	0.9	0.9	0.10	0.10	1.8	0.9965652
2.2	0.92	0.92	0.10	0.10	2	0.9998925
1.9	0.92	0.92	0.10	0.10	1.8	0.9976147
2.1	0.94	0.94	0.05	0.05	2	0.9995477
1.9	0.94	0.94	0.10	0.10	1.8	0.9984959
2.1	0.96	0.96	0.05	0.05	2	0.9999574
1.9	0.96	0.96	0.10	0.10	1.8	0.9992161
2.2	0.98	0.98	0.06	0.06	2	0.9999273
2.3	**0.94**	**0.94**	**0.10**	**0.10**	**2**	**0.9999701**
1.9	0.98	0.98	0.10	0.10	1.8	0.9997866
2.3	0.99	0.99	0.06	0.06	2	0.9998673
1.9	0.99	0.99	0.09	0.09	1.8	0.9995384
1.6	0.99	0.99	0.10	0.10	1.5	0.9905976
2.2	0.99	0.99	0.05	0.05	2	0.9997720
2.4	0.98	0.98	0.05	0.05	2	0.9995455
2.5	0.96	0.96	0.07	0.07	2	0.9998254

By observing the Table 5.1, we can conclude that the optimum CASP-CUSUM Schemes which have the values of ARL and P (A) reach their maximum i.e., 37860.5, 0.999970 respectively, is

$$\begin{bmatrix} B = 2.3 \\ b = 0.94 \\ K = 2 \\ \eta = 0.94 \\ h = 0.10 \\ h' = 0.10 \end{bmatrix}$$

On similar lines we can obtain CASP-CUSUM Schemes when a particular parameter is fixed at a point, for example, if we fix the value of $K = 1.8$, in that case only the maximum value of probability of acceptance P(A)= 0.99921, is

$$\begin{bmatrix} B = 1.9 \\ b = 0.96 \\ K = 1.8 \\ \eta = 0.96 \\ h = 0.10 \\ h' = 0.10 \end{bmatrix}$$

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