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Research Paper

Mathematical Model of Municipal Solid Waste Management System in Aba Metropolis of Abia State Nigeria

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ABSTRACT: *In this study, mathematical model of municipal solid waste management system (MSWMS) is presented for Aba metropolis. The mixed integer programming problem was used in formulating the (MSWMS) model. The formulated model was solved using MATLAB 2015a version. The optimal results show that the best model would include 3 landfills, 322 tons capacity Collection Centers and seven (7) Collection Centers. The proposed model results in a minimum transportation cost of ₦3, 929, 900 per day as against the sum of ₦5, 600, 000 per day which is currently spent on solid waste disposal in Aba according to ASEPA. The study went further to show that any additional increase on the collection center capacity up to 350 tons will result in upward rise and fall of the objective function value. The study finally concluded that in order to improve solid waste management in Aba, the emerging conditions for the adoption and operation of the MSWMS model needs to be addressed through a number of institutional reforms and management designs and arrangement including stakeholder collaborations, encouraging informal sector participation and the development of technological innovations for solid waste management.*

Keywords: *Solid waste, Municipal Solid Waste Management System (MSWMS), Mathematical Model, Landfill.*

I. INTRODUCTION

Solid waste is any material which comes from domestic, commercial and industrial sources arising from human activities which has no value to people who possess it and is discarded as useless. Waste disposal in the early days did not pose difficulty as habitations were sparse and land was plentiful. Waste disposal became problematic with the rise of towns and cities where large numbers of people started to congregate in relatively small areas in pursuit of livelihoods, Shafiul and Manoor (2003). While the population densities in urbanized areas and per capita waste generation increased, the available land for waste disposal decreased proportionately. Solid waste management thus emerged as an essential, special sector for keeping cities healthy and livable. Solid waste management refers to source separation, storage, collection, transportation and final disposal of waste in an environmentally sustainable manner, Puopiel, (2010). Solid waste management is an important environmental health service and an integral part of basic urban services. This is because the health implications of poor waste management can bevery damaging to the people exposed to these unsanitary conditions. Diseases such as cholera, typhoid, dysentery and malaria are all related to the practice of poor waste management. The collection, transfer and disposal of waste have been generally assumed by metropolitan governments in both developed and developing worlds. This constitutes a basic and expected government function. The format varies in most urban areas where solid waste is collected either by a government agency or private contractor, despite the fact that developing countries do spend about 20 to 40 percentages of metropolitan revenues on waste management, they are unable to keep pace with the scope of the problem, Zerbock (2003). In fact, when the governments of African countries were required by the World Health Organization (WHO) to prioritize their environmental health concerns, the result revealed that solid waste was identified as the second most important problem after water quality, (Senkoro, 2003 cited by Zerbock, 2003).

There are two patterns of refuse found on Nigerian streets, the heap of refuse found in collection centers that remain unclear at regular interval and polymers, papers, broken bottles, cans, cartons, food rap leaves among others. This is due to the use and dump anywhere attitude of Nigerians. At times, children and young people carry their refuse in bags and throw them on the streets at night. The responsibility of refuse disposal is conventionally that of the local government. Private contractors also collect refuse from homes by emptying refuse drums kept at strategic places within the compound.

Miller (1994) defines waste as man's unwanted materials that need to be discarded. Adegoke (1990) defines waste as substance and materials which are disposed of or are required to be disposed of according to the provision of the National law. Miller (1993; 1994) defines solid waste as any unwanted or discarded material that is not liquid or gaseous. It may not be generally accepted that solids and indeed waste are "useless, unwanted or discarded(undesirable) materials" given the degree of scavenging on waste heaps in less developed countries like Nigeria by both humans and animals. Urban waste are those materials that are generated, used and has no further value and are thrown away in the environment, these material can be valuable raw materials located at a wrong place, Sharma(2010). Solid waste management is a global issue that is a growing source of concern in developed and developing countries due to increase urbanization; changes in consumer pattern and industrialization which all directly influence solid waste generally. Kadafa et al (2013). Adedibu (1993) is of theview that the nature and composition of solid waste is a product of climatic and business activities in urban centers. He argue further that most of the agricultural produce such as maize, cassava, vegetable, millet are brought unprocessed during the rainy and harvesting seasons from the nearby farms. The composition of refuse generated in an area determines thetype of disposal method suitable for a particular form of waste and the effectiveness of a collection system depends on the cooperation of households and individuals in various sectors of the city in providing containers for storing refuse in accordance with the regulation and regularly placing the materials for collection, Afon (2003). Abumere (1983) links socio-cultural factors to land use pattern such as housing density and eating habits. He further states that solid waste accumulation is a product of chaotic land use pattern, the number of household living and that the eating habit in a house greatly determines the composition of refuse generated. Abila and Kantola (2013) are of the view that municipal waste management problems in Nigeria cut across concern for human health, air and water and land pollution among others. Adewole (2009) argue that continuous indiscriminate disposal of municipal solid waste is accelerating and is linked to poverty, poor governance, urbanization, population growth, poor standard of living and low level of environmental awareness.

The administrative framework for SWM is three tiered in Nigeria, thus consisting of National and State agencies as well as Local government bodies. The Federal Ministry of Environment is the body charged with overseeing the protection of the environment as well as natural resources preservation to the end of achieving sustainable development. They play the role of promulgation of national environmental laws, enforcement and monitoring of the same in addition to ensuring adherence to international environmental guidelines. Operating also on the federal level is the National Environmental Standards and Regulations Agency, in addition to environmental law enforcement roles, the agency is charged with maintenance of effective interaction between national and international actors on issues related to the environment. Up until 1999, environmental laws and regulations were put forward and enforced by the Federal Environmental Protection Agency (FEPA); these subsequently became a role of the Federal Ministry of Environment. In order to ensure protection of the environment is better managed, all states in Nigeria and their local governments have been given the capacity to create related environmental establishments, the sovereignty of which is limited to the state or local government area it has been established, Ogwueleka (2009).

The management of solid waste has been a primary function of the municipal/local government in each state; however, attaining efficiency in the sector has been a major challenge especially in the prominent cities within the country such as Aba, Enugu, Owerri, Port-Harcourt, Kaduna, Lagos and Ibadan where piles of Municipal Solid Waste (MSW) are often observed. Idowu et al (2011), their sources being households, markets and places of commercial activity, Momodu (2011). The problem of Municipal Solid Waste Management (MSWM) has been long standing in the country; households in Enugu (44%), Ibadan (35%) and Kaduna (33%) have been reported to lack access to adequate solid waste management services for over two decades and there appears to be little or no improvement even in the recent times, Aderogba and Afelumo, (2012). The employment of an integrated approach in SWM sector in each state could be an effective way to surmount inherent challenges in the system.Such approach would entail the strategic management of MSW through coordinated development and administration of a blend of new systems as well as significant improvement of current ones, all within a contemporary framework (UNEP, 2005a, p. 30). This would give room for adequate consideration of different actors and aspects in the sector which may be important if efficiency and sustainability are indeed to be achieved. Sustainability in this study, takes into account the environmental perspective of waste management and potential of waste as a resource that could be utilized in a manner that shall be beneficial now and in the future. The industrial wastes include metals, scraps chips and grits from machines, shops, sawdust, paper pieces and glass, Omole and Alakinde (2013).

Kenneth and Huie (1983) also classified solid waste into three categories, namely; garbage, ashes and rubbish. The garbage includes organic matter resulting from preparation and consumption of food. Ashes include remains from cooking and heating processes and the rubbish may either take the form of combustible such as paper, rags,

Gordon (2005) defines municipal solid waste as trash or garbage, according to him it is a combination of the entire city's solid and sometimes semi-solid waste. It includes mainly households or domestic waste, but it can also contain commercial and industrial wastes with the exception of industrial hazardous waste (waste from industrial practices that causes a threat to human or environmental health). He categorizes municipal waste into five. Aba metropolis made up of five Local Government Areas has a population of one million, twenty two thousand, one hundred and ninety two people according to 2006 population and household census by the National Population Commission. Two thousand, One hundred and Twenty Four tons of municipal solid waste is generated per dayin Aba (National Population Commission Priority Table 1). About half of the waste generated is collected and disposed off at some old borrow pits along Enugu-Port Harcourt Expressway in Aba. Limited or no treatment is done at the various borrow pits where some organic material is converted into compost. A limited amount of waste is picked by individuals that sell it to some industries for reuse as raw material; this covers plastic bottles, tins and other metallic objects. A limited amount of organic material is also picked by individuals as animal feeds to pigs, dogs and cattle. Since less than half of the waste is collected and the waste is in the open, much of it litters the streets of Aba metropolis whenever the wind blows and whenever it rains. This might not be unconnected with the incidence of the frequent cholera outbreaks during the raining seasons and the terrible stench from the city areas where the waste accumulated has decayed.

Computer-base models have been widely applied in order to optimize integrated approaches to the different issues of solid waste management over the past four decades. Modeling of integrated waste management systems began as early as 1971 and was initially focused on optimal site positioning, Revelle et al (1971) and Gottinger (1986). Later, in the late 1980's and early 1990's researchers began to consider optimizing resources and recycling activities. A notable advance was the work of Everet and Modak (1996), which developed a complete deterministic linear model for the scheduling of a regional integrated solid waste management strategy. The model approaches the waste management system as a whole, optimizing every aspect of it individually while trying to evaluate its operation as a whole, using multi criteria analysis. The basic concept behind this model is to represent material flows as arcs and sources and destination of these flows as nodes. This enables resource balances to be created in every node, controlling the flow of waste. The model has also been used to perform case studies on real cities, Massimiliano (2001), Badran and El-Haggar (2006).

The total cost of the solid waste management system include the transportation cost of the waste to different facilities such as transfer stations, landfills, incinerators and also the operational and fixed costs of these facilities Badaran and El-Haggar(2006).In the municipality of Genova, Italy, Costi, et al (2004) have proposed a mixed integer non-linear programming decision support model to help decision makers of a municipality in the development of incineration, disposal, treatment and recycling integrated programs. Chang and Chang (1998) have presented a non-linear programming model for municipal solid waste management based on the minimization of an overall cost considering energy and material recovery requirements. Daskalopoulos et al (1998) have presented a mixed integer linear programming model for the management of municipal solid waste streams, taking into account their rates and compositions, as well as their adverse environmental impacts. Using this model, they identify the optimal combination of technologies for handling, treatment and disposal of municipal solid waste in a better economical and more environmentally sustainable way. The single objective is composed of costs per tones of waste treated at the recycling, composting, incinerating plants and landfills.

Fiorucci et al (2003) have presented a mixed integer nonlinear programming decision support model for assisting planners in decisions regarding the overall management of solid waste at a municipal level. By using the model, an optimal number of landfills and treatment plants, optimal quantities and the characteristics of refuse that have to be sent to treatment plants, to landfills and to recycling can be determined. Nganda (2007) developed two mathematical models as tools for solid waste planners in the decisions concerning the overall management of solid waste in a municipality. The models have respectively been formulated as integer and mixed integer linear programming problems. The solid waste management models described above was developed using linear programming and mixed integer linear programming. Hasit and Warner (1981) compared these two techniques when applied to the waste resource allocation programme model. In their scenarios, the number of cost combinations increased rapidly as the number of facilities increased, resulting in higher data requirements and programme handling. Ogwueleka (2009) introduces new decision procedure for solid waste collection problem. The problem objective was to minimize the overall cost, which was essentially based on the distance travelled by vehicle. The study proposed heuristic method to generate feasible solution to an extended Capacitated Arc Routing Problem (CARP) on undirected network. The technique was compared with the existing schedule with respect to cost, time and distance traveled. The adoption of the proposed technique resulted in reduction of the number of existing vehicles by 25% and 16.31% reduction in vehicle distance traveled par day. Daskalopoulos, et al (1998) have presented a mixed integer linear programming model for the management of MSW streams. This is similar to our model. The cost in the objective function of their model caters for the environmental considerations related to the emission of greenhouse gases. This is also similar to our model. Unlike our model, Daskalopoulos, et al (1998) does not cover collection and transportation costs. Regulatory and technical constraints are not considered either.

Badran and El-Haggar (2005) studied optimization of solid waste management systems using operation research methodologies. A mixed integer linear programming model is a problem whose objective covers collection costs from the districts to collection stations, transportation costs from collection stations to their composting plants or to landfills. This is similar to our model. Unlike our model, their model did not cater for environmental considerations related to the emission of greenhouse gases. The model of Chang and Chang (1998) minimizes overall cost through the solution of a nonlinear programming problem. Unlike our model, their model does not cater for regulatory and environmental constraints. We present linear model and go at length in estimation of environmental hazard cost and scavenged fraction.

Halidi (2011) has presented a mixed integer programming of municipal solid waste management in Ilala municipality. He uses the concept of having collection centers. The proposed model results in a least transportation cost T_{sh} 10, 969,252 per day compared with the one given by Ilala municipality of T_{sh} 14,000,000per day. Furthermore, the study shows that any additional increase of the collection centre capacity up to 500 tons will result in a decrease of the objective function value.

II. THE MATHEMATICAL MODEL

Mathematical formulation, description of objective function and constraints of the model will also be presented. We finally present data which are required by the model, which are waste sources data, collection centers data, land-fill data, transportation cost data, hazard cost and scavengers fraction.

2.1 The Model

2.2 Model Assumptions

The assumptions are intended to formulate the mixed integer programming model which was used to model Municipal Solid Waste Management System for Aba metropolis. These assumptions are as follow:

1. It is assumed that all generated solid waste in Aba metropolis are collected from the sources to the collection centres and then transferred to the landfills.
2. In Aba metropolis there is no waste separation at the sources. Waste separation is done at the collection centers where the scavengers are allowed to do the separations.
3. Industrial and institutional wastes are transferred to the nearest collection centre at the expense of their generators.
4. Transportation cost is proportional to both the distance traveled and the carried load.
5. The collection of waste is done at the time when there is no traffic jam.
6. The distances are measured from the centroid of either the streets or the facilities.

2.3 Proposed Model of MSW for Aba Metropolis

The proposed model is formulated taking into consideration of the waste flow in Aba metropolis. A schematic diagram of the waste flow through the different facilities of the proposed MSWMS in Aba metropolis is shown in Figure 2.1. The residential, commercial and municipal service sources will dispose their waste in the collection bins located in front of them. The various households are expected to take the collection bins to the nearest waste sources to them. The waste source is a point where a big waste container (about 7.1 tons and above) is placed for onward delivery to the collection centers. The collection centre is a centroid where wastes from closer sources are deposited for scavenging before taken to the landfills. A transfer vehicle will be responsible for waste transfer from the sources to the collection centers and then to the landfills.

2.4 Model Flow Diagram

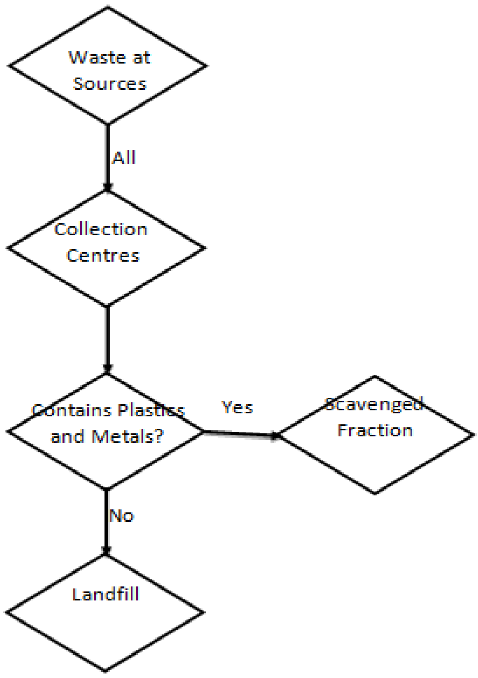

Figure 2.1: Schematic diagram of solid waste flow.

2.5 Model Objective

The study revealed that in Aba metropolis all solid waste are either buried or burnt at the landfill, unlike other municipalities in which some waste are decomposed or recycled. Hence we need to minimize the cost of transporting waste from their sources to land-fill through collection centers. Therefore, the objective of the model is to select between the different potential sites for collection centers in order to minimize the transportation cost of the municipal solid waste. The transportation costs and fixed costs of the facilities is minimized by the mixed integer model.

2.5.1 Decision Variables

Decision variables are the variables within a model that one can control, they are not random variables. There are several decision variables which are the variables that the model is interested in evaluating. These variables are defined as follows:

- x_{ij} is amount (in ton) of daily solid waste to be removed from source i to collection centre j (i = 1,...,m; j = 1,...,n).
- y_{jk} is amount (in ton) of daily solid waste to be removed from collection centre j to land-fill k (j = 1,…n; k = 1,..,K).
- Q_j is a variable which can take value of one or zero. It takes the value one if a collection centre is to be set Up at the location j and zero otherwise, (j = 1,...,n).

2.5.2 Parameters

The parameters are the values that are required by the model as inputs to calculate the decision variables. The proposed model uses the following parameters.

F_j = the fixed daily cost of maintaining the collection center

$Q(0,1)$ = the binary integer, 1 if the collection center is open, and 0 if the collection center is closed

T_{jk} = the transportation cost of waste from the collection center to the Landfill k

Y_{jk} = quantity of waste at the collection center j

L_k = the capacity of the landfill k

X_i = quantity of waste from source i

C_j = the capacity of the collection centre j

N = the number of collection centers to be opened

W_i = all generated waste in tons per day from source i

H_i = the total number of households in a given source i

$i = 1,2,3,\ldots,90,\quad j = 1,2,3,\ldots,15,\ k = 1,2,3$

2.5.3 Mathematical Formulation

The mathematical formulation of the mixed integer model will be as follows: The objective function and the constrained are formulated as

$$\left.\begin{array}{l}
Min.Z(Q,X,Y) = \sum_{j=1}^{15}(F_j + hC_j)Q_j + \sum_{i=1}^{90}\sum_{j=1}^{15} T_{ij}x_{ij} + \sum_{j=1}^{15}\sum_{k=1}^{3} T_{jk}y_{jk} \\
\text{Subject to:} \\
\sum_{j=1}^{15} x_{ij} = W_i, i = 1,2,3,\ldots,90 \\
(1-f)\sum_{i=1}^{90}\sum_{j=1}^{15} x_{ij} = \sum_{j=1}^{15}\sum_{k=1}^{3} y_{jk}, i = 1,2,3,\ldots,90 \\
(1-f)\sum_{i=1}^{90} x_{ij} \le C_j\,(j = 1,2,3,\ldots,15) \\
\sum_{j=1}^{15} y_{jk} \le L_k, k = 1,2,3 \\
\sum_{j=1}^{15} Q_j \le N \\
\sum_{j=1}^{15} Q_j C_j \ge \sum_{i=1}^{90} W_i \\
x_{ij} \ge 0, y_{jk} \ge 0, i = 1,2,3,\ldots,90;\ j = 1,2,3,\ldots,15;\ k = 1,2,3
\end{array}\right\} \qquad (2.1)$$

3. **Data Presentation:** Here, we present data required in this model and they are:

Table 3.1: Amount of Solid Waste Generated/Day from Different Streets/Wards in Aba Metropolis.

	SOURCES	Notation	Waste (Ton/Day)			SOURCES	Notation	Waste (Ton/Day)
S01	Eziama High School	An01	36		S46	Ehi Road Primary Sch.	As21	36
S02	Eziama Central High Sch.	An02	40		S47	Etche Rd Primary Sch	As22	36
S03	Ngwa Cultural Hall	An03	32		S48	Danfodio Rd Primary Sch	As23	24
S04	Old Post Office	An04	37		S49	Aba Town Hall	As24	21
S05	Osusu Primary School	An05	26		S50	Hospital Rd Primary Sch	As25	23
S06	Okigwe Primary School	An06	26		S51	Abayi Com.Primary Sch	Ob01	22
S07	Osusu Secondary School	An07	21		S52	Ehere Comm.Primary Sch	Ob02	32
S08	Holy Ghost Comm.Sch.	An08	17		S53	Umuafor Civic Hall	Ob03	19
S09	St Eugen's Primary School	An09	20		S54	Umuopara Villa Square	Ob04	15
S10	Uratta Council Hall	An10	21		S55	Umuagba Civil Hall	Ob05	11
S11	Stella Maris Sec Sch.	An11	18		S56	Itungwa Cps	Ob06	10
S12	Gulf Course Primary Sch	An12	20		S57	Umuhu Alaoji Hall	Ob07	23
S13	Water Tank Premises	An13	23		S58	Umuokereke-Uhie Court	Ob08	16
S14	Umuola Okpulor Hall	An14	27		S59	Iheoji / Umuojima	Ob09	33

S15	Umuola Egbelu Hall	An15	18		S60	Mboko -Okpulor Hall	Ob10	14
S16	Btc School Premises	An16	40		S61	Umuobikwa	Ob11	14
S17	Old Internal Revenue Premises	An17	42		S62	Ohanze Cps	Ob12	18
S18	Eziobi Primary School	An18	37		S63	Ovom Girls Sec Sch	Ob13	40
S19	Sacred Heart College	An19	32		S64	Umuomasi	Ob14	21
S20	Wilcox Memorial School	An20	37		S65	Umuomai Village Hall	Ob15	17
S21	New Umuahia Road Primary Sch	An21	22		S66	Umuogele Cps Ntighauzo	Ob16	24
S22	Ogbor Hill Primary School	An22	16		S67	Ohanze Agwo-Mkt	Ob17	38
S23	Federal Housing Estate	An23	21		S68	Abiak Village Hall	Ob18	20
S24	Umuogor / Umuasoke Hall	An24	24		S69	Umuokirika Villa Square	Os01	19
S25	Asa Okpulor Council Hall	An25	19		S70	Abayi Ariaria Pry Sch.	Os02	41
S26	Eziukwu Okigwe Rd Pry Sch.	As01	26		S71	Umuojima Ogbu Pry Sch	Os03	20
S27	Eziukwu Ebenator (Omuma)	As02	26		S72	Umungasi Post Office	Os04	36
S28	Eziukwu Asa Okpuaga Hall	As03	23		S73	Ngwa High School	Os05	32
S29	Asa-Osumenyi Hall	As04	31		S74	Okpuala-Aro Villa Hall	Os06	8
S30	Aba Township Primary Sch.	As05	20		S75	Umuihuoma Village Hall	Os07	17
S31	Enyimba- Isiekenesi Hall	As06	41		S76	Okpuala-Ukwu Comm. Sch.	Os08	12
S32	Enyimba- Abaukwu Village Hall	As07	28		S77	Ibibi Uratta Council Hall	Os09	15
S33	Ngwa-Ndoki Road Primary Sch.	As08	37		S78	Umuagbai Comm.Sch	Os10	11
S34	Ngwa-Ihioma Hall	As09	17		S79	Umumba-Umuru Comm Sch	Os11	17
S35	Ngwa Umuagbai East Pry Sch	As10	20		S80	Umuaba Village Hall	Os12	10
S36	Ohazu /Akoli Hall	As11	16		S81	Anigwe Primary School	Ug01	26
S37	Ohazu / Awkuzu Hall (Ngwa Rd)	As12	15		S82	Ugwunagbo Custom Court	Ug02	18
S38	Ohazu / Ihieorji Sec Sch.	As13	15		S83	Amapu Ideobia Pry Sch.	Ug03	22
S39	Igwebuike- Ohabiam Sec Sch.	As14	20		S84	Owere Aba Primary Sch.	Ug04	14
S40	Igwebuikw-Nnentu Villa Hall	As15	17		S85	Akamu Ngwa High Sch.	Ug05	12
S41	Asa Road Primary Sch.	As16	48		S86	Asa Umukwa Primary Sch.	Ug06	10
S42	St.Joseph's Sch. College	As17	40		S87	Asa Nnetu Motor Park Mkt	Ug07	31
S43	Market Road Primary Sch.	As18	28		S88	Alaoji Village Hall	Ug08	17
S44	Old Court Primary Sch.	As19	18		S89	Ihieobeaku Primary Sch	Ug09	8
S45	School Of Health	As20	27		S90	Ngwaiyiekwe Bus Stop	Ug10	20

3.2 Land-fill Data

Three landfills are proposed. The data required for the land-fills are the locations and capacities. The locations of the landfills are Umuhu Alaoji in Obingwa LGA, Opkuala-Aro in Osisioma Ngwa LGA and Ngwaiyiekwe in Ugwunaegbo LGA. The capacities of the landfills should accommodate all solid waste generated in Aba metropolis. Thus, the total solid waste generation in Aba metropolis is 2124 ton/day, and each landfill is 800 tons/day which is greater or equal to the total capacity of the landfills.

3.3 Transportation Cost Matrix

The transportation cost (per ton) for each route is determined by multiplying the distance traveled with transportation cost per ton per kilometer. The transportation cost per ton per kilometer is fixed at N80 which is obtained from ASEPA, Aba office. Table 3.2 shows the transportation cost of solid waste from each candidate collection centres to the landfill.

3.4 Estimation of Environmental hazard cost

The hazard cost per weight of MSW (h) is estimated as follows; The organic matter of MSW constitute a hazard to the environment because they transform to landfill gas which are greenhouse gases (GHGs) that cause global warming. The MSW of Sub-Saharan Africa is rich in organic wastes. The organic matter of Sub-Saharan waste is around 58%, which is higher than the typical content of about 50% in developed countries [Couth & Trois (2011)]. Landfill gas is a mixture of methane (45–60%), carbon dioxide (40–55%) and trace components; for this study we adopt the median of the ranges to give methane content of 52.5% and carbon dioxide content of 47.5%. The anaerobic degradation of one ton of wet MSW, supposed to contain 60% organic matter and 40% moisture would theoretically generate 150 kg of methane, but various studies showed an actual rate of generation of landfill gas range from 39–100 kg methane per ton of waste [Themelis & Ulloa (2007]. A methane generation rate of about 36kg of methane per ton of MSW landfilled has been adopted as conservative estimate which can be used with confidence [Themelis & Ulloa (2007)]. Adopting molar mass of methan and carbone dioxide as 16kg and 44kg respectively means that the corresponding conservative estimate of Landfill gas generation rate is about 36+99=135kg of Landfill gas per ton of MSW. The majority of emissions (85 percent) are priced at less than US$10 per ton of carbon dioxide equivalent, which is considerably lower than the price that economic models have estimated is needed to meet the 2°C climate stabilization goal recommended by scientists [The World Bank and Ecofys (2015)]. For this study we adopt this price as reference that can be used as US$ (χ10) per ton of carbon dioxide equivalent, where χ represents fraction of the actual price in Aba to the reference. Based on the ongoing the environmental hazard cost can now be pegged at h= US$ 1.35χ per ton of MSW. In this study, χ=1 will be adopted and both the Central Bank of Nigeria monetary Exchange rate of 1US$=₦199 and parallel market exchange rate of about 1US$≅₦400 will be used. This means that h=₦268.65 and ₦540 hazard cost per ton of MSW will be used in numerical optimization.
h=₦268.65 and ₦540

3.5 Estimation of Scavenged Fraction

The scavenged fraction (f) of the MSW is estimated as follows; An authoritative detailed study of MSW composition in Aba is not seen but such a study is available for Abuja the Federal Capital Territory. Abuja is a convergence location of all the Nigeria's peoples and thus a microcosm of Nigeria consumption pattern and lifestyle at large. According to Ogwueleka [Toochukwu Chibueze Ogwueleka, Survey of household waste composition and quantitiesin Abuja, NigeriaResources, Conservation and Recycling 77 (2013) 52– 60], the recovery and reuse of valuable materials in Abuja are conducted in a very primitive way, by scavengers. Scavengers salvage plastic, metals, papers. The composition of the waste at source is 63.6% organic waste, 9.7% paper, 8.7% plastics, 3.2% metal, 2.6% glass, 1.6% textile and 10.6% others (unclassified). It was concluded from observing that the scavenger at Aba were mainly after plastics and metal. This means that we adopt a scavenged proportion of about f = 0.119

IV. RESULTS AND ANALYSIS

4.1 Introduction

The developed model was solved using MATLAB 2015a version. MATLAB (matrix laboratory) is a fourth-generation high-level programming language and interactive environment for numerical computation, visualization and programming. MATLAB is developed by MathWorks. It allows matrix manipulations; plotting of functions and data; implementation of algorithms; creation of user interfaces with programs written in other languages, including C, C++, Java and FORTRAN; analyze data; develop algorithms; and create models and applications. It has numerous built-in commands and math functions that help you in mathematical calculations, generating plots, and performing numerical methods. In this section, the results and analysis of the developed model will be presented and discussed

4.2 Results

As pointed out in chapter three, capacities of the collection centres ranges from 142 tons to 350 tons. Thus, in generation of results we considered the cases where the maximum capacity of collection centres are 142, 172, 202, 232, 262, 292, 322 and 350 tons. The parameters of the transportation network are: the number of collection centres is 15; the number of sources is 90; the number of landfills is 3; daily fixed cost of collection centre is N112,500.00; the hazard cost per ton of waste at collection centre is N268.65; scavenged fraction is 0.119; total waste generated daily in Aba metropolis is 2124 tons; and the number of collection centers to be stationed is 15. Details of the discussion for each case are given below:

4.2.1 Case A: Fixed Capacity

Case A1: Maximum Capacity of 142 tons

In this case, the model is left to choose the maximum number of collection centres. An optimal solution is obtained by setting location for waste collection centres at Osusu (AN05), Uratta (AN10), BTC (AN16), Asa

Okpulor (AN25), Eziukwu Asa Okpuaga (AS03), Enyimba-Abaukwu (AS07), Igwebuike-Nnentu (AS15), Ehi Road (AS21), Abayi (OB01), Umuagba (OB05), Ovom (OB13), Abayi Ariaria (OS02), Umuojima Ogbu (OS03), Umuaba (OS12) and Akamu Ngwa (UG05). In the optimal solution, minimum daily transportation cost of waste transfer from sources to collection centres and then to the landfills is N5, 450, 200.00.

Case A2: Maximum Capacity of 172 tons

In this case, the model is left to choose the maximum number of collection centres. An optimal solution is obtained by setting location for waste collection centres at Osusu (AN05), Uratta (AN10), BTC (AN16), Asa Okpulor (AN25), Enyimba-Abaukwu (AS07), Igwebuike-Nnentu (AS15), Ehi Road (AS21), Abayi (OB01), Umuagba (OB05), Abayi Ariaria (OS02), Umuojima Ogbu (OS03), Umuaba (OS12) and Akamu Ngwa (UG05). The two closed collection centres for minimum cost are the Eziukwu Asa Okpuaga (AS03) and Ovom (OB13). In the optimal solution, minimum daily transportation cost of waste transfer from sources to collection centres and then to the landfills is N4,986,400.00.

Case A3: Maximum Capacity of 202 tons

In this case, the model is left to choose the maximum number of collection centres. An optimal solution is obtained by setting location for waste collection centres at Uratta (AN10), BTC (AN16), Asa Okpulor (AN25), Enyimba-Abaukwu (AS07), Ehi Road (AS21), Abayi (OB01), Umuagba (OB05), Abayi Ariaria (OS02), Umuojima Ogbu (OS03), Umuaba (OS12) and Akamu Ngwa (UG05). The four closed collection centres for minimum cost are the Osusu (AN05), Eziukwu Asa Okpuaga (AS03), Igwebuike-Nnentu (AS15) and Ovom (OB13). In the optimal solution, minimum daily transportation cost of waste transfer from sources to collection centres and then to the landfills is N4, 615, 300.00.

Case A4: Maximum Capacity of 232 tons

In this case, the model is left to choose the maximum number of collection centres. An optimal solution is obtained by setting location for waste collection centres at Uratta (AN10), Asa Okpulor (AN25), Enyimba-Abaukwu (AS07), Igwebuike-Nnentu (AS15), Abayi (OB01), Umuagba (OB05), Abayi Ariaria (OS02), Umuojima Ogbu (OS03), Umuaba (OS12) and Akamu Ngwa (UG05). The five closed collection centres for minimum cost are the Osusu (AN05), BTC (AN16), Eziukwu Asa Okpuaga (AS03), Ehi Road (AS21) and Ovom (OB13). In the optimal solution, minimum daily transportation cost of waste transfer from sources to collection centres and then to the landfills is N4, 442, 300.00.

Case A5: Maximum Capacity of 262 tons

In this case, the model is left to choose the maximum number of collection centres. An optimal solution is obtained by setting location for waste collection centres at Asa Okpulor (AN25), Enyimba-Abaukwu (AS07), Igwebuike-Nnentu (AS15), Abayi (OB01), Umuagba (OB05), Abayi Ariaria (OS02), Umuojima Ogbu (OS03), Umuaba (OS12) and Akamu Ngwa (UG05). The six closed collection centres for minimum cost are the Osusu (AN05), Uratta (AN10), BTC (AN16), Eziukwu Asa Okpuaga (AS03), Ehi Road (AS21) and Ovom (OB13). In the optimal solution, minimum daily transportation cost of waste transfer from sources to collection centres and then to the landfills is N4, 270, 300.00.

Case A6: Maximum Capacity of 292 tons

In this case, the model is left to choose the maximum number of collection centres. An optimal solution is obtained by setting location for waste collection centres at Asa Okpulor (AN25), Enyimba-Abaukwu (AS07), Igwebuike-Nnentu (AS15), Abayi (OB01), Umuagba (OB05), Abayi Ariaria (OS02), Umuojima Ogbu (OS03), and Akamu Ngwa (UG05). The seven closed collection centres for minimum cost are the Osusu (AN05), Uratta (AN10), BTC (AN16), Eziukwu Asa Okpuaga (AS03), Ehi Road (AS21), Ovom (OB13) and Umuaba (OS12). In the optimal solution, minimum daily transportation cost of waste transfer from sources to collection centres and then to the landfills is N4, 100, 800.00.

Case A7: Maximum Capacity of 322 tons

In this case, the model is left to choose the maximum number of collection centres. An optimal solution is obtained by setting location for waste collection centres at Asa Okpulor (AN25), Enyimba-Abaukwu (AS07), Abayi (OB01), Umuagba (OB05), Abayi Ariaria (OS02), Umuojima Ogbu (OS03), and Akamu Ngwa (UG05). The eight closed collection centres for minimum cost are the Osusu (AN05), Uratta (AN10), BTC (AN16), Eziukwu Asa Okpuaga (AS03), Ehi Road (AS21), Ovom (OB13), Umuaba (OS12) and Igwebuike-Nnentu

(AS15). In the optimal solution, minimum daily transportation cost of waste transfer from sources to collection centres and then to the landfills is N3, 929, 900.00.

Case A8: Maximum Capacity of 350 tons

In this case, the model is left to choose the maximum number of collection centres. An optimal solution is obtained by setting location for waste collection centres at Asa Okpulor (AN25), Enyimba-Abaukwu (AS07), Abayi (OB01), Umuagba (OB05), Abayi Ariaria (OS02), Umuojima Ogbu (OS03), and Akamu Ngwa (UG05). The eight closed collection centres for minimum cost are the Osusu (AN05), Uratta (AN10), BTC (AN16), Eziukwu Asa Okpuaga (AS03), Ehi Road (AS21), Ovom (OB13), Umuaba (OS12) and Igwebuike-Nnentu (AS15). In the optimal solution, minimum daily transportation cost of waste transfer from sources to collection centres and then to the landfills is N3, 948, 700.00. The results for the different cases when the collection centre capacities are fixed is summarized in Table 4.1, while the binary variables Q_j, the quantities moved from collection centres to landfills and cost of transportation at different values of capacity of collection centres are summarized in Table 4.2 .

Table 4.1: Results for Different Cases when Capacity is fixed

S/N	Case	Capacity of Collection Centre in tons	Number of Collection Centres Required	Transportation cost (₦)
1	A1	142	15	5, 450, 200.00
2.	A2	172	13	4, 986, 400.00
3.	A3	202	11	4, 615, 300.00
4.	A4	232	10	4, 442, 300.00
5.	A5	262	9	4, 270, 300.00
6.	A6	292	8	4, 100, 800.00
7.	A7	322	7	3, 929, 900.00
8.	A8	350	7	3, 948, 700.00

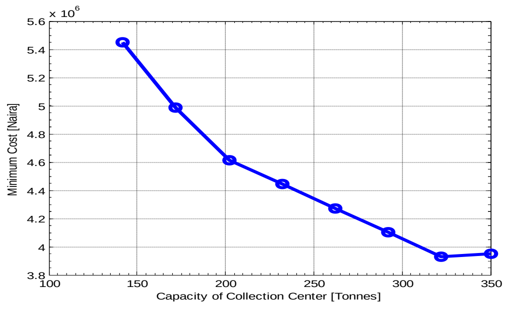

Figure 4.1: A Line graph plot of minimal costs against capacities.

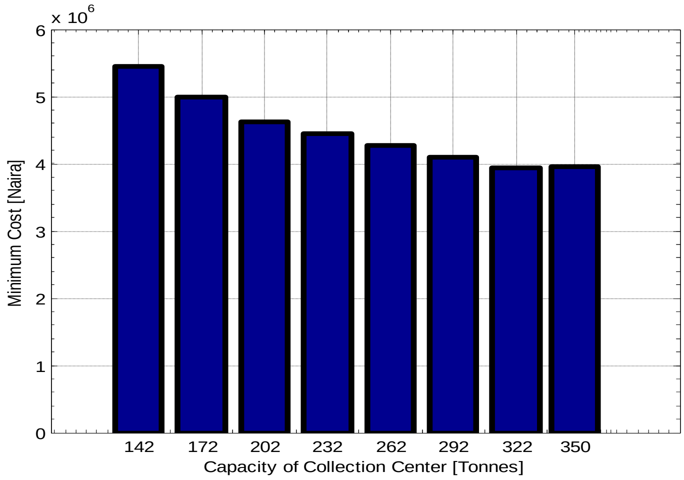

Figure 4.2: A Bar Chart Plot of Minimal Costs against Capacities.

It is seen that cost reduced monotonically with rise in capacity of the collection centre. There seems to be a slight rise beyond C = 322 tons.

Table 4.2: A summary of the binary variables Q_j, the quantities moved from collection centres to landfills and minimum cost of transportation at different values of capacity of collection centres.

Cost	5, 450,200	4, 986, 400	4, 615, 300	4, 442, 300	4, 270, 300	4, 100, 800	3, 929, 900	3, 948, 700
C	142	172	202	232	262	292	322	350
Q1	1.0000	0	0	0	0	0	0	0
Q2	1.0000	0	0	0	0	0	0	0
Q3	1.0000	1.0000	0	0	0	0	0	0
Q4	1.0000	1.0000	0	0	0	0	0	0
Q5	1.0000	1.0000	1.0000	0	0	0	0	0
Q6	1.0000	1.0000	1.0000	1.0000	0	0	0	0
Q7	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0
Q8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0
Q9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Q10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Q11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Q12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Q13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Q14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Q15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
C1-L1	142.0000	140.0000	140.0000	142.0000	20.0000	0	0	0
C1-L2	0	0	0	0	0	0	0	0
C1-L3	0	0	0	0	0	0	0	0
C2-L1	142.0000	172.0000	202.0000	18.7680	0	0	0	0
C2-L2	0	0	0	0	0	0	0	0
C2-L3	0	0	0	0	0	0	0	0
C3-L1	0	0	0	0	0	0	0	0
C3-L2	142.0000	172.0000	2.0000	0	0	0	0	0
C3-L3	0	0	0	0	0	0	0	0
C4-L1	140.0000	171.6000	102.0000	225.2320	184.0000	124.0000	64.0000	8.0000
C4-L2	2.0000	0	100.0000	6.7680	78.0000	0	0	0
C4-L3	0	0	0	0	0	0	0	0
C5-L1	0	0	0	0	0	0	0	0
C5-L2	0	0	0	0	0	0	0	0
C5-L3	28.7680	0	0	0	0	0	0	0
C6-L1	0	0	0	0	0	0	0	0
C6-L2	0	0	0	0	0	0	0	0

C6-L3	142.0000	172.0000	202.0000	232.0000	262.0000	290.7680	200.7680	116.7680
C7-L1	0	0	0	0	0	0	0	0
C7-L2	0	0	0	0	0	0	0	0
C7-L3	0	0	0	0	0	0	0	0
C8-L1	0	0	0	0	0	0	0	0
C8-L2	0	0	0	0	0	0	0	0
C8-L3	142.0000	134.7680	54.7680	0	0	0	0	0
C9-L1	142.0000	172.0000	202.0000	232.0000	262.0000	292.0000	322.0000	350.0000
C9-L2	0	0	0	0	0	0	0	0
C9-L3	0	0	0	0	0	0	0	0
C10-L1	142.0000	172.0000	202.0000	232.0000	262.0000	292.0000	322.0000	350.0000
C10-L2	0	0	0	0	0	0	0	0
C10-L3	0	0	0	0	0	0	0	0
C11-L1	0	0	0	0	0	0	0	0
C11-L2	0	0	0	0	0	0	0	0
C11-L3	142.0000	0	0	0	0	0	0	0
C12-L1	0	0	0	0	0	0	0	0
C12-L2	142.0000	172.0000	202.0000	232.0000	262.0000	292.0000	322.0000	350.0000
C12-L3	0	0	0	0	0	0	0	0
C13-L1	0	0	0	0	0	0	0	0
C13-L2	142.0000	172.0000	202.0000	232.0000	262.0000	292.0000	3222.0000	350.0000
C13-L3	0	0	0	0	0	0	0	0
C14-L1	0	0	0			0	0	0
C14-L2	142.0000	172.0000	202.0000	232.0000	40.7680	142.0000	0	0
C14-L3	0	0	0	0	0	0	0	0
C15-L1	0	0	0	0	0	0	0	0
C15-L2	0	0	0	0	0	0	0	0
C15-L3	142.0000	172.0000	202.0000	232.0000	262.0000	292.0000	322.0000	350.0000

5. Summary, Conclusion and Recommendations

5.1 Summary

We have been able to present a Mathematical model of municipal solid waste management system (MSWMS) for Aba metropolis using data collected from ASEPA. The mixed integer programming problem was used in formulating the (MSWMS) model. The model is solved using Linear Programming toolbox of MATLAB 2015a software for Windows. It would require extra time in entering the data, which is dependent on the user typing skills as they are typed in the code. The optimal results show that the best model would include 3 landfills, 322 tons capacity Collection Centers and seven (7) Collection Centers.

5.2 Conclusion

In this research, municipal solid waste management in Aba metropolis is modeled with the concept of having collection centres. It should be noted that the proposed model results in a least transportation cost of N3, 929, 900.00 per day compared with the one given by ASEPA, Aba zonal office of N5, 600, 000.000 per day. Further, the study shows that any additional increase of the collection centre capacity up to 350 tons will result in upward rise and fall of the objective function value. The main limitation in this study is the quality of the data about the waste sources distribution and the detailed maps. The waste source is assumed to be located at the centroid of the waste source area. Moreover, the distance between different places is assumed to be the distance between their respectively centroid. This assumption has resulted in a solution presented in this dissertation, which has been called the best solution. The actual optimal solution will be reached only with highly detailed data about waste source locations and distributions. The result of the presented model is based on some assumptions in the absence of the available data and could change with more accurate data.

5.3 Recommendations

In order to improve solid waste management in Aba, the emerging conditions for the adoption and operation of the MSWMS model needs to be addressed through number of institutional reforms and management designs and arrangements including; stakeholder collaborations, encouraging informal sector participation and the development of technological innovations for solid waste management. The future works is also discussed at the end of this section.

5.3.1 The Need for Municipal Authority and Private Sector Collaboration

There is the need for efficient collaboration between the municipal authority and the private sector. Municipal solid waste management should not be the sole responsibility of the municipal authority but a collaborative effort of the parties. The municipal authority should put in place mechanisms such as stakeholders forum, performance review meetings. This may facilitate the process of data and information sharing on solid waste management among the parties responsible for solid waste management in Aba. This is essential for experience sharing and avoidance of duplication of efforts.

5.3.2 Recognition of the Private Informal Sector Operators

In order to ensure the adoption and practice of the MSWMS model in Aba, there is the need to recognize the contribution of the private informal sectors such as scavengers and itinerant waste buyers in urban solid waste management. The private informal sector needs to be organized into associations and groups so that programs can be designed to build their capacities and also assist them with protective equipment to efficiently participate in the solid waste management process. Through the formation of co-operative societies or micro-enterprises, it is often possible to considerably increase the job stability and earnings of such informal sector workers and to enhance the effectiveness of their contribution to waste management.

5.3.3 The Need for Technological Innovations

In order to improve solid waste management in Aba, the municipal authority and private companies need to formulate strategies and implement technological innovations necessary for effecting improved separation at source, resource recovery, recycling and disposal of solid waste in Aba. Some of the known technologies observed in Indian cities such as incineration, conversion to bio-gas, refuse derived fuel and composting can as well be adopted and practiced in Aba.

5.4 Future Work

The main aim of this work is not the final result, but the application of Operations Research methodologies to model the MSW in Aba using the concept of collection centers. An important area for future research is to include the recycling of te collected waste in formulation of the model. This impact requires detailed information on the cost of recycling per ton of waste which was not available during this study. Inclusion of land-fill as among the collection centers where by the sources nearby it could send their wastes and a decision support system (DSS) for Aba metropolis where various parameters can be switched for decision making, can be undertaken as a continuation of this work.

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