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Research Paper

Minimum Spacings between 3-PrimeFactors Numbers till 1 Trillion

Neeraj Anant Pande

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ABSTRACT : *Numbers with exactly 3 prime divisors to them are considered. They fall in category of 3-PrimeFactors numbers. The trends in extreme densities of these types of numbers are known. Here we analyse minimum spacings between successive 3-PrimeFactors numbers in both ways, viz., in whole range of inspection which is 1 trillion for this work and in blocks of various sizes of base powers, i.e., powers of 10 in various ranges. Minimum in-block spacings between successive 3-PrimeFactors numbers, number of occurrences of such minimum spacings, number of blocks containing pairs of closed placed successive 3-PrimeFactors numbers and first and last starters 3-PrimeFactors numbers which maintain minimum distance with their corresponding successors in the whole range or respective sized block in various ranges are determined which throw light on gaps between successive 3-PrimeFactors numbers.*

Keywords : *Prime number, k-PrimeFactors number, 3-PrimeFactors number, Minimum spacing*
Mathematics Subject Classification (2010) - 11A51, 11N05, 11N80

I. INTRODUCTION

Number theory is primarily study of integers, although for exploring the properties of the same, one has to expand inspection domain beyond them to rationals, irrationals, i.e., all reals and at times even complex numbers. But the main focus is on integers. Integers, that to if we limit ourselves to positive integers, seem to be simplest type of numbers. But there are occasions where they exhibit most complicated behavior; and one such instance is that of prime numbers [1]. They as well as their types are so random-looking that their behavior is needed to be studied by inspecting all of them in higher and higher ranges [3], [4].

1.1 k-PRIMEFACTORS NUMBERS

Prime numbers is a special positive integer greater than 1 which has only 1 indivisible divisor (itself). This concept is generalized by author [6].

Definition (*k*-PrimeFactors Number) : For any integer $k \geq 0$, a positive integer having k number of prime factors, which need not be necessarily distinct, is called as *k*-PrimeFactors number.Infinitude of primes allows their infinite multiplicative combinations taken any $k \geq 1$ at a time and hence there are infinitely many *k*-PrimeFactors numbers for each $k \geq 1$. The case of $k = 0$ is odd man out as there is only one 0-PrimeFactor number, viz., 1.Of these, 1-PrimeFactor numbers, which are usual primes [3] and 2-PrimeFactors numbers have been analysed in quite detail for their low [6] and high [7] densities, minimum [8] and maximum [9] spacings between successive candidates, digits in their units places [10] and units & tens places [11].

1.2 Primefactors Numbers

The value of $k = 3$ is taken up now. That leads to following.

Definition (3-PrimeFactors Number) : A positive integer having exactly 3 prime divisors, which need not be necessarily distinct, is called as 3-PrimeFactors number.

First few 3-PrimeFactors numbers are :
8, 12, 18, 20, 27, 28, 30, ⋯

Each of them has precisely 3 prime factors : $8 = 2^3$, $12 = 2^2 \times 3$, $18 = 2 \times 3^2$, $20 = 2^2 \times 5$, $27 = 3^3$, $28 = 2^2 \times 7$, $30 = 2 \times 3 \times 5$ and so on.

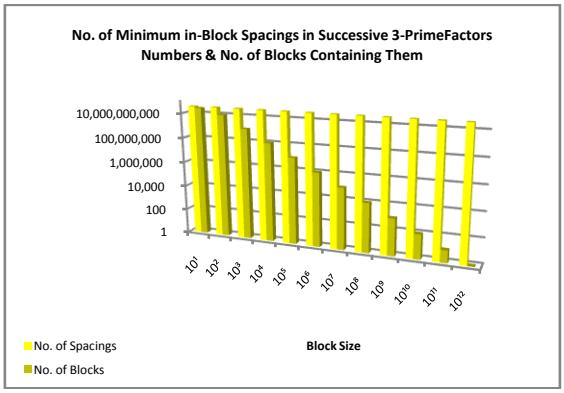
The details of lowest [12] and highest [13] occurrence percentages of 3-PrimeFactors numbers in different ranges for different block-sizes are now available.The huge database of 3-PrimeFactors numbers required for carrying out this analysis could be made available by using most efficient algorithms [2].

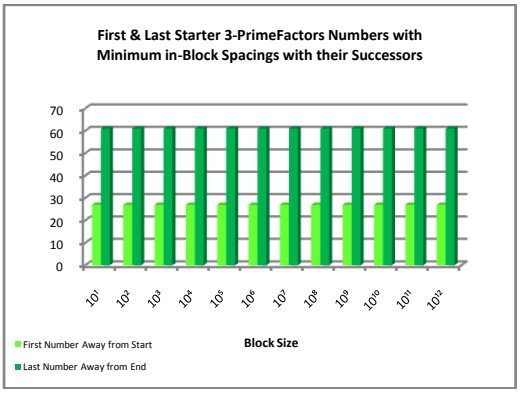
1.3 Minimum Spacing Between Successive 3-Primefactors Numbers In Blocks Of Sizes 10n

The selected range was up to 1 trillion. This range was divided into smaller blocks of sizes of powers of 10 - like 10, 100, 1000 etc. till 1 trillion itself. For each block size, minimum spacing between successive 3-PrimeFactors numbers in each sized block, number of times such minimum spacing occurs, First and last 3-PrimeFactors numbers keeping minimum spacings with immediate next 3-PrimeFactors number, and number of blocks in which such minimally spaced successive pairs occur are determined to be as following.

Sr. No.	Block-Size	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	10^1	1	34,850,174,142	27	999,999,999,938	27,583,741,772
2	10^2	1	36,029,848,844	27	999,999,999,938	9,664,840,449
3	10^3	1	36,029,848,844	27	999,999,999,938	1,000,000,000
4	10^4	1	36,029,848,844	27	999,999,999,938	100,000,000
5	10^5	1	36,029,848,844	27	999,999,999,938	10,000,000
6	10^6	1	36,029,848,844	27	999,999,999,938	1,000,000
7	10^7	1	36,029,848,844	27	999,999,999,938	100,000
8	10^8	1	36,029,848,844	27	999,999,999,938	10,000
9	10^9	1	36,029,848,844	27	999,999,999,938	1,000
10	10^{10}	1	36,029,848,844	27	999,999,999,938	100
11	10^{11}	1	36,029,848,844	27	999,999,999,938	10
12	10^{12}	1	36,029,848,844	27	999,999,999,938	1

As seen, the minimum spacing between successive 3-PrimeFactors numbers is 1 and it has occurred quite frequently. Compared to their total number, viz., 209214982911 till 1 trillion, the figure of 36029848844 is just above 17%.





The first and last starter 3-PrimeFactors number having minimum spacing with its successor is same for all block sizes.

IV.A. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10

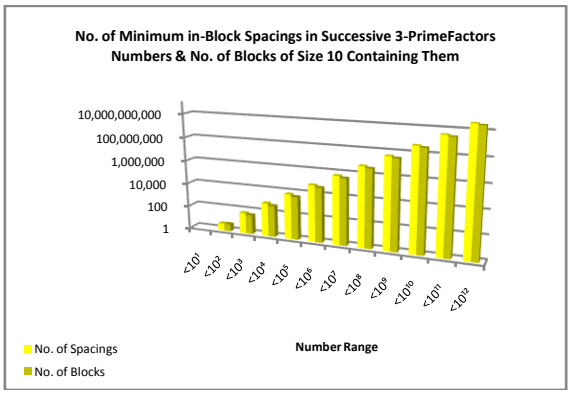
As in case of 2-PrimeFactors numbers [8], for fixed block-sizes, minimum spacings between 3-PrimeFactors numbers in various ranges is undertaken.The beginner block size is 10, block 0 representing number range 0 to 9, 10 representing range 10 to 19 and so on.

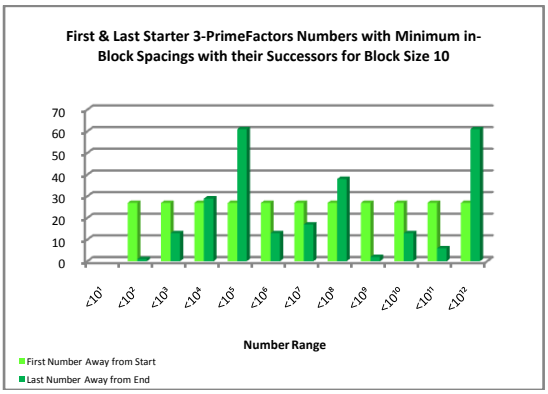
Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^1$	-	-	-	-	-
2	$<10^2$	1	4	27	98	4
3	$<10^3$	1	59	27	986	44
4	$<10^4$	1	688	27	9,970	467
5	$<10^5$	1	6,677	27	99,938	4,542
6	$<10^6$	1	61,694	27	999,986	43,046
7	$<10^7$	1	561,801	27	9,999,982	401,553
8	$<10^8$	1	5,082,560	27	99,999,961	3,724,830
9	$<10^9$	1	46,011,566	27	999,999,997	34,488,595
10	$<10^{10}$	1	417,709,493	27	9,999,999,986	319,539,629
11	$<10^{11}$	1	3,807,307,502	27	99,999,999,993	2,965,687,034

*Corresponding Author: Neeraj Anant Pande

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
12	$<10^{12}$	1	34,850,174,142	27	999,999,999,938	27,583,741,772

In the first range 0 to 9, there is no occurrence of any pair of successive 3-PrimeFactors numbers for checking minimum spacing between them. Although the very first 3-PrimeFactors number 8 lies in this range, but its successor 12 doesn't come in it and hence this pair cannot be considered as member of this range. For all higher ranges, the minimum spacing between successive 3-PrimeFactors numbers is minimum possible, i.e., 1. It begins quite early with fifth pair. So while the first minimum spacing pair is at a fixed distance from 0, the last pair keeps occurring a bit near and far from range-end.



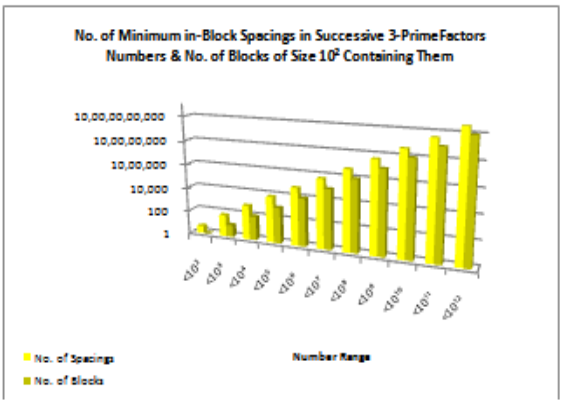


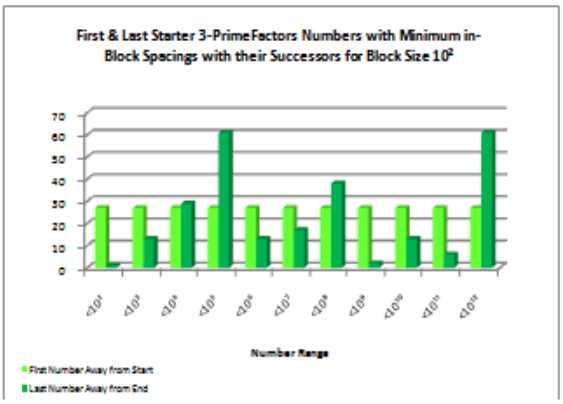
Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^2

Next block-size under consideration is 102, i.e., 100. For this case, block 0 will indicate number range 0 to 99, block 100 will indicate range 100 to 199 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^2					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^2$	1	4	27	98	1
2	$<10^3$	1	63	27	986	10
3	$<10^4$	1	727	27	9,970	100
4	$<10^5$	1	7,014	27	99,938	1,000
5	$<10^6$	1	64,556	27	999,986	9,984
6	$<10^7$	1	585,725	27	9,999,982	99,598
7	$<10^8$	1	5,284,711	27	99,999,961	993,118
8	$<10^9$	1	47,753,279	27	999,999,997	9,887,379
9	$<10^{10}$	1	432,841,730	27	9,999,999,986	98,292,953
10	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	975,520,526
11	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	9,664,840,449

These values are depicted graphically as follows.



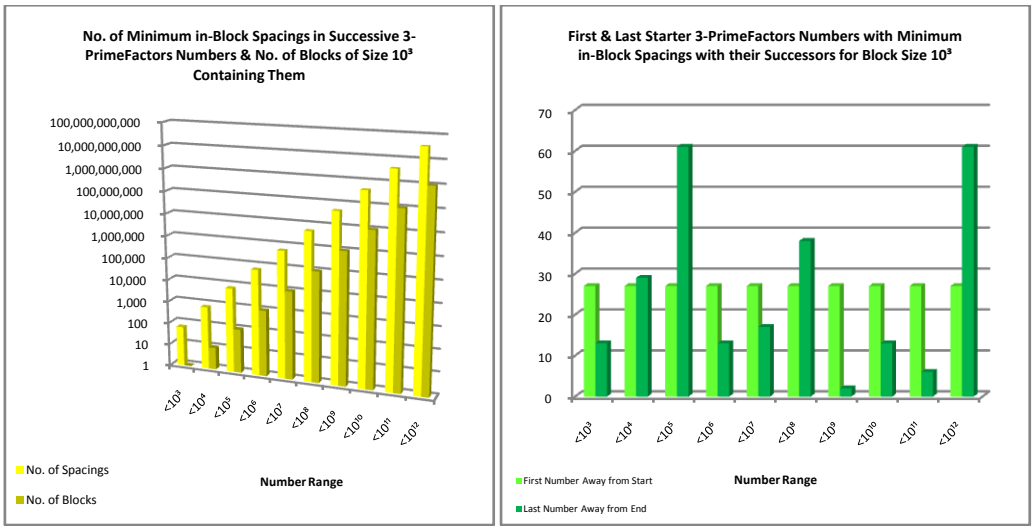


IV.B. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^3

Higher one is block-size 10^3, i.e., 1000, where block 0 gives number range 0 to 9999, block 10000 gives range 10000 to 19999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^3					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^3$	1	63	27	986	1
2	$<10^4$	1	727	27	9,970	10
3	$<10^5$	1	7,014	27	99,938	100
4	$<10^6$	1	64,556	27	999,986	1,000
5	$<10^7$	1	585,725	27	9,999,982	10,000
6	$<10^8$	1	5,284,711	27	99,999,961	100,000
7	$<10^9$	1	47,753,279	27	999,999,997	1,000,000
8	$<10^{10}$	1	432,841,730	27	9,999,999,986	10,000,000
9	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	100,000,000
10	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	1,000,000,000

Naturally, with increase in the block-size, there is decrease in the number of blocks containing successive 3-PrimeFactors numbers having minimum spacings between them.

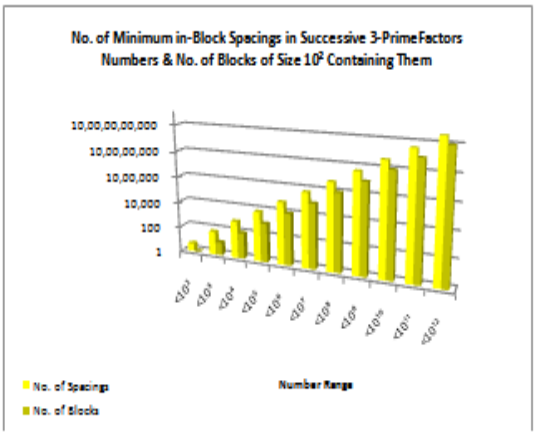


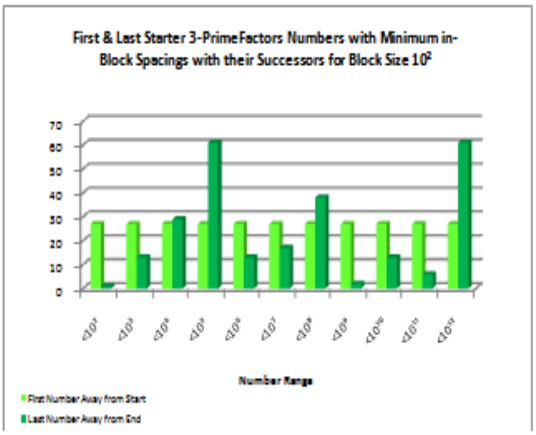
IV.C. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^4

Now let's consider block size 10^4, i.e., 10000, block 0 being number range 0 to 9999, block 10000 being range 10000 to 19999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^4					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^4$	1	727	27	9,970	1
2	$<10^5$	1	7,014	27	99,938	10
3	$<10^6$	1	64,556	27	999,986	100
4	$<10^7$	1	585,725	27	9,999,982	1,000
5	$<10^8$	1	5,284,711	27	99,999,961	10,000
6	$<10^9$	1	47,753,279	27	999,999,997	100,000
7	$<10^{10}$	1	432,841,730	27	9,999,999,986	1,000,000
8	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	10,000,000
9	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	100,000,000

Each block in each range, in fact, from previous block-size itself, has started containing pairs of successive 3-PrimeFactors numbers with minimum spacing.



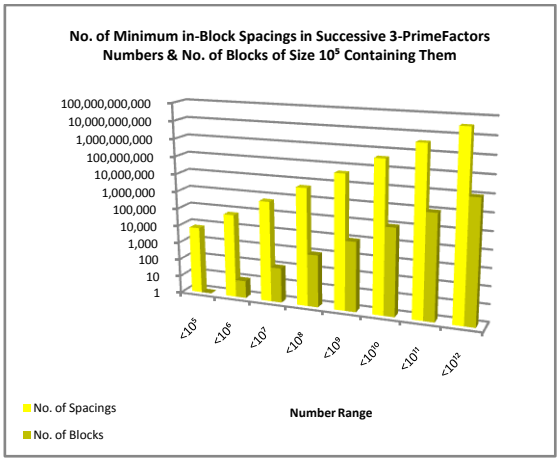


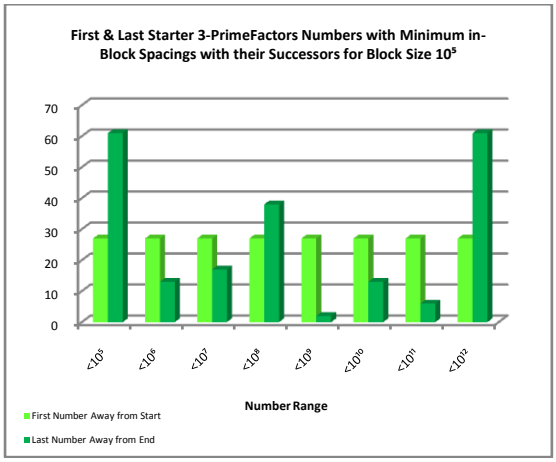
IV.D. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^5

Further blocks of size 10^5, i.e., 100000 are under consideration, for which block 0 means number range 0 to 99999, block 100000 means range 100000 to 199999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^5					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^5$	1	7,014	27	99,938	1
2	$<10^6$	1	64,556	27	999,986	10
3	$<10^7$	1	585,725	27	9,999,982	100
4	$<10^8$	1	5,284,711	27	99,999,961	1,000
5	$<10^9$	1	47,753,279	27	999,999,997	10,000
6	$<10^{10}$	1	432,841,730	27	9,999,999,986	100,000
7	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	1,000,000
8	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	10,000,000

The values remain same now with decrease in number of blocks.





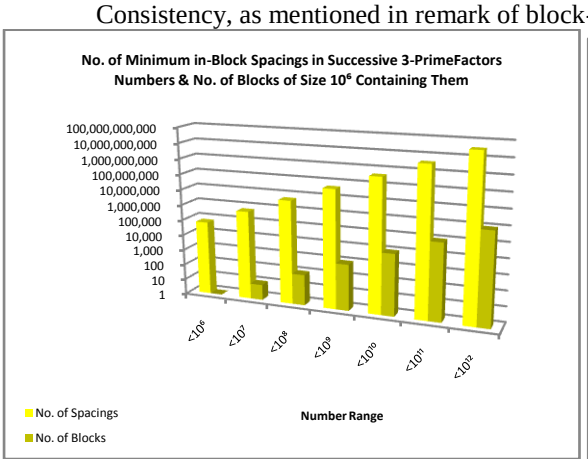
IV.E. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^6

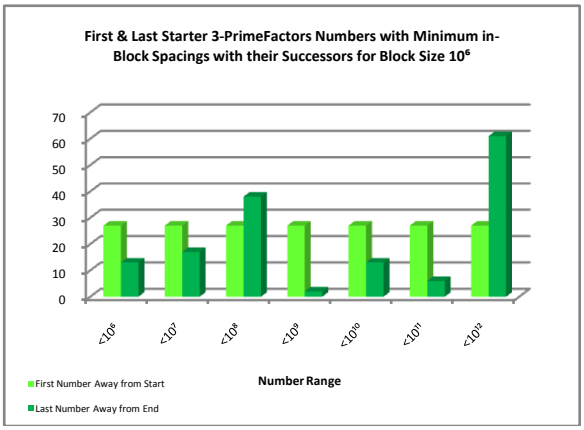
Next blocks of size 10^6, i.e., 1000000 are under consideration; block 0 refers to number range 0 to 999999, block 1000000 refers to range 1000000 to 1999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^6					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^6$	1	64,556	27	999,986	1
2	$<10^7$	1	585,725	27	9,999,982	10
3	$<10^8$	1	5,284,711	27	99,999,961	100
4	$<10^9$	1	47,753,279	27	999,999,997	1,000
5	$<10^{10}$	1	432,841,730	27	9,999,999,986	10,000

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^6					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
6	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	100,000
7	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	1,000,000

Consistency, as mentioned in remark of block-size of previous subsection, continues.

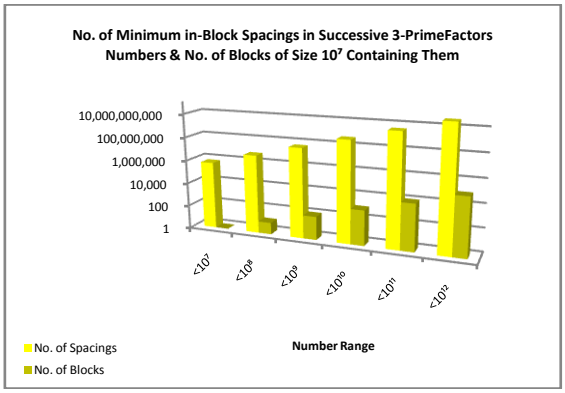


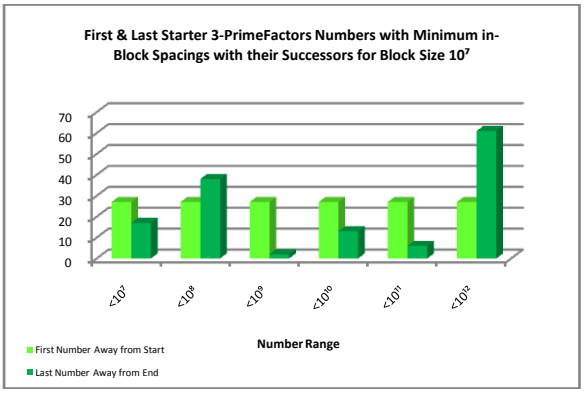


IV.F. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^7

Higher block size is 10^7, i.e., 10000000, block 0 corresponding to number range 0 to 9999999, block 10000000 corresponding to range 10000000 to 19999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^7					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^7$	1	585,725	27	9,999,982	1
2	$<10^8$	1	5,284,711	27	99,999,961	10
3	$<10^9$	1	47,753,279	27	999,999,997	100
4	$<10^{10}$	1	432,841,730	27	9,999,999,986	1,000
5	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	10,000
6	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	100,000



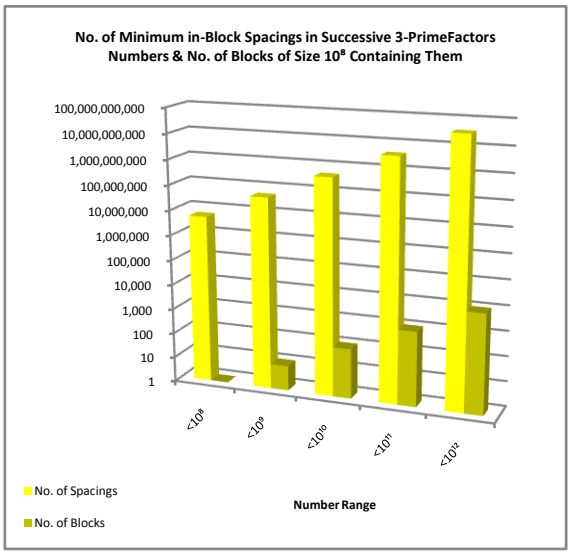


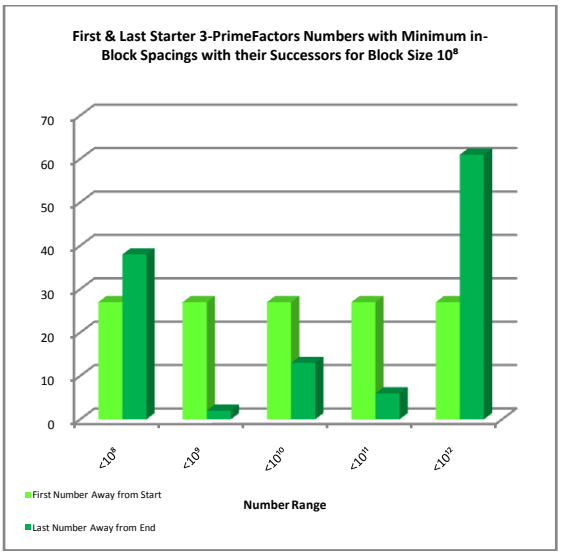
IV.G. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^8

We go ahead with block size is of 10^8, i.e., 100000000. Here block 0 denotes number range 0 to 99999999, block 100000000 denotes range 100000000 to 199999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^8					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^8$	1	5,284,711	27	99,999,961	1
2	$<10^9$	1	47,753,279	27	999,999,997	10
3	$<10^{10}$	1	432,841,730	27	9,999,999,986	100
4	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	1,000

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^8					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
5	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	10,000

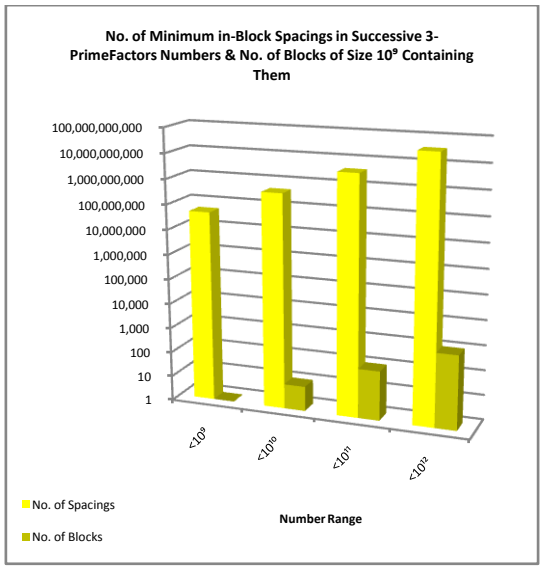


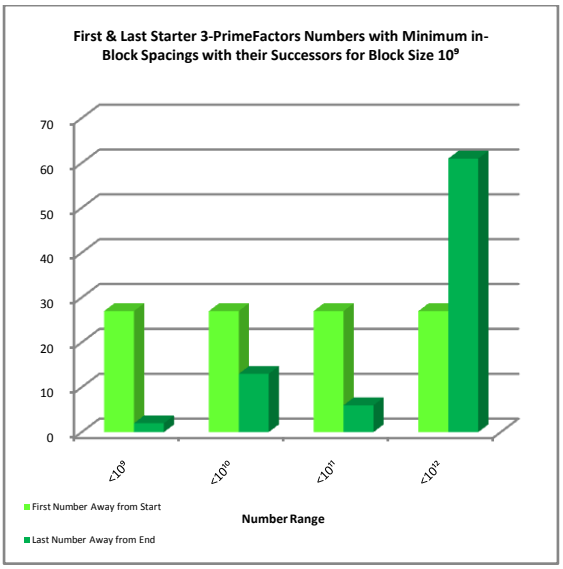


IV.H. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^9

Next one is block size 10^9, i.e., 1000000000; block 0 signifies number range 0 to 999999999, block 1000000000 signifies range 1000000000 to 1999999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^9					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^9$	1	47,753,279	27	999,999,997	1
2	$<10^{10}$	1	432,841,730	27	9,999,999,986	10
3	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	100
4	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	1,000

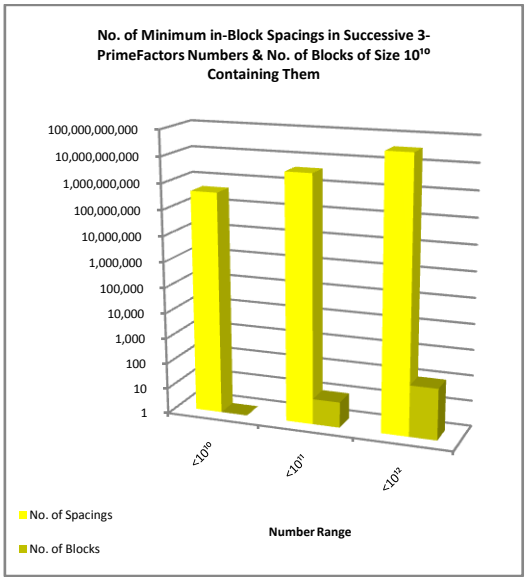


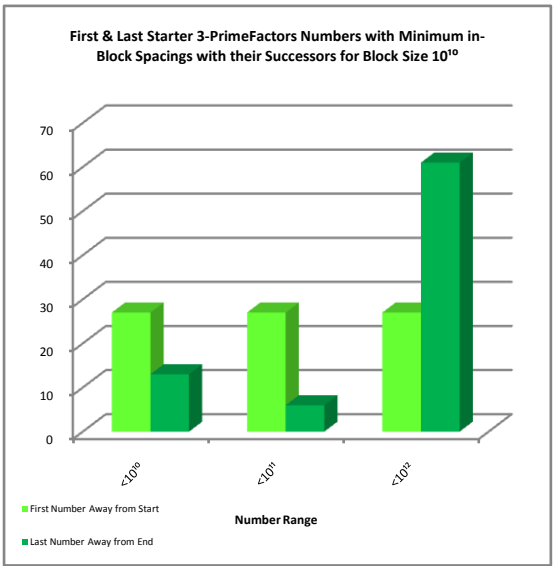


IV.I. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^{10}

Block-size 10^{10}, i.e., 10000000000 is next up in queue. Block 0 happens to be number range 0 to 9999999999, block 10000000000 happens to be range 10000000000 to 19999999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^{10}					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^{10}$	1	432,841,730	27	9,999,999,986	1
2	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	10
3	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	100

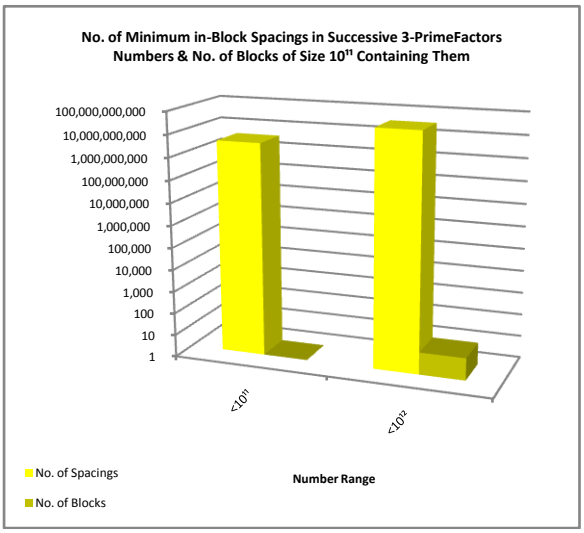


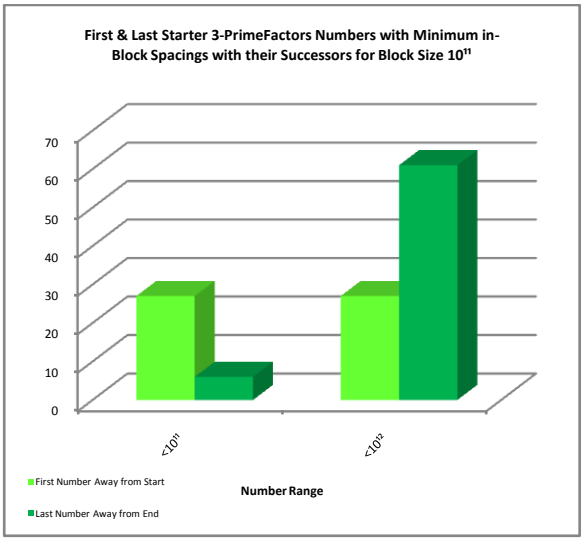


IV.J. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^{11}

Now its turn of block size 10^{11}, i.e., 100000000000; where block 0 stands for number range 0 to 99999999999, block 100000000000 stands for range 100000000000 to 199999999999 and so on.

Sr. No.	Minimum In-Block Spacings in Successive 3-PrimeFactors Numbers for Block of Size 10^{11}					
	Range	Minimum In-Block Spacing	Number of Minimum Spacings	First Number with Minimum In-Block Spacing	Last Number with Minimum In-Block Spacing	Number of Blocks with Minimum Spacings
1	$<10^{11}$	1	3,940,283,058	27	99,999,999,993	1
2	$<10^{12}$	1	36,029,848,844	27	999,999,999,938	10

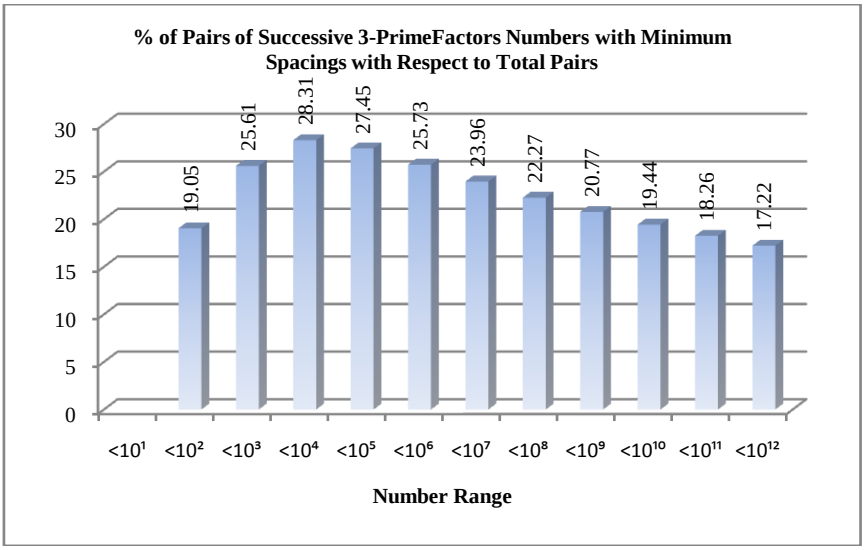




IV.K. Minimum Spacings between Successive 3-PrimeFactors Numbers in Blocks of Size 10^{12}

And finally the last block size is 10^{12}, that is the whole number range 1 trillion under consideration. Trivially there is unique block that can be talked about and hence it is the block with all attributes like the one

with minimum spacings between successive 3-PrimeFactors numbers etc. Minimality between successive pairs of 3-PrimeFactors numbers is often seen feature. It occurs as early as just after first 4 pairs and keeps coming frequently with following percentages.



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