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Research Paper

Powermean Labelings of Some Union And Product Graphs

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ABSTRACT: *A graph $G = (V, E)$ is called a Power mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, \ldots, q+1$ in such way that when each edge $e = uv$ is labeled with*

$$f(e = uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil$$

or

$$f(e = uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

then the edge labels are distinct. Here f is called a Power mean labeling of G. We investigate Power mean labeling of some standard graphs.

Keywords: *Power mean graph, $nK_3; n \geq 1, nK_3 \cup P_m; m, n \geq 1, n \geq 1$ and $m \geq 3$ and planar grids $P_m \times P_3$ and $P_m \times P_n$.*

I. INTRODUCTION

Herein we consider finite and undirected graphs. Let G = (V, E) be a graph with p vertices and q edges. One may refer to Gallian[2] and Acharya et al.[1] for a detailed survey of graph labeling. For all other standard terminology and notations, we follow Harary [3]. Somasundaram and Ponraj [6] introduced and studied [9]mean labeling for some standard graphs. Sandhya and Somasundaram[5] introduced Harmonic mean labeling of graphs and Sandhya et al. [4] studied the technique in detail. Somasundaram et al. [7] introduced the concept of Geometric mean labeling of graphs and studied their labeling in [8]. In this contribution, we define Power mean labeling and study some standard graphs like nK3; n _ 1, nK3 [Pm;m, n _ 1, n _ 1 and m _ 3 and planar grids Pm × P3 and Pm × Pn for power mean labeling. We provide illustrative examples to support our study.

II. DEFINITION AND RESULTS

Now we introduce the main concept of this paper

Definition 2.1. A graph $G = (V, E)$ with p vertices and q edges is said to be a Power Mean Graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, 3, \ldots, q+1$ is such a way that when each edge $e = uv$ is labeled with

$$f(e = uv) = \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil$$

or

$$f(e = uv) = \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor$$

then the resulting edge labels are distinct.In this case, f is called Power mean labeling of G .

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Remark 2.1. If G is a Power mean labeling graph, then 1 must be a label of one of the vertices of G , since an edge should get label 1.

Remark 2.2. If $p > q+1$, then the graph $G = (p,q)$ is not a Power mean graph, since it doesn't have sufficient labels from $\{1,2,3,...,q+1\}$ for the vertices of G.

The following Proposition will be used in the edge labelings of some standard graphs to get Power mean labeling.

Proposition 2.1. *Let a, b and i be positive integers with $a < b$. Then*

(i) $a < (a^b b^a)^{\frac{1}{a+b}} < b,$

(ii) $i < (i^{1+2}(i+2)^i)^{\frac{1}{2i+2}} < (i+1),$

(iii) $i < (i^{i+3}(i+3)^i)^{\frac{1}{2i+3}} < (i+2),$

(iv) $i < (i^{i+4}(i+4)^i)^{\frac{1}{2i+4}} < (i+2),$ *and*

(v) $(1^i i^1)^{\frac{1}{i+1}} = i^{\frac{1}{i+1}} < 2.$

Proof. **(i)** Since $a^{a+b} = a^a a^b < b^a a^b < b^a b^b = b^{a+b}$, we get the inequality in Proposition 2.1.(i). That is, the Power mean of two numbers lies between the numbers a and b. This leads to infer that if vertices u, v have labels $i, i+1$ respectively, then the edge uv may be labeled i or $i+1$ for Power mean labeling.

(ii) As a proof of this inequality, we see

$$\begin{aligned} i^{i+2}(i+2)^i &< i^2[i(i+2)]^i, \\ &< i^2(i+1)^{2i}, \\ &\quad \text{since } i(i+2) < (i+1)^2, \\ &< (i+1)^2(i+1)^{2i}, \\ &= (i+1)^{2i+2}. \end{aligned}$$

This leads to $[(i^{i+2}(i+2)^i)^{\frac{1}{2i+2}}] < i+1$.

Therefore, if u, v have labels $i, i+2$ respectively, then the edge uv may be labeled i or $i+1$.

(iii) Next we have

$$\begin{aligned} i^{i+3}(i+3)^i &= i^3[i(i+3)]^i, \\ &< i^3(i+2)^{2i}, \text{ since } i(i+3) < (i+2)^2, \\ &< (i+2)^3(i+2)^{2i}, \\ &= (i+2)^{2i+3}. \end{aligned}$$

This leads to $[i^{i+3}(i+3)^i]^{\frac{1}{2i+3}} < (i+2)$. Hence, if u, v have labels $i, i+3$ respectively, then the edge uv may be labeled $i+1$ without ambiguity.

(iv) Now

$$\begin{aligned} i^{i+4}(i+4)^i &= i^4[i(i+4)]^i, \\ &< i^4(i+2)^{2i}, \text{ since } i(i+4) < (i+2)^2 \\ &< (i+2)^4(i+2)^{2i}, \\ &= (i+2)^{2i+4}. \end{aligned}$$

Therefore

$$[i^{i+4}(i+4)^i]^{\frac{1}{2i+4}} < i+2.$$

Hence if u, v have labels $i, i+4$ respectively , then the edge uv may be labeled $i+1$.

(v) Now

$$\begin{aligned} 2^{i+1} &= (i+1)^{i+1}, \\ &= 1 + {}^{(i+1)}C_1 + \cdots + {}^{(i+1)}C_{i+1}, \\ &\geq 1 + 1 + \cdots + (i+2) \text{ terms}, \\ &\geq i+2 > i. \end{aligned}$$

Therefore $(1^i i^1)^{\frac{1}{i+1}} = i^{\frac{1}{i+1}} < 2$. Thus we observe that if u, v are labeled $1, i$ respectively, then the edge uv may be labeled 1 or 2 . ■

Definition 2.2. *Two graphs G_1 and G_2 have disjoint vertex sets V_1, V_2 and edge sets E_1, E_2 respectively. Their* **union** *$G = G_1 \cup G_2$ has ,as expected, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. The union of m copies of G is denoted by mG.*

Definition 2.3. *The* **Cartesian product** *of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G(V, E)$ with $V = V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 \times G_2$ whenever ($u_1 = v_1$ and u_2 is adjacent to v_2) or ($u_2 = v_2$ and u_1 is adjacent to v_1). It is denoted by $G_1 \times G_2$.*

2.1 *Power mean labeling for* nK_3 , $n \geqslant 1$

Theorem 2.1. *For $n \geq 1$, nK_3 is a Power mean graph where n is a number of copies of K_3.*

Proof. Let the vertex set of nK_3 be $V = \{V_1 \cup V_2 \cup V_3 \cup ... \cup V_n\}$ where $V_i = \{v_i^1, v_i^2, v_i^3\}; 1 \leq i \leq n$ and the edge set be $E = \{E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n\}$ where $E_i = \{e_i^1, e_i^2, e_i^3\}; 1 \leq i \leq n$. Define a function:

$$f : V(nK_3) \longrightarrow \{1, 2, 3, \ldots, q+1 = 3n+1\}$$

by

$f(v_i^j) = 3(i-1) + j$; $1 \leq i \leq n$; $1 \leq j \leq 3$.

By Proposition 2.1.(i),(ii) and (v), we get the set of distinct labels for the edges of nK_3 as $\{1, 2, 3, \ldots, 3n\}$. Thus nK_3 is a Power mean graph for $n \geqslant 1$. ■

Example 2.1. *An illustrative example for Power mean labeling of $3K_3$ is in Figure 2.1.*

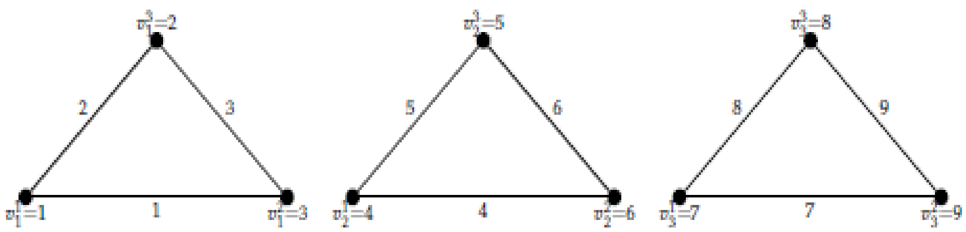

Figure 2.1: 3K3

2.2 Power mean graph for $nK_3 \cup P_m$, $m, n \geq 1$

Theorem 2.2. *The graph $nK_3 \cup P_m$ is a Power mean graph for $m \geq 1, n \geq 1$ where n is a number of copies of K_3.*

Proof. Let the vertex set of nK_3 be $U = U_1 \cup U_2 \cup U_3 \cup \ldots \cup U_n$ where $U_i = \{u_i^1, u_i^2, u_i^3\}; 1 \leq i \leq n$ and the edge set be $E = E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n$ where $E_i = \{e_i^1, e_i^2, e_i^3 \ ; \ 1 \leq i \leq n\}$.
Let P_m be the path with vertices is $\{v_1, v_2, v_3, \ldots, v_m\}$. The number of vertices of $nK_3 \cup P_m$ is $3n + m$. The number of edges of $nK_3 \cup P_m$ is $3n + m - 1$.

Define a function

$$f : V(nK_3 \cup P_m) \longrightarrow \{1, 2, 3, ..., q + 1 = 3n + m\}$$

for vertex labeling as

(i) $f(u_i^j) = 3(i-1) + j \ ; 1 \leq i \leq n \ ; 1 \leq i \leq 3$ and

(ii) $f(v_i) = 3n + i \ ; 1 \leq i \leq m \ ; 1 \leq i \leq m.$

By Proposition 2.1.(i), (ii) and (v), we get the set of labels for the edges as

(i) $E(u_i^1 u_i^2) = f(u_i^1) \ ; 1 \leq i \leq n,$

(ii) $E(u_i^2 u_i^3) = f(u_i^3) \ ; 1 \leq i \leq n,$ and

(iii) $E(u_i^1 u_i^3) = \dfrac{f(u_i^3) + f(u_i^1)}{2} \ ; 1 \leq i \leq n.$

Therefore, the set of labels of edges of nK_3 is $\{1, 2, 3, ..., 3n\}$ and the set of labels of edges of P_m is $\{3n + 1, 3n + 2, ..., 3n + m - 1\}$. As the edge labels are distinct, $nK_3 \cup P_m$ is a Power mean graph for $m \geq 1, n \geq 1$. ■

Example 2.2. *Power mean labeling for the graph $3K_3 \cup P_5$ with 14 vertices and 13 edges is given below Figure 2.2.The set of all vertex labels $3K_3 \cup P_5$ is $\{1, 2, 3, ..., 14\}$. The set of all edge labels $3K_3 \cup P_5$ is $\{1, 2, 3, ..., 13\}$.*

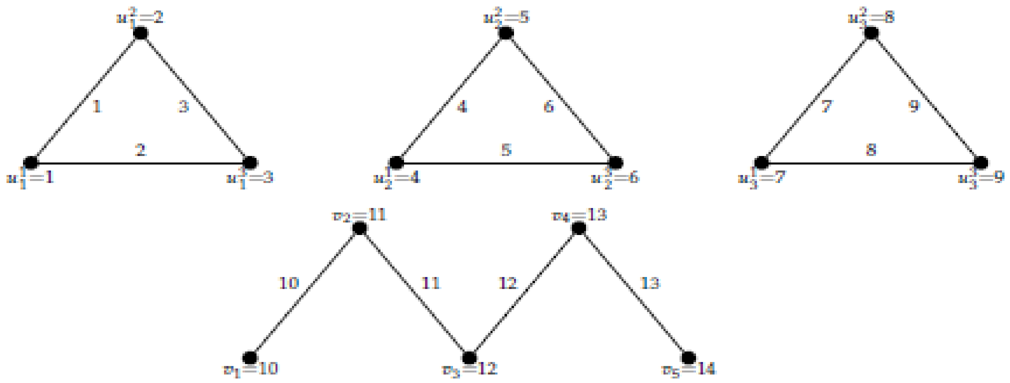

Figure 2.2: 3K3 P5

2.3 Power mean labeling for $nK_3 \cup C_m$, $n \geqslant 1, m \geqslant 3$

Theorem 2.3. *The graph $nK_3 \cup C_m$ is a Power mean graph for $n \geq 1, m \geq 3$*

Proof. Let the vertex set of nK_3 be $U = \{U_1 \cup U_2 \cup U_3 \ldots \cup U_n\}$ where $U_i = \{u_i^1, u_i^2, u_i^3\}$ $1 \leq i \leq n$ and the edge set be $E = \{E_1 \cup E_2 \cup E_3 \cdots \cup E_n\}$ where $E_i = \{e_i^1, e_i^2, e_i^3\}$. The vertex set of C_m be the cycle is $v_1, v_2, v_3, \ldots, v_m, v_1$. The number of vertices of $nK_3 \cup C_m$ is $3n + m$. The number of edges of $nK_3 \cup C_m$ is $3n + m$.

Define a function

$$f : V(nK_3 \cup C_m) \longrightarrow \{1, 2, 3, \ldots, q + 1 = 3n + m + 1\}$$

by

(i) $f(u_i^j) = 3(i-1) + j$; $1 \leq i \leq n$, $1 \leq j \leq 3$ and

(ii) $f(v_i) = 3n + i$; $1 \leq i \leq m$

By Proposition 2.1.(i),(ii) and (v), we get the set of labels for the edges as

(i) $E(u_i^1 u_i^2) = f(u_i^1)$, $1 \leq i \leq n$,

(ii) $E(u_i^2 u_i^3) = f(u_i^3)$, $1 \leq i \leq n$, and

(iii) $E(u_i^1 u_i^3) = \dfrac{f(u_i^1) + f(u_i^3)}{2}$, $1 \leq i \leq n$.

The set of labels of the edges of nK_3 is $\{1, 2, 3, \ldots, 3n\}$, the set of labels of the edge of C_m is $\{3n+1, 3n+2, 3n+3, \ldots, 3n+m\}$ and the set of labels of the edge $nK_3 \cup C_m$ is $\{1, 2, 3, \ldots, q+1\}$. Since all the edge labels are different, $nK_3 \cup C_m$ is Power mean labeling graph.

∎

Example 2.3. *An example of Power mean labeling for $nK_3 \cup C_m$ is in Figure 2.3.*

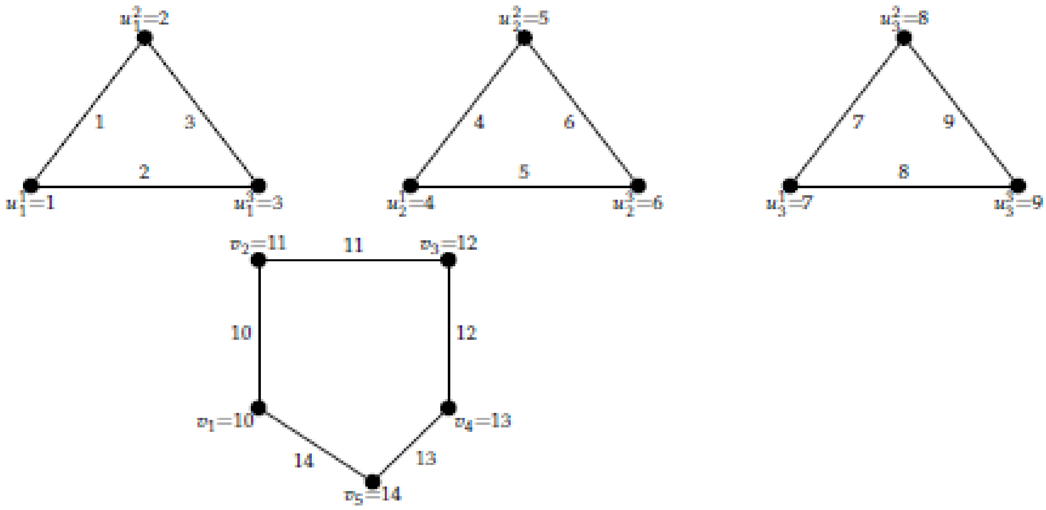

Figure 2.3: 3K3 [C5

III. PRODUCT GRAPHS

In this section, the Power mean labeling of planar grid is investigated. Power mean labelings of planar grids are obtained for n = 3, 4 .

3.1 *Power mean labeling for planar grid* $P_m \times P_3$

Theorem 3.1. *The planar grid* $P_m \times P_3$ *is a Power mean graph for* $m \geq 2$

Proof. Let the vertex set of $P_m \times P_3$ be $V(P_m \times P_3) = \{a_{ij}, 1 \leq i \leq m, 1 \leq j \leq 3\}$ and edge set be $E(P_m \times P_3) = \{a_{i(j-1)}a_{ij}, 1 \leq i \leq m, 2 \leq j \leq 3\} \cup \{a_{(i-1)j}a_{ij}, 2 \leq i \leq m, 1 \leq j \leq 3\}$ .The graph $P_m \times P_n$ has $3m$ vertices and $5m-3$ edges.

Define a function

$$f : V(P_m \times P_3) \longrightarrow \{1, 2, 3, \ldots, q+1\}$$

as

(i) $f(a_{11}) = 1,$

(ii) $f(a_{12}) = 3,$

(iii) $f(a_{13}) = 5,$

(iv) $f(a_{ij}) = f(a_{(i-1)2}) + 2 + j$, $i = 2$, $1 \leq j \leq 3$ and

(v) $f(a_{ij}) = f(a_{(i-1)3}) + 2 + j$, $3 \leq i \leq n$, $1 \leq j \leq 3.$

By Proposition 2.1.(i),(ii),(iii),(iv) and (v), the labelings of the edges are

(i) $E(a_{11}a_{12}) = 1,$

(ii) $E(a_{12}a_{13}) = 3,$

(iii) $E(a_{ij}a_{i(j+1)}) = 5(i-1) + j$, $3 \leq i \leq m$, $1 \leq i \leq 2,$

(iv) $E(a_{11}a_{21}) = 2$ and

(v) $E(a_{ij}a_{(i+1)j}) = 5(i-1) + 2 + j$, $1 \leq i \leq m-1$, $2 \leq j \leq 3.$

As the edge labels are distinct, the planar grid $P_m \times P_3$ is a Power mean graph for $m \geq 2$. ∎

Example 3.1. Figure 3.1 is an example for Power mean labeling of P4 ×P3 which has 12 vertices and 17 edges.

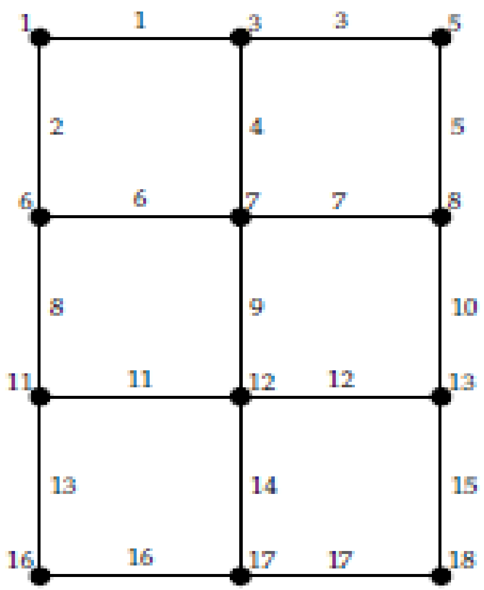

Figure 3.1: P4 × P3

3.2 *Power mean labeling for planar grid* $P_m \times P_4$

Theorem 3.2. *The Planar grid* $P_m \times P_4$ *is a Power mean graph for* $m \geq 2$.

Proof. Let the vertex set of $P_m \times P_n$ be $V(P_m \times P_n) = \{a_{ij}, 1 \leq i \leq m, 1 \leq j \leq 3\}$ and edge set be $E(P_m \times P_n) = \{a_{ij}a_{i(j+1)}, 1 \leq i \leq m, 1 \leq j \leq 3\} \cup \{a_{ij}a_{(i+1)j}, 1 \leq i \leq m-1, 1 \leq j \leq 4\}$. The graph $P_m \times P_n$ has $4m$ vertices and $7m-4$ edges.
Define a function

$$f : V(P_m \times P_n) \longrightarrow \{1, 2, 3, \ldots, q+1 = 7m-3\}$$

as

(i) $f(a_{11}) = 1$,

(ii) $f(a_{ij}) = 2j - 1$, $i = 1$, $j = 2$ and 3,

(iii) $f(a_{ij}) = j + 2$, $i = 1$, $j = 4$, and

(iii) $f(a_{ij}) = f(a_{(i-1)4}) + 3 + j$, $2 \leq i \leq m$, $1 \leq j \leq 4$.

By Proposition 2.1.(i),(ii),(iii),(iv) and (v), we get the set of labels for the edges as

(i) $E(a_{ij}a_{i(j+1)}) = 2j - 1$, $i = 1$, $1 \leq j \leq 3$,

(ii) $E(a_{ij}a_{i(j+1)}) = 7(i-1) + j$, $2 \leq i \leq m$, $1 \leq j \leq 3$,

(iii) $E(a_{ij}a_{(i+1)j}) = 2j$, $i = 1$, $1 \leq j \leq 3$, and

(iv) $E(a_{ij}a_{(i+1)j}) = 7(i-1) + 3 + j$, $2 \leq i \leq m-1$, $1 \leq j \leq 4$.

As the edge labels are distinct, $P_m \times P_4$ is a Power mean graph for $m \geq 2$. ■

III. CONCLUSION

In this paper we have proved that $nK_3; n \geq 1, nK_3 \cup P_m; m, n \geq 1, n \geq 1$ and $m \geq 3$ and planar grids $P_m \times P_3$ and $P_m \times P_4$ graphs are amenable for Power Mean labeling and illustrated examples are provided to support our investigation.

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