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Research Paper

An Optimal Eighth-Order of King-Type Family Iterative Method for Solving Nonlinear Equations

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ABSTRACT: *In this paper, we present anew family of optimal eighth order of methods, the new method developed combining of king's fourth order method with Newton's method as a third step. Using the divided difference and three real-valued functions in the third step to reduce the number of function evaluations, and increasing the order to be optimal.*
KEYWORDS: nonlinear *equation, iterative method, order of convergence, optimal eight order.*

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I. INTRODUCTION

Nonlinear equations are at the heart of many problems of nonlinear science and the exact solutions are not always possible to find. We construct iterative methods to find a simple root of a nonlinear equation, $f(x) = 0$, where $f(x) : I \subset R \to R$is a scalar function on an open interval I.Many iterative method have been used for solving nonlinear equation, for example, well knew the classical Newton's method (NM)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \qquad (1)$$

One-parameter family of fourth-order methods have developed by King method (KM)[1], which is written as :

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$x_{n+1} = y_n - \frac{f(x_n)+\beta f(y_n)}{f(x_n)+(\beta-2)f(y_n)}\frac{f(y_n)}{f'(x_n)}. \qquad (2)$$

Where $\beta \in R$ is a parameter. In particular, the famous Ostrowski's method [2] is a member of this family when$\beta = 0$,

$$x_{n+1} = y_n - \frac{f(x_n)}{f(x_n)-2f(y_n)}\frac{f(y_n)}{f'(x_n)}. \qquad (3)$$

Kung and Traub's method [3] who conjectured that an iterationmethod based on n evaluation of $f$orits derivatives could achieve optimal convergence order 2^{n-1}. Newton's method is optimal second-order method, King's method (2) andOstrowski's method (3)areportion of the optimal fourth-order methods, since they just perform three functional evaluations per step. Recently, based on Ostrowski's or King's methods the weight functions have been used in the third step to increase the order of convergence, see for example, [4-6].Many researchers have developed eighth-order, see [7-10].

II. CONSTRUCTION OF THE OPTIMAL FAMILY OF ITERATIVE METHOD

In order to construct new methods, we consider an iteration scheme of using king method (2) and adding Newton method (1) as a third step, we have

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$z_n = y_n - \frac{f(x_n)+\beta f(y_n)}{f(x_n)+(\beta-2f)(y_n)}\frac{f(y_n)}{f'(x_n)},$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}. \qquad (4)$$

This method of order eight and has five functional evaluation, which is not optimal so we will reduce the number of functions evaluation using divided deference, [2]

$f'(z) \approx \frac{f[x_n,z_n]f[y_n,z_n]}{f[x_n,y_n]}$. (5)

Now, we present a new family of optimal eighth-order king-type iterative methods using weighted functions to increasing the order of convergence, as the following:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$z_n = y_n - \frac{f(x_n)+\beta f(y_n)}{f(x_n)+(\beta-2f)(y_n)}\frac{f(y_n)}{f'(x_n)},$$

$$x_{n+1} = z_n - \{A(t_1)+B(t_2)+C(t_3)\} \times \frac{f(z_n)f[x_n,y_n]}{f[x_n,z_n]f[y_n,z_n]}. \quad (6)$$

Where $A(t_1), B(t_2), C(t_3)$ are three real-valued weight function, and

$t_1 = \frac{f(z)}{f(x)}, t_2 = \frac{f(y)}{f(x)}, t_3 = \frac{f(z)}{f(y)}$.

Theorem1.: Let μ be a simple root on the open interval in R for the function $f: I \to R$.Also initial approximationx_0 is sufficiently close to μ of$f$. Then the method (6) gives optimal eighth-order of convergence if satisfies the following conditions:

$$A(0) = 1, A'(0) = 1,$$

$$B(0) = -1, B'(0) = 0, B''(0) = 0, B'''(0) = -12\beta, |B^{(4)}(0)| < \infty,$$

$$C(0) = 1, C'(0) = 0, |C''(0)| < \infty. \quad (7)$$

Proof: Let $e_n = x_n - \mu$ be the error. Using the Taylor expansion, we have

$f(x) = f'(\mu)(e + c_2e^2 + c3e^3 + c_4e^4 + c_5e^5 + c_6e^6 + c_7e^7 + c_8e^8 + c_9e^9)$. (8)

Where $c_k = \frac{f^{(k)}(\mu)}{k!f'(\mu)}$, $k = 2,3,\ldots$

$f'(x) = f'(\mu)(1 + 2c_2e + 3c_3e^2 + 4c_4e^3 + 5c_5e^4 + 6c_6e^5 + 7c_7e^6 + 8c_8e^7 + 9c_9e^8)$. (9)

Diving (8) by (9),we get

$\frac{f(x)}{f'(x)} = e - c_2e^2 + (2c_2^2 - 2c_3)e^3 + \cdots + (-64c_2^7 + 304c_2^5c_3 - \cdots + 31c_4c_5 - 7c_8)e^8 + O(e^9)$(10)

Now, from (10), we have

$y_n = (\mu + c_2e^2 + \cdots + (64c_2^7 - 304c_2^5c_3 + \cdots + 31c_4c_5 + 7c_8)e^8 + O(e^9)$. (11)

From (11), we obtain.

$f(y) = f'(\mu)(c_2e^2 + (-2c_2^2 + 2c_3)e^3 + (5c_2^3 - 7c_2c_3 + 3c_4)e^4) + \cdots + O(e^9)$. (12)

Combining (8), (9), (10) and (12), we have

$z_n = \mu + (2\beta c_2^6 - 2\beta c_2^4c_3 + c_2^6 - 2c_2^4c_3 + c_2^2c_3^2)e^7 + (2\beta^2c_2^7 + 2\beta^2c_2^5c_3 - 14\beta c_2^7 + \cdots + 19c_2^5c_3 -$ *3c24c417c23c32+3c22c3c4+4c2c33e8*(13)

From (13), we get

$$f(z_n) = f'(\mu)((2\beta c_2^6 - 2\beta c_2^4c_3 + c_2^6 - 2c_2^4c_3 + c_2^2c_3^2)e^7 + (2\beta^2c_2^7 + 2\beta^2c_2^5c_3 - 14\beta c_2^7 + \cdots + 19c_2^5c_3 - 3c_2^4c_4 - 17c_2^3c_3^2 + 3c_2^2c_3c_4 + 4c_2c_3^3)e^8 + O(e^9)).$$

Using the Taylor series expansion, we get

$A(t_1) = A(0) + A'(0)t_1 + \cdots + O(t_1^9)$.

$B(t_2) = B(0) + B'(0)t_2 + \frac{1}{2!}B''(0)t_2^2 + \frac{1}{3!}B'''(0)t_2^3 + \frac{1}{4!}B''''(0)t_2^4 + \cdots + O(t_2^9)$.

$C(t_3) = C(0) + C'(0)t_3 + \cdots + O(t_3^9)$

Furthermore,

$f[x_n,y_n] = f'(\mu)\left(1 + c_2e + (c_2^2 + c_3)e^2 + + \cdots + O(e^9)\right)$. (14)

$f[y_n,z_n] = f'(\mu)\left(1 + c_2^2e^2 + (-2c_2^3 + 2c_2c_3)e^3 + \cdots + O(e^9)\right)$. (15)

$f[x_n,z_n] = f'(\mu)\left(1 + c_2e^2 + (-2c_2^3 + 3c_2c_3 + c_4)e^3 + \cdots + O(e^9)\right)$. (16)

Finally, using (14), (15), (16) and

$A(0) = 1, A'(0) = 1,$

$B(0) = -1, B'(0) = 0, B''(0) = 0, B'''(0) = -12\beta,$

$C(0) = 1, C'(0) = 0.$

Substituting in third step (7) we get, the error expression

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$e_{n+1} = \mu + \left(-\frac{1}{2}C''(0)c_2^7 - \frac{1}{12}B^{(4)}(0)\beta c_2^7 - 6C''(0)\beta^2 c_2^7 - 4C''(0)\beta^3 c_2^7 - 3C''(0)\beta c_2^7 + \frac{3}{2}C''(0)c_2^5 c_3 -\right.$
32C''0c23c32+12C''0c2c33−124B40c27+ 124B40c25∗c3+
6C''0β2c25c3+6C''0βc25c3−3C''0βc23c32+4β3c27 − 2β2c27+…+2βc24c4−
c22c3c4+3c27−7c25c3+c24c4+4c23c32e8+Oe9.
Where $|C''(0)| < \infty$, $\left|B^{(4)}(0)\right| < \infty$.
Which shows that the order of convergence of the family (7)is exactly eight for any value of $\beta \in R$. This theorem is proved.
Any method from (6) has four evaluations per iteration and has eight-order convergence also satisfies the conditions (7).

Method 1 (OSM1):
If the functions $\mathrm{A}(\mathrm{t}_1), \mathrm{B}(\mathrm{t}_2), \mathrm{C}(\mathrm{t}_3)$ are define by:
$A(t_1) = 1 + t_1,$
$B(t_2) = -1 - 2\beta t_2^a\ , a \geq 3, a \in R$
$C(t_3) = 1 + t_3^p, p \geq 2, p \in R.$
Satisfy the conditions in theorem1, then

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$
$$z_n = y_n - \frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2f)(y_n)} \frac{f(y_n)}{f'(x_n)},$$
$$x_{n+1} = z_n - \left\{-t_3^p - 2\beta t_2^a + t_1 + 1\right\} \times \frac{f(z_n) f[x_n, y_n]}{f[x_n, z_n] f[y_n, z_n]}, \qquad (17)$$
$$where\ t_1 = \frac{f(z)}{f(x)}\ , t_2 = \frac{f(y)}{f(x)}\ , t_3 = \frac{f(z)}{f(y)}.$$

Method 2 (OSM2):
If the functions $\mathrm{A}(\mathrm{t}_1), \mathrm{B}(\mathrm{t}_2), \mathrm{C}(\mathrm{t}_3)$ are define by:
$A(t_1) = \cos(t_1) + \sin(t_1),$
$B(t_2) = -1 + t_2^a(1 - 2\beta)\ , a \geq 3\ and\ a \in R,$
$C(t_3) = t^q e^{t3} + 1, q \geq 1, q \in R.$
Satisfy the conditions in theorem1, then

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$
$$z_n = y_n - \frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2f)(y_n)} \frac{f(y_n)}{f'(x_n)},$$
$$x_{n+1} = z_n - \{\cos(t_1) + \sin(t_1) + t_2^a(1 - 2\beta) + t^q e^{t_3}\} \times \frac{f(z_n) f[x_n, y_n]}{f[x_n, z_n] f[y_n, z_n]}, \qquad (18)$$
$$where\ t_1 = \frac{f(z)}{f(x)}\ , t_2 = \frac{f(y)}{f(x)}\ , t_3 = \frac{f(z)}{f(y)}.$$

Method 3 (OSM3):
If the functions $\mathrm{A}(\mathrm{t}_1), \mathrm{B}(\mathrm{t}_2), \mathrm{C}(\mathrm{t}_3)$ are define by:
$A(t_1) = 1 + \sin(t_1),$
$B(t_2) = -1 + t_2^a(\sin(t_2) + 2\beta), a \geq 3, a \in R,$
$C(t_3) = \cos(t_3).$
Satisfy the conditions in theorem1, then

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$
$$z_n = y_n - \frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2f)(y_n)} \frac{f(y_n)}{f'(x_n)},$$
$$x_{n+1} = z_n - \{\sin(t_1) + \cos(t_3) + t_2^a(\sin(t_2) + 2\beta)\} \times \frac{f(z_n) f[x_n, y_n]}{f[x_n, z_n] f[y_n, z_n]}, \qquad (19)$$
$$where\ t_1 = \frac{f(z)}{f(x)}\ , t_2 = \frac{f(y)}{f(x)}\ , t_3 = \frac{f(z)}{f(y)}.$$

Method 4 (OSM4):
If the functions $\mathrm{A}(\mathrm{t}_1), \mathrm{B}(\mathrm{t}_2), \mathrm{C}(\mathrm{t}_3)$ are define by:
$A(t_1) = \mathrm{e}^{\mathrm{t}_1},$
$B(t_2) = -1 + 2\beta t_2^a + t_2^a \mathrm{e}^{\mathrm{t}_2}, a \geq 3,$
$C(t_3) = 1.$

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Satisfy the conditions in theorem1, then

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$z_n = y_n - \frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2f)(y_n)} \frac{f(y_n)}{f'(x_n)},$$

$$x_{n+1} = z_n - \{e^{t_1} + 2\beta t_2^a + t_2^a e^{t_2}\} \times \frac{f(z_n) f[x_n, y_n]}{f[x_n, z_n] f[y_n, z_n]}, \quad (20)$$

$$where\ t_1 = \frac{f(z)}{f(x)}, t_2 = \frac{f(y)}{f(x)}, t_3 = \frac{f(z)}{f(y)}$$

III. NUMERICAL RESULTS

We use the method OSM1to solve some non-linear equations and compare with the othermethods. We specifically take $a = 3$ and$\beta = -1, -0.5, 0, 1, 0.5$ for the method (OSM1), (17), and $\beta = 0$ for the methods (OSM2),(18),(OSM3), (19), (OSM4), (20).All computation were done using MATLAB. We have used as stopping criteria that$|x_{n+1} - x_n| \leq 10^{-15} and\ |f(x_n)| \leq 10^{-15}$.

Table 1:test function and exact root.

$f_1(x) = \log(x) + \sqrt{x} - 5$	$\mu = 8.30943269423157$
$f_2(x) = x^3 + 4x^2 - 15$	$\mu = 1.63198080556606$
$f_3(x) = \sin(x)^2 - x^2 + 1$	$\mu = 1.40449164821534$
$f_4(x) = sin(x) - \frac{x}{3}$	$\mu = 2.27886266007583$
$f_5(x) = 2xcos(x) + x - 3$	$\mu = -3.53225169153648$

Table 1 shows us numerical example with deferent root.We compare the family of king's method (2), β=2,

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$x_{n+1} = y_n - \frac{f(x_n) + 2f(y_n)}{f(x_n)} \frac{f(y_n)}{f'(x_n)}.$$

Kung and Traub's method optimal eight order (KTM), [9],

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$z_n = y_n - \frac{f(x_n)}{f'(x_n)} \frac{f(x_n) f(y)}{\left(f(x_n) - f(y_n)\right)^2},$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(x_n)}.$$

Sharma method optimal eight order (SM), [5],

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$z_n = y_n - \frac{f(y)}{f'(x_n)} \frac{f(x_n)}{f(x_n) - 2f(y_n)},$$

$$x_{n+1} = z_n - \left[1 + \frac{f(z_n)}{f(x_n)} + \left(\frac{f(z_n)}{f(x_n)}\right)^2\right] \frac{f(z_n) f[x_n, y_n]}{f[x_n, z_n] f[y_n, z_n]}.$$

Modified King method optimal eight order (MKM), [5],

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$z_n = y_n - \frac{f(y)}{f'(x_n)} \Delta_n, \Delta_n = \frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2f)(y_n)},$$

$$x_{n+1} = z_n - t_n(z_n - y_n).$$

Shown in Table 2 the definition of Efficiency index(EI), [8] as $p^{\frac{1}{n}}$ where p is the order and the n is total number of evaluation per iterative.

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Table 2:Order of convergence, number of function evaluations and Efficiency of different methods.

Method	p	n	EI
NM	2	2	$\sqrt[2]{2} \approx 1.1414$
KN	4	3	$\sqrt[3]{4} \approx 1.587$
KTM	8	4	$\sqrt[4]{8} \approx 1.682$
SM	8	4	$\sqrt[4]{8} \approx 1.682$
OSM	8	4	$\sqrt[4]{8} \approx 1.682$

Shown in Table 3,the number of iterations (Iter) and the computational order of convergence (COC) which can be approximated using the formula :

$$COC \approx \frac{\ln(x_n - x_{n-1})/\ln(x_{n-1} - x_{n-2})}{\ln(x_{n-1} - x_{n-2})/\ln(x_{n-2} - x_{n-3})}.$$

IV. CONCLUSION

In this pepper, a new family of variants of king's method is obtained. The convergence orders of these methods are eight. New proposed families are found by approximating $f'(z)$ using divided difference and weighted functions toreduce the functions evaluations and to increase the order of convergence. The new family of variants has an efficiencyindex equal to $\sqrt[4]{8} = 1.682$.Comparing the other methods using numerical example to explain the convergence of new methods

Table3:Numerical Comparing

Method	Iter	COC	$\|f(x_n)\|$	$\|x_{n+1} - x_n\|$
		$f_1(x), x_0 = 11.9$		
NM	6	2.0	0	0
KN	4	4.0	0	0
MKM	3	8.0	0	0
SM	3	8.0	0	0
KTM,$a = 0$	3	8.0	0	0
OSM1, $\beta = 1$	3	8.0	0	0
OSM1, $\beta = 0.5$	3	8.0	0	0
OSM1, $\beta = 0$	3	8.0	0	0
OSM1, $\beta = -0.5$	3	8.0	0	0
OSM1, $\beta = -1$	3	8.0	0	0
OSM2, $\beta = 0$	3	8.0	0	0
OSM3, $\beta = 0$	3	8.0	0	0
OSM4, $\beta = 0$	3	8.0	0	0
		$f_2(x), x_0 = 2$		
NM	6	2.0	2.2682686e-1007	0
KN	4	4.0	2.2682686e-1007	0
MKM	3	8.0	2.2682686e-1007	0
SM	3	8.0	2.2682686e-1007	0
KTM,$a = 0$	3	8.0	2.2682686e-1007	0
OSM1, $\beta = 1$	3	8.0	2.2682686e-1007	0
OSM1, $\beta = 0.5$	3	8.0	2.2682686e-1007	0
OSM1, $\beta = 0$	3	8.0	2.2682686e-1007	0
OSM1, $\beta = -0.5$	3	8.0	2.2682686e-1007	0
OSM1, $\beta = -1$	3	8.0	2.2682686e-1007	0
OSM2, $\beta = 0$	3	8.0	2.2682686e-1007	0
OSM3, $\beta = 0$	3	8.0	2.2682686e-1007	0
OSM4, $\beta = 0$	3	8.0	2.2682686e-1007	0
		$f_3(x), x_0 = 1.6$		
NM	6	2.0	0	0
KN	5	4.0	0	0
MKM	3	8.0	0	0
SM	3	8.0	0	0
KTM,$a = 0$	3	8.0	0	0
OSM1, $\beta = 1$	3	8.0	0	0
OSM1, $\beta = 0.5$	3	8.0	0	0
OSM1, $\beta = 0$	3	8.0	0	0
OSM1, $\beta = -0.5$	3	8.0	0	0
OSM1, $\beta = -1$	3	8.0	0	0
OSM2, $\beta = 0$	3	8.0	0	0
OSM3, $\beta = 0$	3	8.0	0	0
OSM4, $\beta = 0$	3	8.0	0	0

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Method	Iter	COC	$\lvert f(x_n)\rvert$	$\lvert x_{n+1}-x_n\rvert$
$f_4(x), x_0 = 2$				
NM	6	2.0	1.4176679e-1008	0
KN	4	4.0	1.4176679e-1008	0
MKM	3	8.0	1.4176679e-1008	0
SM	3	8.0	1.4176679e-1008	0
KTM,$a = 0$	3	8.0	1.4176679e-1008	0
OSM1, $\beta = 1$	3	8.0	1.4176679e-1008	0
OSM1, $\beta = 0.5$	3	8.0	1.4176679e-1008	0
OSM1, $\beta = 0$	3	8.0	1.4176679e-1008	0
OSM1, $\beta = -0.5$	3	8.0	1.4176679e-1008	0
OSM1, $\beta = -1$	3	8.0	1.4176679e-1008	0
OSM2, $\beta = 0$	3	8.0	1.4176679e-1008	0
OSM3, $\beta = 0$	3	8.0	1.4176679e-1008	0
OSM4, $\beta = 0$	3	8.0	1.4176679e-1008	0
$f_5(x), x_0 = -4.8$				
NM	8	2.0	0	0
KN	5	4.0	0	0
MKM	3	8.0	0	0
SM	3	8.0	0	0
KTM,$a = 0$	3	8.0	0	0
OSM1, $\beta = 1$	3	8.0	0	0
OSM1, $\beta = 0.5$	3	8.0	0	0
OSM1, $\beta = 0$	3	8.0	0	0
OSM1, $\beta = -0.5$	3	8.0	0	0
OSM1, $\beta = -1$	3	8.0	0	0
OSM2, $\beta = 0$	3	8.0	0	0
OSM3, $\beta = 0$	3	8.0	0	0
OSM4, $\beta = 0$	3	8.0	0	0

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