Quest Journals
Journal of Research in Applied Mathematics
Volume 5 ~ Issue 3 (2019) pp: 05-15
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Rapid Prototyping System selection using multi criteria decision making methods

Dr. V. P. Darji

ABSTRACT: *Rapid prototyping (RP) systems are material processing fully automatic techniques. They offer a number of competitive advantages over traditional manufacturing processes and are particularly useful for rapid product development. Commonly used RP systems are stereolithography, Selective Laser Sintering (SLS), Fused Deposition Modeling (FDM), laminated object manufacturing and 3D printing etc. Several issues in RP systems such as new processes, material properties, part surface quality, build time, applications and tooling are active issues. The present work is attempted to introduce and validate the application of a new multi criteria decision making methods for the selection of best RP systems.*
***Keywords**: Rapid Prototyping systems; Extended TODIM; ARAS; a hybrid method.*

Received 05 Aug, 2019; Accepted 20 Aug, 2019 © The author(s) 2019.
Published with open access at www.questjournals.org

I. INTRODUCTION

An important recent advance in manufacturing is rapid prototyping, a process by which a solid physical model of a part is made directly from a three-dimensional CAD drawing. Also called desktop manufacturing or free-form fabrication and developed in the mid-1980s, rapid prototyping entails several different techniques that allow making a prototype, that is , a first full- scale model of a product. In order to appreciate the importance and economic impact of rapid prototyping, let us consider a design that is in its conceptual stage. First, through a three-dimensional CAD system, the design is viewed in its entirety and at different angles on the cathode-ray tube. Before that particular product is made, a prototype is manufactured and studied thoroughly from esthetic, technical, and functional aspects, using materials such as plastics or metals [1].

Making a prototype has traditionally involved actual manufacturing processes using a variety of tooling and machines, and usually taking anywhere from weeks to months, depending on part complexity. Rapid prototyping reduces this time significantly, as well as cost, by using various consolidation processes such as resin curing, sintering, deposition, and solidification techniques. Generally used for prototype production, these techniques are being developed further so that they can also be used for low-volume production [1].

The next section presents the selected multiple attribute decision making methods and their computational details.

II. SELECTED MULTIPLE ATTRIBUTE DECISION MAKING METHODS

The present paper unfolds the application of three selected multiple attribute decision making method for ranking rapid prototyping system selection. The methods considered are: Extended TODIM, ARAS and a hybrid method combining SWARA and WASPAS.

2.1.Extended TODIM method

Step 1: Transformation of three formats of attributes values

For the convenience of analysis and computation, it is necessary to transform different formats of attribute values into the same format. According to the existing literature [2], three formats of attribute values (crisp number, interval numbers and fuzzy numbers) are to be converted into the format of random variables with cumulative distribution functions. The transformation process and calculation formulae of each format are described in the sessions [2].

Crisp number: If x_{ij} is a crisp number, i.e., $x_{ij} = x_{ij}^{'}$, it can be regarded as particular random variable [3]. Its cumulative distribution function is

$$F_{ij}(x)=\begin{cases}0, & x< x'_{ij},\\ 1, & x\geq x_{ij'},\end{cases}\quad i\in M,\quad j\in N^{k} \tag{1}$$

The interval and fuzzy numbers are treated in the proposed method. But the existing paper does not take into account these two data format.

Step 2: Calculation of gains and losses

To calculate the gain and loss of each alternative relative to the others, firstly, the calculation formulae for the superior and inferior values about the comparison of two cumulative distribution functions are given as described below:

Let x_{ij} and x_{ij} be the attribute value of alternatives A_i and A_k concerning attributes C_j respectively, $i,k\in M, j\in N$. Let $F_{ij}(x)$ and $F_{kj}(x)$ be the cumulative distribution functions of x_{ij} and x_{ij}, respectively. For the benefit attributes, the superior and inferior values of $F_{ij}(x)$ relative to $F_{kj}(x)$ are respectively expressed by

$$B_s\left(F_{ij}(x),F_{kj}(x)\right)=\int_{\Omega_{ik}^{j}}\left[F_{kj}(x)-F_{ij}(x)\right]dx \qquad i,k\in M,\quad j\in N_b, \tag{2}$$

$$B_i\left(F_{ij}(x),F_{kj}(x)\right)=\int_{\Theta_{ik}^{j}}\left[F_{ij}(x)-F_{kj}(x)\right]dx, \qquad i,k\in M,\quad j\in N_b, \tag{3}$$

Where $\Omega_{ik}^{j}=\left\{x\mid F_{ij}(x)<F_{kj}(x), x\in\left[a_{ik}^{j*},b_{ik}^{j*}\right]\right\}$, $\Theta_{ik}^{j}=\left\{x\mid F_{ij}(x)>F_{kj}(x), x\in\left[a_{ik}^{j*},b_{ik}^{j*}\right]\right\}$, and $a_{ik}^{j*}=\min\{a_{ij},a_{kj}\}$, $b_{ik}^{j*}=\max\{b_{ij},b_{kj}\}$.

Here, $a_{ij}=b_{ij}=x'_{ij}$ and $a_{kj}=b_{kj}=x'_{kj}$ for $j\in N^k; a_{ij}=x_{ij}^{l}, b_{ij}=x_{ij}^{u}, a_{kj}=x_{kj}^{l}$ and $b_{kj}=x_{kj}^{u}$ for $j\in N^l; a_{ij}=\alpha_{ij}, b_{ij}=\gamma_{ij}, a_{kj}=\alpha_{kj}$ and $b_{kj}=\gamma_{kj}$ for $j\in N^F$.

Correspondingly, for the cost attributes, the superior and inferior values of $F_{ij}(x)$ relative to $F_{kj}(x)$ are respectively expressed by

$$C_s\left(F_{ij}(x),F_{kj}(x)\right)=\int_{\Theta_{ik}^{j}}\left[F_{ij}(x)-F_{kj}(x)\right]dx, \qquad i,k\in M,\quad j\in N_c, \tag{4}$$

$$C_i\left(F_{ij}(x),F_{kj}(x)\right)=\int_{\Omega_{ik}^{j}}\left[F_{kj}(x)-F_{ij}(x)\right]dx \qquad i,k\in M,\quad j\in N_c, \tag{5}$$

Here, a graphical exposition of above equations is not required as the data format is only crisp type. $D(F_{ij}(x), F_{kj}(x))$ and $T(F_{ij}(x),F_{kj}(x))$ are also the superior and inferior values of alternative A_i relative to A_k, respectively. The gain of alternative A_i relative to alternative A_k concerning attribute C_j, G_{ik}^{j} is expressed by,

$$G_{ik}^{j} = D(F_{ij}(x),\ F_{kj}(x)) \qquad i,k\in M,\quad j\in N. \tag{6}$$

Correspondingly, the loss of A_i relative to A_k, L_{ik}^{j} is expressed by

$$L_{ik}^{j} = -T(F_{ij}(x),\ F_{kj}(x)) \qquad i,k\in M,\quad j\in N. \tag{7}$$

It is clear from the above Eqs. (6) and (7), that $G_{ik}^{j}\geq 0$ and $L_{ik}^{j}\leq 0$. Based on the above analysis, gain matrix $G_j=\left[G_{ik}^{j}\right]_{mxm}$ and loss matrix $L_j=\left[L_{ik}^{j}\right]_{mxm}$ concerning attributes C_j can be constructed, respectively, i.e.,

$$G_j = \left[G_{ik}^j\right]_{mxm} = \begin{bmatrix} G_{11}^j & G_{12}^j & \cdots & G_{1m}^j \\ G_{21}^j & G_{22}^j & \cdots & G_{2m}^j \\ \vdots & \vdots & \cdots & \vdots \\ G_{m1}^j & G_{m2}^j & \cdots & G_{mm}^j \end{bmatrix}, \quad \& \quad L_j = \left[L_{ik}^j\right]_{mxm} = \begin{bmatrix} L_{11}^j & L_{12}^j & \cdots & L_{1m}^j \\ L_{21}^j & L_{22}^j & \cdots & L_{2m}^j \\ \vdots & \vdots & \cdots & \vdots \\ L_{m1}^j & L_{m2}^j & \cdots & L_{mm}^j \end{bmatrix}, \ j \in N$$

Where $G_{ii}^j = L_{ii}^j = 0$ for $\forall i \in M$.

Step 3: Construct normalized matrices for gain and loss matrices. Since gains or losses concerning different attributes are generally incommensurate, they need to be normalized so as to transform them into comparable values. This is achieved by normalizing every element in matrix $G_j = \left[G_{ik}^j\right]_{mxm}$ or $L_j = \left[L_{ik}^j\right]_{mxm}$ into a corresponding element in matrix $Y_j = \left[Y_{ik}^j\right]_{mxm}$ or $Z_j = \left[Z_{ik}^j\right]_{mxm}$ using the following formulae [2]:

$$Y_{ik}^j = \frac{G_{ik}^j - G_j^{\min}}{G_j^{\max} - G_j^{\min}}, \qquad i,k \in M, \quad j \in N, \tag{8}$$

$$Z_{ik}^j = \frac{L_{ik}^j - L_j^{\max}}{L_j^{\max} - L_j^{\min}}, \qquad i,k \in M, \quad j \in N, \tag{9}$$

Where $G_j^{\max} = \max\left\{G_{ik}^j \mid i,k \in M\right\}, G_j^{\max} = \max\left\{G_{ik}^j \mid i,k \in M\right\}$ and $L_j^{\max} = \max\left\{L_{ik}^j \mid i,k \in M\right\}$ and $L_j^{\min} = \min\left\{L_{ik}^j \mid i,k \in M\right\}, \ j \in N$. Here, $Y_{ik}^j \in [0,1]$ and $Z_{ik}^j \in [-1,0]$.

Step 4: Construct dominance degree matrix. Based on the classical TODIM method, the dominance degree of alternative A_i over alternative A_k concerning attribute C_j can be calculated.

For continuous values, the gain and loss of alternative A_i relative to A_k, G_{ik}^j and L_{ik}^j, may exist simultaneously, thus dominance degrees for the gain and loss should be first calculated respectively, and then be aggregated. The dominance degree for the gain $\Phi_{ik}^{j(+)}$ is given by Fan et. al. [2] as follow:

$$\Phi_{ik}^{j(+)} = \sqrt{\frac{w_j Y_{ik}^j}{w_r \sum_{j=1}^{n} (w_j / w_r)}}, \qquad i,k \in M, \ j \in N \tag{10}$$

And the dominance degree for the loss, $\Phi_{ik}^{j(-)}$, is given by

$$\Phi_{ik}^{j(-)} = \frac{-1}{\theta} \sqrt{\frac{-Z_{ik}^j w_r}{w_j} \sum_{j=1}^{n} (w_j / w_r)}, \qquad i,k \in M, \ j \in N \tag{11}$$

Where $w_r = \max\left\{w_j \mid j \in N\right\}$, and θ is the attenuation factor of the loss. θ denotes the degree of loss aversion of the DM, θ>0. The greater θ is, the lower the degree of loss aversion is. Obviously, $0 \le \Phi_{ik}^{j(+)} < 1$ and $\Phi_{ik}^{j(-)} \le 0$.

Further, dominance degrees $\Phi_{ik}^{j(+)}$ and $\Phi_{ik}^{j(-)}$ are aggregated, i.e.,

$$\Phi_{ik}^j = \Phi_{ik}^{j(+)} + \Phi_{ik}^{j(-)}, \qquad i,k \in M, \ j \in N \tag{12}$$

Thus, the dominance degree matrix concerning attribute C_j, Φ_j can be constructed, i.e.,

$$\Phi_j = \left[\Phi_{ik}^j\right]_{mxm} = \begin{bmatrix} \Phi_{11}^j & \Phi_{12}^j & \cdots & \Phi_{1m}^j \\ \Phi_{21}^j & \Phi_{22}^j & \cdots & \Phi_{2m}^j \\ \vdots & \vdots & \cdots & \vdots \\ \Phi_{m1}^j & \Phi_{m2}^j & \cdots & \Phi_{mm}^j \end{bmatrix}, \ j \in N,$$

Where $\Phi_{ii}^j = 0$ for $\forall i \in M$, $j \in N$.

Step 5: Construct overall values of each alternative. Based on matrix Φ_j, the overall dominance degree matrix, Δ, is constructed, i.e.,

$$\Delta = [\delta_{ik}]_{mxm} = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1m} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \delta_{m1} & \delta_{m2} & \cdots & \delta_{mm} \end{bmatrix},$$

Where δ_{ik} is the overall dominance degree of alternative A_i over alternative A_k, i.e.,

$$\delta_{ik} = \sum_{j=1}^{n} \Phi_{ik}^{j}, \qquad i,k \in M, \tag{13}$$

Step 6: Calculate the overall value of each alternative based on matrix Δ. The overall value of alternative A_i, $\xi(A_i)$ can be calculated, i.e.,

$$\xi(A_i) = \frac{\sum_{k=1}^{m} \delta_{ik} - \min_{i \in M} \left\{ \sum_{k=1}^{m} \delta_{ik} \right\}}{\max_{i \in M} \left\{ \sum_{k=1}^{m} \delta_{ik} \right\} - \min_{i \in M} \left\{ \sum_{k=1}^{m} \delta_{ik} \right\}}, \qquad i \in M \tag{14}$$

Step 7: Determine the ranking order of alternatives according to the overall values obtained in step 6. It is clear that, $0 \le \xi(A_i) \le 1$ and the greater $\xi(A_i)$ is the better alternative A_i will be. Therefore, in accordance with a descending order of the overall values of all the alternatives, we can determine the ranking of all the alternatives or select the desirable alternative (s).

2.2 ARAS method

ARAS describes an alternative under consideration, to the sum of the values of normalized and weighted criteria. The steps of procedure are explained below [6]:

Step 1: Formulation of decision making matrix for the data having m alternatives (rows) and n criteria describing each alternative (colums). x_{ij} is value representing the performance value of the i alternative in terms of the j criterion and w_j be the criteria weights.

Step 2: The criteria, whose preferable values are minima, are normalized by first using Eq. (15) followed by Eq. (16) and the criteria, whose preferable values are maxima are normalized using Eq. (16).

$$xij = \frac{1}{xij} \tag{15}$$

$$xij = \frac{xij}{\sum_{i=0}^{m} xij} \tag{16}$$

Step 3: Normalized-weighted values of all the criteria are calculated as by

$$X'_{ij} = X_{ij} * W_j \tag{17}$$

Step 4: The values of optimality functions of i alternative is S_i can be given by

$$S_i = \frac{xij}{\sum_{j=1}^{n} x'ij} \tag{18}$$

The biggest value is the best, and the least one is the worst. The optimality function S_i has a direct and proportional relationship with x_{ij} and weights of the criteria and their relative influence on the final result.

Step 5: The degree of the alternative utility is determined by a comparision of the variant with the ideally best S_0. The utility degree U_i of an alternative A_i as:

$U_i = S_i/S_o$. (19)

The utility values are in the interval [0,1] and can be used for the ranking of alternatives.

2.3 Weighted Sum Product ASsessment (WASPAS)

2.3.1Weighted Sum Product Assessment (WASPAS) method

The WASPAS method was developed by Zavadskas et al. (2012) and applied for dealing with civil engineering problems by Zavadskas et al. (2013a, b).

Step 1: In general, given MADM problem is defined on m alternatives and n decision attribute. w_j denotes the relative significance of the attribute and x_{ij} is the performance value of alternative when it is evaluated in terms of attribute .

The linear normalization of the initial criteria values x_{ij} is applied and dimensionless values $\bar{x}_{ij}$ are obtained:

$$\bar{x}_{ij} = \frac{x_{ij}}{\max\limits_{j} x_{ij}}, \tag{20}$$

if $\max\limits_{j} x_{ij}$ value is preferable or

$$\bar{x}_{ij} = \frac{\min\limits_{j} x_{ij}}{x_{ij}}, \tag{21}$$

if $\min\limits_{j} x_{ij}$ value is preferable.

Step 2: The relative importance of alternative, denoted as Q_j is calculated applying the joint generalized attribute of the weighted aggregation of additive and multiplicative methods:

$$Q_j^{(1)} = \sum\nolimits_{i=1}^{m} \bar{x}_{ij} w_i \tag{22}$$

$$Q_j^{(2)} = \prod\nolimits_{i=1}^{m} (\bar{x}_{ij})^{w_i} \tag{23}$$

Step 3: According to the Weighted Sum Model (WSM) and Weighted Product Model (WPM), the total relative importance of alternative i, denoted as Q_j. Then the optimal values of weighted coefficient λ_j can be calculated using Eq. (24).

$$\lambda_j = \frac{\sigma^2(Q_j^{(2)})}{\sigma^2(Q_j^{(1)}) + \sigma^2(Q_j^{(2)})} \tag{24}$$

Variances $\sigma^2(Q_j^{(1)})$ and $\sigma^2(Q_j^{(2)})$ should be calculated as:

$$\sigma^2(Q_j^{(1)}) = \sum_{i=1}^{m} w_i^2 \sigma^2(\bar{x}_{ij}) \tag{25}$$

$$\sigma^2(Q_j^{(2)}) = \sum_{i=1}^{m} \left(\frac{\prod_{i=1}^{m} (\bar{x}_{ij})^{w_i} w_i}{(\bar{x}_{ij})^{w_i} (\bar{x}_{ij})^{(1-w_i)}} \right)^2 \sigma^2(\bar{x}_{ij}) \tag{26}$$

Step 4: The ranking of alternatives is presented:

$$Q_j = \lambda_j \sum_{i=1}^{m} \bar{x}_{ij} w_i + (1-\lambda_j) \prod_{i=1}^{m} (\bar{x}_{ij})^{wi} \tag{27}$$

where λ_j is the weighted coefficient. Alternatives are ranked according to Q_j.

The applicability of three selected multiple attribute decision making methods are presented in the next section.

III. RAPID PROTOTYPING SYSTEM SELECTION

3.1 Example 1: Rapid prototyping system selection

3.1.1 Extended TODIM and computations steps

The computational details are presented for the selection of best rapid prototyping system selection.

Step 1: Transform format of attribute values into the format of random variables with cumulative distribution functions using Eq. (1). The qualitative and quantitative data for the RP system is given in Table 1. The qualitative data is transformed into crisp number as per the Byun and Lee [12]. The crisp data format of attribute values are transformed into the format of random variables with cumulative distribution functions.

Step 2: Construction of gain matrix $G_j = [G_{ik}^j]_{mxn}$ and loss matrix $L_j = [L_{ik}^j]_{mxn}$ using Eqs. (6)-(7), $j \in N$. For the comparison purpose the qualitative data of two attributes (B, C) are transformed into crisp number and shown in Table 2. Therefore, the calculations for gain and loss of each alternative relative to others become simple and need not to carry out the graphical exposition for the superior and inferior values. Let the gain and loss of each alternative relative to the others are G_1, G_2, G_3, G_4, G_5, G_6 and L_1, L_2, L_3, L_4, L_5, L_6 respectively for all six attributes.

The gain matrices are:

$$G_1 = \begin{bmatrix} 0 & 30 & 5 & 65 & 0 & 480 \\ 0 & 0 & 0 & 35 & 0 & 450 \\ 0 & 25 & 0 & 60 & 0 & 475 \\ 0 & 0 & 0 & 0 & 0 & 415 \\ 25 & 55 & 30 & 95 & 0 & 505 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} G_2 = \begin{bmatrix} 0 & 6 & 14.5 & 13.5 & 0 & 9 \\ 0 & 0 & 8.5 & 7.5 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 3 & 9 & 17.5 & 16.5 & 0 & 12 \\ 0 & 0 & 5.5 & 4.5 & 0 & 0 \end{bmatrix} G_3 = \begin{bmatrix} 0 & 25 & 35 & 40 & 35 & 60 \\ 0 & 0 & 10 & 15 & 10 & 35 \\ 0 & 0 & 0 & 5 & 0 & 25 \\ 0 & 0 & 0 & 0 & 0 & 20 \\ 0 & 0 & 0 & 5 & 0 & 25 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 4 \\ 3.5 & 0 & 0 & 0 & 2.5 & 7.5 \\ 5 & 1.5 & 0 & 0 & 4 & 9 \\ 5 & 1.5 & 0 & 0 & 4 & 9 \\ 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} G_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.08 & 0.08 & 0 & 0 & 0.08 & 0 \\ 0.155 & 0.155 & 0.075 & 0 & 0.155 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.61 & 0.61 & 0.53 & 0.455 & 0.61 & 0 \end{bmatrix} G_6 = \begin{bmatrix} 0 & 0 & 0.245 & 0 & 0 & 0 \\ 0 & 0 & 0.245 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.09 & 0.09 & 0.335 & 0 & 0 & 0 \\ 0.09 & 0.09 & 0.335 & 0 & 0 & 0 \\ 0.245 & 0.245 & 0.45 & 0.155 & 0.155 & 0 \end{bmatrix}$$

The loss matrices are:

$$L_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & -25 & 0 \\ -30 & 0 & -25 & 0 & -55 & 0 \\ -5 & 0 & 0 & 0 & -30 & 0 \\ -65 & -35 & -60 & 0 & -95 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -480 & -450 & -475 & -415 & -505 & 0 \end{bmatrix} L_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & -3 & 0 \\ -6 & 0 & 0 & 0 & -9 & 0 \\ -14.5 & -8.5 & 0 & -1 & -17.5 & -5.5 \\ -13.5 & -7.5 & 0 & 0 & -16.5 & -4.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -9 & -3 & 0 & 0 & -12 & 0 \end{bmatrix} L_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -25 & 0 & 0 & 0 & 0 & 0 \\ -35 & -10 & 0 & 0 & 0 & 0 \\ -40 & -15 & -5 & 0 & -5 & 0 \\ -35 & -10 & 0 & 0 & 0 & 0 \\ -60 & -35 & -25 & -20 & -25 & 0 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} 0 & -3.5 & -5 & -5 & -1 & 0 \\ 0 & 0 & -1.5 & -1.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.5 & -4 & -4 & 0 & 0 \\ -4 & -7.5 & -9 & -9 & -5 & 0 \end{bmatrix} L_5 = \begin{bmatrix} 0 & 0 & -0.08 & -0.155 & 0 & -0.61 \\ 0 & 0 & -0.08 & -0155 & 0 & -0.61 \\ 0 & 0 & 0 & -.0.75 & 0 & -0.53 \\ 0 & 0 & 0 & 0 & 0 & -0.455 \\ 0 & 0 & -0.08 & -0.155 & 0 & -0.61 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} L_6 = \begin{bmatrix} 0 & 0 & 0 & -0.09 & -0.09 & -0.245 \\ 0 & 0 & 0 & -0.09 & -0.09 & -0.245 \\ -0.245 & -0.245 & 0 & -.0.335 & -0.355 & -0.45 \\ 0 & 0 & 0 & 0 & 0 & -0.155 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.155 \end{bmatrix}$$

Step 3: The normalized matrices $Y_j = [Y_{ik}^j]_{mxn}$ and $Z_j = [Z_{ik}^j]_{mxn}$ using Eqs. (8)-(9) $j \in N$. The twelve normalized matrices ($Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Z_1, Z_2, Z_3, Z_4, Z_5, Z_6$) are constructed, respectively, i.e.,

- For beneficial attributes:

$$Y_1 = \begin{bmatrix} 0 & 0.0594 & 0.0099 & 0.1287 & 0 & 0.9505 \\ 0 & 0 & 0 & 0.0693 & 0 & 0.8911 \\ 0 & 0.0495 & 0 & 0.1188 & 0 & 0.9406 \\ 0 & 0 & 0 & 0 & 0 & 0.8218 \\ 0.0495 & 0.1089 & 0.0594 & 0.1881 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad Y_2 = \begin{bmatrix} 0 & 0.3429 & 0.8286 & 0.7714 & 0 & 0.5143 \\ 0 & 0 & 0.4857 & 0.4286 & 0 & 0.1714 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.05714 & 0 & 0 & 0 \\ 0.1714 & 0.5143 & 1 & 0.9429 & 0 & 0.6857 \\ 0 & 0 & 0.3143 & 0.2571 & 0 & 0 \end{bmatrix}$$

$$Y_3 = \begin{bmatrix} 0 & 0.4167 & 0.5833 & 0.6667 & 0.5833 & 1 \\ 0 & 0 & 0.1667 & 0.25 & 0.1667 & 0.5833 \\ 0 & 0 & 0 & 0.0833 & 0 & 0.4167 \\ 0 & 0 & 0 & 0 & 0 & 0.3333 \\ 0 & 0 & 0 & 0.0833 & 0 & 0.4167 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad Y_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & & 0.4444 \\ 0.3889 & 0 & 0 & 0 & 0.2778 & 0.8333 \\ 0.5556 & 0.1667 & 0 & 0 & 0.4444 & 1 \\ 0.5556 & 0.1667 & 0 & 0 & 0.4444 & 1 \\ 0.1111 & 0 & 0 & 0 & 0 & 0.5556 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Y_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1501 & 0.1501 & 0 & 0 & 0.1501 & 0 \\ 0.3001 & 0.3001 & 0.1501 & 0 & 0.3001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0.8504 & 0.7002 & 1 & 0 \end{bmatrix} \quad Y_6 = \begin{bmatrix} 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1667 & 0.1667 & 0.6667 & 0 & 0 & 0 \\ 0.1667 & 0.1667 & 0.6667 & 0 & 0 & 0 \\ 0.5 & 0.5 & 1 & 0.3333 & 0.3333 & 0 \end{bmatrix}$$

- For cost (non-beneficial) type attributes are:

$$Z_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.0495 & 0 \\ -0.0594 & 0 & -0.0495 & 0 & -0.1089 & 0 \\ -0.0099 & 0 & 0 & 0 & -0.0594 & 0 \\ -0.1287 & -0.0693 & -0.1188 & 0 & -0.1881 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.9505 & -0.8911 & -0.9406 & -0.8218 & -1 & 0 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.1714 & 0 \\ -0.3429 & 0 & 0 & 0 & -0.5143 & 0 \\ -0.8286 & -0.4857 & 0 & -0.0571 & -1 & -0.3143 \\ -0.7714 & -0.4286 & 0 & 0 & -0.9429 & -0.2571 \\ 0 & 0 & 0 & 0. & 0 & 0 \\ -0.5143 & -0.1714 & 0 & 0 & -0.6857 & 0 \end{bmatrix}$$

$$Z_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.4167 & 0 & 0 & 0 & 0 & 0 \\ -0.5833 & -0.1667 & 0 & 0 & 0 & 0 \\ -0.6667 & -0.25 & -0.0833 & 0 & -0.0833 & 0 \\ -0.5833 & -0.1667 & 0 & 0 & 0 & 0 \\ -1 & -0.5833 & -0.4167 & -0.3333 & -0.4167 & 0 \end{bmatrix}$$

$$Z_4 = \begin{bmatrix} 0 & -0.3889 & -0.5556 & -0.5556 & -0.1111 & 0 \\ 0 & 0 & -0.1667 & -0.16667 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2778 & -0.4444 & -0.4444 & 0 & 0 \\ -0.4444 & -0.8333 & -1 & -1 & -0.5556 & 0 \end{bmatrix}$$

$$Z_5 = \begin{bmatrix} 0 & 0 & -0.1501 & -0.3001 & 0 & -1 \\ 0 & 0 & -0.1501 & -0.3001 & 0 & -1 \\ 0 & 0 & 0 & -0.1501 & 0 & -0.8504 \\ 0 & 0 & 0 & 0 & 0 & -0.7002 \\ 0 & 0 & -0.1501 & -0.3001 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad Z_6 = \begin{bmatrix} 0 & 0 & 0 & -0.1667 & -0.1667 & -0.5 \\ 0 & 0 & 0 & -0.1667 & -0.1667 & -0.5 \\ -0.5 & -0.5 & 0 & -0.6667 & -0.6667 & -1 \\ 0 & 0 & 0 & 0 & 0 & -0.3333 \\ 0 & 0 & 0 & 0 & 0 & -0.3333 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 4: The dominance degree matrix $\phi_j = [\phi_{ik}^j]_{mxn}$ using Eqs. (10)-(12), $j \in N$. The dominance degree matrices concerning attributes are:

$$\Phi_1 = \begin{bmatrix} 0 & 0.1377 & 0.0562 & 0.2026 & -0.394 & 0.5506 \\ -0.432 & 0 & -0.394 & 0.1487 & -0.584 & 0.5332 \\ -0.176 & 0.1257 & 0 & 0.1947 & -0.432 & 0.5478 \\ -0.635 & -0.466 & -0.61 & 0 & -0.768 & 0.512 \\ 0.1257 & 0.1864 & 0.1377 & 0.245 & 0 & 0.5648 \\ -1.726 & -1.671 & -1.7171 & -1.605 & -1.771 & 0 \end{bmatrix}$$

$$\Phi_2 = \begin{bmatrix} 0 & 0.3307 & 0.5141 & 0.4961 & -0.733 & 0.405 \\ 0.5332 & -1.0367 & 0 & 0.3697 & -1.27 & 0.2338 \\ -1.6116 & -1.234 & 0 & -.423 & -1.771 & -0.993 \\ -1.5551 & -1.159 & 0.135 & 0 & -1.719 & -0.898 \\ 0.2338 & 0.405 & 0.5648 & 0.5484 & 0 & 0.4677 \\ -1.2697 & -0.733 & 0.3166 & 0.2864 & -1.466 & 0 \end{bmatrix}$$

$$\Phi_3 = \begin{bmatrix} 0 & 0.2318 & 0.2743 & 0.2933 & 0.2743 & 0.3592 \\ -1.797 & 0 & 0.1466 & 0.1796 & 0.1466 & 0.2743 \\ -2.126 & -1.137 & 0 & 0.1037 & 0 & 0.2318 \\ -2.273 & -1.392 & -0.804 & 0 & -0.804 & 0.2074 \\ -2.126 & -1.137 & 0 & 0.1037 & 0 & 0.2318 \\ -2.784 & -2.126 & -1.797 & -1.607 & -1.797 & 0 \end{bmatrix}$$

$$\Phi_4 = \begin{bmatrix} 0 & -1.736 & -2.075 & -2.075 & -0.928 & 0.2394 \\ 0.224 & 0 & -1.137 & -1.137 & 0.1893 & 0.3279 \\ 0.2677 & 0.1466 & 0 & 0 & 0.2394 & 0.3592 \\ 0.2677 & 0.1466 & 0 & 0 & 0.2394 & 0.3592 \\ 0.1197 & -1.467 & -1.856 & -1.856 & 0 & 0.2677 \\ -1.856 & -2.542 & -2.784 & -2.784 & -2.075 & 0 \end{bmatrix}$$

$$\Phi_5 = \begin{bmatrix} 0 & 0 & -1.699 & -2.402 & 0 & -4.385 \\ 0 & 0 & -1.699 & -2.402 & 0 & -4.385 \\ 0.0883 & 0.0883 & 0 & -1.699 & 0.0883 & -4.044 \\ 0.1249 & 0.1249 & 0.0883 & 0 & 0.1249 & -3.67 \\ 0 & 0 & -1.699 & -2.402 & 0 & -4.385 \\ 0.228 & 0.228 & 0.2103 & 0.1908 & 0.228 & 0 \end{bmatrix} \quad \Phi_6 = \begin{bmatrix} 0 & 0 & 0.1612 & -1.79 & -1.79 & -3.1009 \\ 0 & 0 & 0.1612 & -1.79 & -1.79 & -3.1009 \\ -3.101 & -3.1010 & 0 & -3.581 & -3.581 & -4.3853 \\ 0.0931 & 0.0931 & 0.1862 & 0 & 0 & -2.5319 \\ 0.0981 & 0.0981 & 0.1862 & 0 & 0 & -2.5319 \\ 0.1612 & 0.1612 & 0.228 & 0.1317 & 0.1317 & 0 \end{bmatrix}$$

Here, in the calculation process, W_r = max{0.319, 0.319, 0.052, 0.052, 0.129, 0.129}= 0.319. Taking $\theta = 1$, which means that the losses will contribute with their real value to the global value [2].

Step 5: By using Eq. (13), overall dominance degree matrix $\Delta = [\delta_{ik}]_{mxn}$ is built, i. e.,

$$\Delta = \begin{bmatrix} 0 & -1.036 & -2.768 & -5.276 & -3.571 & -5.932 \\ -3.041 & 0 & -2.528 & -4.631 & -3.308 & -6.117 \\ -6.659 & -5.111 & 0 & -5.404 & -5.455 & -8.283 \\ -3.978 & -2.653 & -1.004 & 0 & -2.927 & -6.021 \\ -1.554 & -1.92 & -2.666 & -3.361 & 0 & -5.385 \\ -7.247 & -6.683 & -5.544 & -5.388 & -6.749 & 0 \end{bmatrix}$$

Step 6: By using Eq. (14), the overall value of each alternative can be obtained. These overall values are $\xi(A_1) = 0.7791$, $\xi(A_2) = 0.7167$, $\xi(A_3) = 0.0417$, $\xi(A_4) = 0.8988$, $\xi(A_5) = 1.0002$, and $\xi(A_6) = 0.00007$.

Step 7: According to the overall values, the ranking order of the six candidate rapid prototyping systems is determined. The ranking gives Quadra as the first choice for the given purpose, whereas the Z402 is ranked last.

3.1.2 Additive Ration ASsessment (ARAS) method and computational steps

Step 1: The decision making matrix for the ranking procedure is formed. The differentiation for minima and maxima is carried out. The matrix formed to help in further calculations.

Step 2 & 3: By using Eqs. (15) & (16), the normalization for minima and maxima are calculated respectively. Table 3 shows the normalized values for all RP systems with respect to their respective criterion. The weighted normalized values are tabulated in Table 4.

Step 4: The optimality functions of each alternative is found by using Eq. (17) and tabulated in Table 5. The highest value of optimality gives the best alternative. These values directly influence the final result of ranking.

RP Systems	Min	Min	Min	Min	Max	Max
1	120	6.5	0.745	0.5	65	5
2	150	12.5	0.745	0.5	40	8.5
3	125	21	0.665	0.745	30	10
4	185	20	0.59	0.41	25	10
5	95	3.5	0.745	0.41	30	6
6	600	15.5	0.135	0.255	5	1
Weight	0.319	0.319	0.052	0.052	0.129	0.129

Step 4: The optimality functions of each alternative is found by using Eq. (17) and tabulated in Table 5. The highest value of optimality gives the best alternative. These values directly influence the final result of ranking.
Step 5: To obtain ideally best solution the final utility degree for each alternative is calculated.

3.1.3 Weighted Sum Product ASsessment (WAAPAS) method and computational steps

Step 1: The data shown in Table 6 is normalized using Eqs. (20) and (21) for beneficial and non-beneficial attributes respectively.

Step 2: Weighted normalized decision matrix is using Eqs. (22) and (23) are formed for $Q_j^{(1)}$ and $Q_j^{(2)}$ and for individual attribute are calculated and represented in Tables 7 and 8 respectively.

Step 3: By using Eqs. (24) and (25) the variances are calculated and then by Eq. (23) the weighted coefficient is determined for each alternative.

Step 4: By using Eq. (26) total relative significance of alternatives are determined. The final results are tabulated in Table 9.

The results of various ranking methods are quite accurate and match with the results of the previous researchers. Quadra is the best choice for the given component whereas Z402 is the worst option for the selection to produce a given part.

Table 1. The ratings of attributes on major Rapid Prototyping (RP) systems

RP System	Accuracy (μm) (A)	Surface Roughness (μm) (R)	Tensile strength (MPa) (S)	Elongation % (E)	Cost of the Part (C)	Built time (B)
SLA3500	120	6.5	65	5	Very High	Medium
SLA2500	150	12.5	40	8.5	Very High	Medium
FDM8000	125	21	30	10	High	Very High
LOM1015	185	20	25	10	Slightly High	Slightly Low
Quadra	95	3.5	30	6	Very High	Slightly Low
Z402	600	15.5	5	1	Very Very Low	Very Low

Table 2. Objective data of the RP system selection attributes

RP System	Accuracy (μm) (A)	Surface Roughness (μm) (R)	Tensile strength (MPa) (S)	Elongation % (E)	Cost of the Part (C)	Built time (B)
SLA3500	120	6.5	65	5	0.745	0.5
SLA2500	150	12.5	40	8.5	0.745	0.5
FDM8000	125	21	30	10	0.665	0.745
LOM1015	185	20	25	10	0.665	0.745
Quadra	95	3.5	30	6	0.745	0.41
Z402	600	15.5	5	1	0.135	0.255

T**able 3** . Normalized decision matrix for minima criterion and summation

RP system	A	R	C	B	S	E
SLA3500	0.0083	0.1538	1.3423	2	65	5
SLA2500	0.0067	0.08	1.3423	2	40	8.5
FDM8000	0.008	0.0476	1.5038	1.3423	30	10
LOM1015	0.0054	0.05	1.6949	2.4390	25	10
Quadra	0.0105	0.2857	1.3423	2.4390	30	6
Z402	0.0017	0.0645	7.4074	3.9216	5	1
$\sum_{i=1}^{6} x_{ij}$	0.0406	0.6817	14.6329	14.1419	195	40.5

Table 4.. Weighted normalized values of all criterion for RP system selection

RP system	A	R	C	B	S	E
SLA3500	0.0655	0.0719	0.0048	0.0074	0.043	0.01593
SLA2500	0.0524	0.0374	0.0048	0.0074	0.0265	0.0271
FDM8000	0.0629	0.0222	0.0053	0.0049	0.0198	0.0319
LOM1015	0.0425	0.0234	0.0060	0.0090	0.0165	0.0319
Quadra	0.0827	0.1337	0.0048	0.0090	0.0198	0.0191
Z402	0.0131	0.0302	0.02632	0.0144	0.0033	0.0032

Table 5. Optimality values of all attribute for RP system selection

RP system
SLA3500
SLA2500
FDM8000
LOM1015
Quadra
Z402

Table 6. Normalized data for the example 1

RP system	A	R	S	E	C	B
SLS3500	0.7916	0.5384	0.1812	0.51	1	0.5
SLA2500	0.6333	0.28	0.1812	0.51	0.6154	0.85
FDM8000	0.76	0.16667	0.2030	0.3423	0.4615	1
LOM1015	0.5135	0.175	0.2288	0.6220	0.3846	1
Quadra	1	1	0.1812	0.6220	0.4615	0.6
Z402	0.1583	0.22581	1	1	0.0769	0.1

Table 7 Weighted normalized decision matrix for $Q_j^{(1)}$

RP system	A	R	S	E	C	B
SLS3500	0.2525	0.1718	0.0094	0.0265	0.129	0.0645
SLA2500	0.2020	0.0893	0.0094	0.0265	0.0794	0.1097
FDM8000	0.2424	0.0532	0.01065	0.01778	0.0595	0.129
LOM1015	0.1638	0.0558	0.0119	0.0323	0.0496	0.129
Quadra	0.319	0.319	0.0094	0.0321	0.0595	0.0774
Z402	0.0505	0.0720	0.052	0.052	0.0099	0.0129

Table 8 Weighted normalized decision matrix for $Q_j^{(2)}$

RP system	A	R	S	E	C	B
SLS3500	0.9282	0.8208	0.9150	0.9656	1	0.9145
SLA2500	0.8644	0.6663	0.9150	0.9656	0.93932	0.9793
FDM8000	0.9162	0.5646	0.9204	0.9458	0.9051	1
LOM1015	0.8085	0.5735	0.9262	0.9756	0.8840	1
Quadra	1	1	0.9150	0.9756	0.9051	0.9362
Z402	0.5555	0.6222	1	1	0.7183	0.7430

Table 9. Final result of WASPAS method for example 1

RP systems	Optimal λ_j	Relative Significance Q_j	Rank
SLS3500	0.44	0.6324	3
SLA2500	0.44	0.4893	4
FDM8000	0.33	0.4422	5
LOM1015	0.4	0.3992	2
Quadra	0.39	0.7799	1
Z402	0.38	0.20910	6

Table 10. Ranking comparison using various methods for example 1

RP Systems	WASPAS Method	Extended TODIM	Modified TOPSIS	GTMA method
			(Byun and Lee, 2005)	(Rao, 2007)
SLS3500	3	3	2	2
SLA2500	4	4	3	3
FDM8000	5	5	4	5
LOM1015	2	2	5	4
Quadra	1	1	1	1
Z402	6	6	6	6

Dr. V. P. Darji" Rapid Prototyping System selection using multi criteria decision making methods" Quest Journals Journal of Research in Applied Mathematics , vol. 05, no. 03, 2019, pp. 05-15