Quest Journals
Journal of Research in Applied Mathematics
Volume 5 ~ Issue 3 (2019) pp: 25-30
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

Pre-Predator Model With A Double Mutualist In The Open Ecological Space

Ortiz. L[a], Ferreira. R [b], Sánchez. S[a], Oliveira. K[b], Z. Ribeiro[b] ,Lacort. M[b], Libório. A[c], RuizA. I.[b]
a: Faculty of Mathematics and Computation, U O.
b: University of the State of Amazonas
c: Federal InstituteofBrasilia

ABSTRACT: *This is a study of the Predator-Predator model as a form of coexistence of different species of animals in the open nature, indicating the moments of growth and decreases, which leads to a cyclical behavior in its development. The model is presented for when there is a mutualistic pair and a predator that feeds only on one of these species, giving conclusions of future trends;here the existence of the quasi-normal form is demonstrated which facilitates the study of the process reaching conclusions of the future development regarding the coexistence.*
KEY WORDS: *Ecology, Prey, Predator,Mutualist.*

Received 04 Feb., 2020; Accepted 20 Feb., 2020 © The author(s) 2020.
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I. INTRODUÇÃO

Ecology is the science that studies living beings and their interactions with the environment in which they live. This science is of utmost importance because the results of their studies provide data that reveal whether animals and ecosystems are in perfect harmony. At a time when deforestation and the extinction of several species are under way, the work of ecologists is of the utmost importance.

In [9] the authors deal with the problem of coexistence of species in the open space, where especially for the case of a prey and a predator we saw the cyclic behavior in its numerical variation.

In [8] we dealt with the case of three species where a native predator, a native prey and a predator prey appear, here we deal with the problem of a predator that feeds on one of the third species that mutualistic.

The problem of coexistence between different species in an open ecological space is addressed in [3], and [4], where the different types of coexistence and one can favor or hinder the development of the other.

Many types of biological systems have been modeled mathematically with the purpose of realizing a better study of the natural interaction that exists between different species; in particular the prey-predator model has a relevant position due to the applicability not only of biology where it practically governs the coexistence of different species in open space, but also because it can be applied in other areas including economics. Here, in addition to the highly publicized Lorka-Volterra models, we will analyze lesser known ones in addition to their qualitative study.

The model was discovered independently by Lotka and Volterra, and for this reason it is known as a model Lotka-Volterra or model predator-prey that describes the evolution of prey and predators very well when they are located in an isolated ecosystem. Nevertheless, we have to clarify that two distinct populations in the same environment have several ways of surviving, for example:

- Mutual competence, that is to say compete for the same food source, tend to cause the extinction of a population of them, and the other tends to take advantage of the maximum capacity of environmental resources.
- Interdependence, that is to say the two populations provide some food resources, live peacefully among them, and tend to a state of equilibrium.
- The law of the jungle, is to say a population survives depending on the abundance of natural resources, called prey; however, the other population lives depending on the populations of prey, called the predator. The two elements are composed by the prey-predator model. - The parasitic life, where one species feed on the other without killing it, but which by all means shaves its quality of life.

[10] refers to the mathematical modeling of several processes between them, dealing with the Prey-Predator model, which includes the possibility of system integration that simulates this interaction between two species.

In [11] the interaction of different species is treated in an open medium, indicating in particular a model for the coexistence between a prey and a predator. In addition, it draws a parallel in the economy coming to some conclusions of the process. The prey-predator model has been extensively treated using different techniques, here it may be included,[6].

Another focus on the Lootka-Volterra model is presented in [1]. In the master's dissertation [7] a very exhaustive study of the prey-predator model is made.

The treatment that we will make in this case corresponds with other models presented in the researches of diseases, especially the case of sicklemia, quite treated and with a large number of already developed models; we will only mention some of these works, in [12] and [13], the qualitative study of different models in autonomous and non-autonomous form of the formation of polymers in the blood is treated.

Following these ideas from these previous works here is simulated the interaction between two species being simplified the referred system to arrive at conclusions of this process of coexistence in the open nature.

In nature the most frequent is the competence between different species in the struggle for survival, appearing here the prey-predator model developed by Lotka 1924; Volterra, 1926; Gause, 1934; Kostitzin, 1939. [2].

Drawing on the work of Lotka, the models that consider the population classified by age groups have been developed, in order to solve the limitations of the models that treat all the individuals of the population identically.

One of the most commonly used classical mathematical models is the dynamic system consisting of two elements (usually two species of animals) interacting in such a way that one (predator) species feeds on the other (prey). A typical example is the system consisting of foxes and rabbits, but it can be transferred without loss of generality to any other context, for example, that formed by sellers and buyers applicable to the Economy.

Foxes feed on rabbits and grass rabbits that we assume will never run out. When there are many rabbits, the population of foxes will increase since food is abundant, but there will come a time when the rabbit population will decline as foxes are abundant. By not having the foxes, enough food their population will decrease, which will again favor the rabbit population. That is to say, if they produce cycles of growth and decrease of both to the populations. Is there a mathematical model that explains this periodic behavior?

On the other hand, in the second decade of the 20th century the Italian biologist Umberto D'Ancona studied and compiled data on catches of fish of some types in the Mediterranean, on the one hand, seals (sharks, rachis, etc.), and other fish that were eaten by the previous ones (sardines, anchovies, etc.), in other words, one prey (the edible fish) and the other predator (seals).

One of the first reasons he thought was related to the First World War. In fact, at that time the first great war developed and this forced less boats to go fishing, and therefore, by reducing the intensity of fishing, this caused an increase in the number of predatory fish (seals). However, this argument had a problem and it was also that the number of edible fish had increased. In fact, if the intensity of fishing is small, then this fact benefits the predators more than the prey. The pertinent question was why?

Briefly, two questions were raised:

•. How to explain the cyclical behavior of the evolution of two populations, where one species feed on the other?

•. Why does a low catch intensity favor predator more than prey?

A detailed study of these types of systems is analyzed in the authors' work [5], which characterizes the behavior of the Lotka-Volterra systems under the hypothesis that the prey grows exponentially in the absence of predators and the predator disappears in absence of prey, studying the behavior of the trajectories in an environment of the equilibrium positions, one can perceive the existence of closed orbits due to the periodicity of the solutions.

Among the models of interaction between species the classic prey-predator model can be highlighted, whose mathematical formulation is composed of Malthusian models and the law of mass action. The analogy can be easily observed in epidemiological models. The prey-predator model also known as the Lotka-Volterra model has also been the starting point for the development of new techniques and mathematical theories.

Predation is a very fundamental type of interaction in nature, where predators catch prey for their food. We can imagine that this relationship is beneficial only to the predator, but from the ecological point of view this is important to regulate the population density of both prey and predator.

Predators remove individuals from the population, consuming them; the ease of catching the prey depends greatly on the size relationship between the prey population and the predator. The greater the population of prey, the greater the possibility of its capture.

Predation occurs when an organism kills and feeds on beings of another species; the animal that killed it is called a predator, which already fed on the prey. Predators are usually found in smaller quantities and have characteristics that favor prey capture; among these characteristics, we can mention the sharp claws, speed and agility.

II. MODEL FORMULATION:

In the study of the interaction of different species in nature lately, mathematical modeling has been very important, since this has allowed us to make predictions regarding the future behavior of this coexistence process, determining if any species could at any time be endangered. extinction; All of this is important, because this way you know when you can introduce some artificial element such as fishing or house as the case may be.

The Predator-Predator model simulates the interaction between species in an open ecological space, in which case we will treat the case of three species. We will consider that there are two mutualistic species. The first one has only feeding limitations or problems with space only with an excessive increase of their species. concentration; the second constitutes prey, for which the only food source is acquired through its relation to the first species; there is a predator that feeds on only this second species of the mutualist pair; Let's admit that during the process, the medium will not have changes.

For the development of the model we will consider the following variables:

$x = x(t)$ - Concentration of the first species at the moment t.

$y = y(t)$ - Prey concentration, mutualistic species of the first, at the moment t.

$z = z(t)$ - Predator concentration of second species in time t.

$$\begin{cases} \frac{dx}{dt} = a_1x + a_2xy - a_3x^2 \\ \frac{dy}{dt} = -b_1y + b_2xy - b_3yz \quad \textbf{(1)} \\ \frac{dz}{dt} = -c_1z + c_2yz \end{cases}$$

With the initial conditions $x(0) = x_0$, $y(0) = y_0$, $z(0) = z_0$; in the model we are considering that the coefficients are all positive. This system has fourequilibrium positions as follows:

$P_1(0,0,0)$; $P_2\left(\frac{b_1}{b_3}, \frac{a_3b_1 - a_1b_2}{a_2b_2}, 0\right)$; $P_3\left(\frac{a_1}{a_2}, 0, \frac{a_1b_2 - a_3b_1}{a_3b_3}\right)$, $P_4\left(\frac{a_2c_1 + a_1c_2}{a_3c_2}, \frac{c_1}{c_2}, \frac{a_1b_2c_2 - a_3b_1c_2 + a_2b_2c_1}{a_3b_3c_2}\right)$

It is evident that the equilibrium position $(0,0,0)$, it is unstable because the first approximation method can be applied directly, where there is a positive eigenvalue; which indicates that the species will not have concentrations close to zero; as for the second and third position of equilibrium, having a null component would cause an ecological imbalance, which would not be very interesting to study, but could be done the same way we will do for the other.

To study the fourth equilibrium position, we need to perform a coordinate transformation to bring the origin to that position, so one would have to do the coordinate transformation:

$$\begin{cases} \boldsymbol{x = x_1 + h} \\ \boldsymbol{y = y_1 + k} \\ \boldsymbol{z = z_1 + l} \end{cases} \qquad \textbf{(2)}$$

At where

$$(h, k, l) = \left(\frac{a_1c_2 + a_2c_1}{a_3c_2}, \frac{c_1}{c_2}, \frac{a_2b_2c_1 - a_3b_1c_2 + a_1b_2c_2}{a_3b_3c_2}\right)$$

By deriving transformation (2) and taking system (1) into consideration, we obtain the system,

$$\begin{cases} \frac{dx_1}{dt} = -\frac{x_1}{c_2}(a_1c_2 + a_2c_1) + \frac{a_2y_1}{a_3c_2}(a_1c_2 + a_2c_1) + a_2x_1y_1 - a_3x_1^2 \\ \frac{dy_1}{dt} = \frac{b_2c_1}{c_2}x_1 + \frac{y_1}{a_3c_2}(a_1b_2c_2 - a_3b_1c_2 + a_2b_2c_1) - \frac{b_3c_1}{c_2}z_1 + b_2x_1y_1 - b_3y_1z_1 \quad (3) \\ \frac{dz_1}{dt} = \frac{y_1}{a_3b_3}(a_1b_2c_2 - a_3b_2c_1 + a_2b_2c_1) + 2c_2yz \end{cases}$$

The characteristic equation of the system (3) has the form,

$$\lambda^3 + n_1\lambda^2 + n_2\lambda + n_3 = 0$$

At where

$n_1 = \frac{1}{a_3c_2}(a_1b_2c_2 + a_2b_2c_1 - a_3b_1c_2 - a_3a_1c_2 - a_3a_2c_1)$,

$n_2 = \frac{1}{a_3b_3c_2^2}[a_1b_2b_3c_1c_2^2 + a_2b_2b_3c_1^2c_2 - a_3b_1b_3c_1c_2^2 - a_1^2b_2b_3c_2^2 - a_1a_2b_2b_3c_2^2 + a_1a_3b_1b_3c_2^2 - 2a_1a_2b_2b_3c_1c_2 - a_2^2b_2b_3c_1c_2 + a_2a_3b_1b_3c_1c_{2\_}a_2^2b_2b_3c_1^2]$,

$n_3 = \frac{1}{a_3b_3c_2^2}(a_1^2b_2b_3c_1c_2^2 + 2a_1a_2b_2b_3c_1^2c_2 + a_2^2b_2b_3c_1^2 - a_1a_3b_1b_3c_1c_2^2 - a_2a_3b_1b_3c_1^2c_2)$.

For all the eigenvalues of the matrix of the linear part of the system (3) to have a negative real part it is necessary and sufficient that all the smallest diagonals of the following Hurwitz matrix (H) are positive.

$$H = \begin{bmatrix} n_1 & 1 & 0 \\ n_3 & n_2 & n_1 \\ 0 & 0 & n_3 \end{bmatrix}$$

It follows that,

$n_1 > 0$, $n_2 > 0$, $n_3 > 0$ and $n_1n_2 > n_3$.

This allows us to determine the stability of the fourth equilibrium position of the system (3) as a function of its coefficients, thus obtaining the following result.

Theorem1: If the following inequalities $n_1 > 0$, $n_2 > 0$, $n_3 > 0$ and $n_1n_2 > n_3$ are satisfied, then the solutions of system (3) converge to $(0,0,0)$ and therefore the solutions of system (1) converge to the equilibrium position (h,k,l).

From this it can be concluded that the concentrations of the species remain in the vicinity of the point (h,k,l).

III. QUASI-NORMAL FORM.

If in the characteristic equation one has that, $n_i > 0$, $i = 1,2,3$ and $n_3 = n_1n_2$ then the matrix of the linear part of the system has a pair of pure imaginary eigenvalues and a negative real eigenvalue, that is to say we are in the presence of a critical case, in which, $\lambda_1 = \sigma i$, $\lambda_2 = -\sigma i$ and $\lambda_3 < 0$, for which it is necessary to simplify the system and apply the Qualitative Theory of Differential Equations .

By means of a non-degenerate linear transformation $X = QY$, the system (3) can be reduced to the shape,

$$\begin{cases} y_1' = \sigma i y_1 + Y_1(y_1, y_2, y_3) \\ y_2' = -\sigma i y_2 + Y_2(y_1, y_2, y_3) \\ y_3' = \lambda_3 y_3 + Y_3(y_1, y_2, y_3) \end{cases} \quad \textbf{(4)}$$

Theorem 2: There is the exchange of variables,

$$\begin{cases} y_1 = z_1 + \hbar_1(z_1, z_2) + \hbar_1^0(z_1, z_2, z_3) \\ y_2 = z_2 + \hbar_2(z_1, z_2) + \hbar_2^0(z_1, z_2, z_3) \\ y_3 = z_3 + \hbar_3(z_1, z_2) \end{cases} \quad \textbf{(5)}$$

Which transforms the system (4) in almost normal form,

$$\begin{cases} z_1' = \sigma i z_1 + z_1 P(z_1 z_2) \\ z_2' = -\sigma i z_2 + z_2 \overline{P}_2(z_1 z_2) \\ z_3' = \lambda_3 z_3 + Z_3(z_1, z_2, z_3) \end{cases} \quad \textbf{(6)}$$

At where $z_2 = \overline{z}_1$, P and $\overline{P}$ complex, in addition, $\boldsymbol{h_1^0}$, $\boldsymbol{h_2^0}$ and Z_3 annul for $z_3 = 0$.

Demonstration: By deriving the transformation (5) along the trajectories of systems (4) and (6) we obtain the system of equations,

$$\begin{cases}(p_1 - p_2 - 1)\sigma i \hbar_1 + z_1 P = Y_1 - \frac{\partial \hbar_1}{\partial z_1} z_1 P - \frac{\partial \hbar_1}{\partial z_2} z_2 \overline{P} - \frac{\partial \hbar_1^0}{\partial z_1} z_1 P \\ - \frac{\partial \hbar_1^0}{\partial z_2} z_2 \overline{P} - \frac{\partial \hbar_1^0}{\partial z_3}(\lambda_3 z_3 + Z_3) \\ p_1 - p_2 + 1) i\sigma \hbar_2 + z_2 \overline{P} = Y_2 - \frac{\partial \hbar_2}{\partial z_1} z_1 P - \frac{\partial \hbar_2}{\partial z_2} z_2 \overline{P} - \frac{\partial \hbar_2^0}{\partial z_1} z_1 P \\ - \frac{\partial \hbar_2^0}{\partial z_2} z_2 \overline{P} - \frac{\partial \hbar_2^0}{\partial z_3}(\lambda_3 z_3 + Z_3) \\ [(p_1 - p_2) - \lambda_3]\hbar_3 + Z_3 = Y_3 - \frac{\partial \hbar_3}{\partial z_1} z_1 P - \frac{\partial \hbar_3}{\partial z_2} z_2 \overline{P} \end{cases} \quad \textbf{(7)}$$

To determine the series that intervene in the systems and the transformation, we will separate the coefficients of the power of degree $p = (p_1, p_2, p_3)$ in the following two cases:

Case I) Making in the system (7)$z_2 = z_3 = 0$,is to say to the vector$p = (p_1, 0,0)$results the system,

$$\begin{cases}(p_1 - p_2 - 1)\sigma i \hbar_1 + z_1 P = Y_1 - \frac{\partial \hbar_1}{\partial z_1} z_1 P - \frac{\partial \hbar_1}{\partial z_2} z_2 \overline{P} \\ p_1 - p_2 + 1) i\sigma \hbar_2 + z_2 \overline{P} = Y_2 - \frac{\partial \hbar_2}{\partial z_1} z_1 P - \frac{\partial \hbar_2}{\partial z_2} z_2 \overline{P} \\ [(p_1 - p_2) i\sigma - \lambda_3]\hbar_3 = Y_3 - \frac{\partial \hbar_3}{\partial z_1} z_1 P - \frac{\partial \hbar_3}{\partial z_2} z_2 \overline{P} \end{cases} \quad \textbf{(8)}$$

The system (8) allows to determine the coefficients of the series:
$P, \overline{P}, h_1, h_2$ and h_3, where, for being the resonant case, we deduce the form indicated for P and$\overline{P}$, and the remaining series are determined uniquely.

Case II) The case when$z_3 \neq 0$of the system (7)

$$\begin{cases} Y_1 = \frac{\partial \hbar_1^0}{\partial z_1} z_1 P + \frac{\partial \hbar_1^0}{\partial z_2} z_2 \overline{P} + \frac{\partial \hbar_1^0}{\partial z_3}(\lambda_3 z_3 + Z_3) \\ Y_2 = \frac{\partial \hbar_2^0}{\partial z_1} z_1 P + \frac{\partial \hbar_2^0}{\partial z_2} z_2 \overline{P} + \frac{\partial \hbar_2^0}{\partial z_3}(\lambda_3 z_3 + Z_3) \\ Z_3 = Y_3(z_1 + \hbar_1 + \hbar_1^0, z_2 + \hbar_2 + \hbar_2^0, z_3 + \hbar_3) \end{cases} \quad \textbf{(9)}$$

Because the series of the system (6) are known expressions, the system (8) allows to calculate the series $\boldsymbol{h_1^0}$,$\boldsymbol{h_2^0}$ and Z_3 .This proves the existence of the exchange of variables.

In the system (6) the functions$P$ and $\overline{P}$ admit the following development in series of powers:

$$\boldsymbol{P(z_1 z_2) = \sum_{n=k}^{\infty} a_n (z_1 z_2)^n + \sum_{n=l}^{\infty} b_n (z_1 z_2)^n}$$

Theorem 3: If $a_k < 0$, then the trajectories of the system (6) are asymptotically stable, otherwise they are unstable.

Demonstration: Consider the positive defined Lyapunov function,
$\boldsymbol{V(z_1, z_{2,}, z_3) = z_1 z_2 + z_3^2}$

The function is such that its derivative along the trajectories of the system (10) has the following expression,
$\boldsymbol{V'(z_1, z_{2,}, z_3) = a_k (z_1 z_2)^{k+1} + 2\lambda z_3^2 + R(z_1, z_2. z_3)}$

This function is defined as negative because in R potencies of degrees higher than those indicated in the initial part of the expression of the derivative of$V$, this allows us to state that the equilibrium position is asymptotically stable; In this case, it can be seen that the variation of the amount of drug in the organism is negative, so if the conditions do not change, it decreases with time; and the patient will be baseline.

IV. CONCLUSION

1. Due to the characteristics of the problem considered, it is natural that the critical case analyzed appears.
2. The almost normal form allows great difficulties to make a qualitative studio of the system trajectories.
3. Theorem one gives conditions that guarantee a balance between the three species participating in the process.
4. Theorem two gives the following methodology to simplify the original system, transforming it into almost normal form, in order to find a better treatment to the studied process.
5. If $a_k < 0$species concentrations will remain in equilibrium converging to the equilibrium position(h, k, l).
6. If the above conditions are not met, no equilibrium can be guaranteed as to the concentrations of the three participating species, which could lead to excessive growth in one of them and extinction of one of the others.

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Maria Rosaria Capobianco, et.al."Some Remarks about the Hilbert Transform " Quest Journals Journal of Research in Applied Mathematics , vol. 05, no. 03, 2019, pp. 25-30.