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Research Paper

Unique Metro Domination of Square of Cycles

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Abstract: *A dominating set D of G which is also a resolving set of G is called a metro dominating set. A metro dominating set D ofa graph G(V,E) is a unique metro dominating set (in short an UMD-set) if|N(v)∩D|=1for each vertex $v \in V - D$andthe minimum cardinality of an UMD-set of G is the unique metro domination number of G denoted by $\boldsymbol{\gamma_{\mu\beta}(G)}$. In this paper, wedetermine unique metro domination number of $\boldsymbol{C_n}^{\mathbf{2}}$graphs.*
Keywords: *Domination, metric dimension, metro domination, unique metro domination*.

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I. INTRODUCTION

All the graphs considered in this paper are simple, connected and undirected. The length of a shortest path between two vertices u and v in a graph G is called the distance between u and v and is denoted by $d(u, v)$. For a vertex v of a graph, $N(v)$ denote the set of all vertices adjacent to v and is called open neighborhood of v. Similarly, the closed neighborhood of v is defined as $N[v] = N(v) \cap \{v\}$. Let $G(V, E)$ be a graph. For each ordered subset $S = \{v_1, v_2, v_3, \ldots, v_k\}$ of V, each vertex $v \in V$ can be associated with a vector of distances denoted by $\Gamma(v/S) = (d(v_1, v), d(v_2, v), \ldots d(v_k, v))$. The set S is said to be a resolving set of G, if$\Gamma(v/S) \neq \Gamma(u/S)$, for every $u, v \in V - S$. A resolving set of minimum cardinality is a *metric basis* and cardinality of a metric basis is the *metric dimension* of G. The k-tuple, $\Gamma(v/S)$ associated to the vertex $v \in V$ with respect to a metric basis S, is referred as a code generated by S for that vertex v. If $\Gamma(v/S) = (c_1, c_2, \ldots, c_k)$, then $c_1, c_2, c_3, \ldots, c_k$ are called components of the code of v generated by S and in particular c_i, $1 \leq i \leq k$, is called i^{th}-component of the code of v generated by S.

A dominating set D of a graph $G(V, E)$ is the subset of V having the property that for each vertex $v \in V - D$, there exists a vertex $u \in D$ such that uv is in E. A dominating set D of G which is also a resolving set of G is called a *metro dominating set*. A metro dominating set D of a graph $G(V, E)$ is a *unique metro dominating set* (in short an $UMD - set$) if $|N(v) \cap D|$ =1 for each vertex $v \in V - D$ and the minimum of cardinalities of UMD-sets of G is the *unique metro domination number* of G denoted by $\gamma_{\mu\beta}(G)$.

Consider $C_n, n \geq 4$ labelled as $v_1, v_2, \ldots, v_n$ in anticlockwise direction. Join v_i to v_{i+2} for $1 \leq i \leq n-2$, v_{n-1} to v_1 and $v_n$to v_2. The resulting graph is denoted by C_n^2.

Lemma 1*: For any positive integer n,* $\gamma_{\mu\beta}(C_n^2) \geq \left\lceil \frac{n}{5} \right\rceil$.

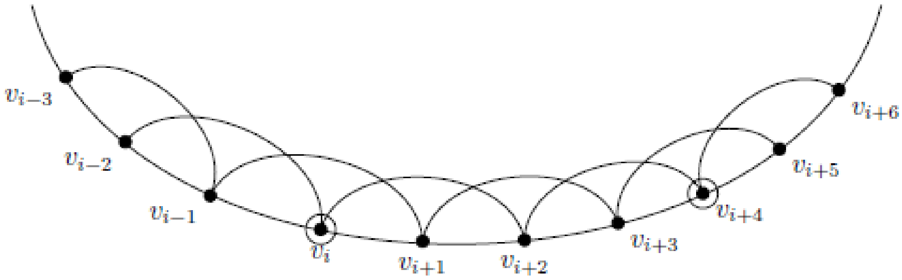
Proof: A vertex v_i dominates five vertices$v_i, v_{i-1}, v_{i-2}, v_{i+1}, v_{i+2}$.Therefore, if D is a minimal dominating set then $|D| \geq \frac{n}{5}$. Hence we have $\gamma(C_n^2) \geq \lceil \frac{n}{5} \rceil$.

Definition 1. [8] *Consider a set S with two or more vertices of the graph G and let v_i and v_j be twodistinct vertices of S. Further, let P and P' denote two distinct $v_i v_j$ -paths in G. If either P or P', say Pcontains only two vertices of S namely $v_i$and v_j, then we refer to v_i and v_j as neighboring vertices of S.Then the set of all the vertices of $P - \{v_i v_j\}$ is called a gap of S determined by v_i and v_j and denoted by$\eta_S(v_i, v_j)$if the following hold*
1.v_i, v_j are neighboring vertices in S and
2. No vertex in S is adjacent to any vertex in $P - \{v_i, v_j\}$.

Defnition 2. [8]*The minimum number of vertices in the gap is called the order of the gap, denoted by* $o(\eta_S(v_i, v_j))$

Lemma 2:*For* $n = 5k, \gamma(C_n^2) = \left\lceil \frac{n}{5} \right\rceil$.
Proof:Let $n = 5k$. Then $D = \{v_3, v_8, v_{13}, \ldots, v_{5k-2}\}$ is a minimal dominating set. Further $|D| = k = \frac{n}{5}$. D is a dominating set of minimum cardinality, From Lemma 1, $\gamma(C_n^2) = k = \frac{n}{5}$.
When $n = 5k + 1$, then by Lemma 1, $|D| > k$. If $v_i \in D$ then D contains $v_i, v_{i+5}, v_{i+10}, \ldots v_{i+n-6}$. The gap between v_{i+n-6} and v_i with respect to C_n is of order 5. Hence we include one more vertex into D. Any one of $v_{i+n-5}, v_{i+n-4}, v_{i+n-3}, v_{i+n-2}, v_{i+n-1}$ can be included in D. Thus $|D| = k + 1$ and $\frac{n}{5} < |D| < \frac{n}{5} + 1$.
Now consider the cases, $n = 5k + 2, 5k + 3, 5k + 4$. In these cases D contains $v_i, v_{i+5}, v_{i+10}, \ldots, v_{i+n-6}$ and the gap between v_{i+n-6} and v_i with respect to C_n is of order greater than 4. Therefore one more vertex to be included in D. Thus, $|D| = k + 1$ and $\frac{n}{5} < |D| < \frac{n}{5} + 1$.Hence we have $\gamma(C_n^2) = \left\lceil \frac{n}{5} \right\rceil$.

Lemma 3: *Let D be a unique dominating for* C_n^2. *If* v_i *and* $v_j \in D$ *have a gap of order less than 4 with respect to* C_n *then* $D = V$.
Proof: Let $v_i, v_j \in D,\ j > i$ and gap between v_i and v_j with respect to C_n is of order less than 4. Suppose the order of the gap is 3. Then v_{i+2} is dominated by v_i and $v_{i+4} = v_j$.Hence domination is not unique. We include v_{i+2} in D. But if $v_{i+2} \in D$ then v_{i+1} and v_{i+3} are not uniquely dominated. Hence we include v_{i+1} and v_{i+3} in D. Again v_{i-1} is dominated by v_i and v_{i+1} and likewise v_{i+5} is dominated by v_{i+4} and v_{i+3}.So we have to include v_{i-1} and v_{i+5} in D. Proceeding like this we conclude that all gaps will be reduced to 0 order. Hence D=V. Similarly if the gap is of order 2 or order 1, then D is not a UMD set and to make D an UMD set, we need to have D=V.



Lemma 4:*Let D be a unique dominating for* C_n^2, *then the gap with respect to* C_n *between any two vertices in D is of order less than 5. If* v_i *and* $v_j \in D$ *have a gap of order less than 4 with respect to* C_n *then* $D = V$.
Proof: Let $v_i, v_j \in D,\ j > i$. If the gap is of order 5, then $j = i + 6$. We observe that v_{i+3} is not dominated by any vertex in D . Hence D is not a dominating set, a contradiction . Similarly if gap is of order greater than 5, we get a contradiction.

If gap between v_i and v_j with respect to C_n is of order 4, then $j = i + 5$ and $v_{i+1}, v_{i+2}, v_{i+3}, v_{i+4}$ are uniquely dominated by v_i and v_j. Hence we have

Lemma 4: If all gaps are of order 4, then D is a unique dominating set .
Proof: Let $D = \{v_{5p+1} | p \in \mathbb{N}, 0 \le p < k\}$. Then D is a unique dominating set for $C_n^2, n = 5k, k \ge 3$. We observe that $d(v_1, v_j) = d(v_1, v_{j+1}) = \frac{j-i+1}{2} = d(v_1, v_{n-j+2}) = d(v_1, v_{n-j+1})$ where $j \in \mathbb{N}$ is even. However $d(v_6, v_{n-j+2}) = \left(\frac{6+j}{2}\right) - 1$ and $d(v_6, v_{n-j+1}) = \frac{6+j}{2}$. Hence $d(v_6, v_{n-j+2}) \ne d(v_6, v_{n-j+1})$. When $j > 6, d(v_6, v_j) = \left(\frac{j-6}{2}\right) + 1$. Hence $d(v_6, v_j) \ne d(v_6, v_{j+1})$. Therefore codes generated by $\{v_1, v_6\}$ are distinct to

all the vertices $v_j, 6 < j \leq n$. For v_2, v_3, v_4 and v_5 codes generated by $\{v_1, v_6, v_{11}\}$ are (1,2,5),(1,2,4),(2,1,4) and (2,1,3). Hence $\{v_1, v_6, v_{11}\}$ resolves all vertices of $C_n^2, n \geq 15$.

Therefore $D = \{v_{5p+1} | p \in \mathbb{N}, 0 \leq p < k\}$ is a UMD set and hence we have

Theorem 3:

$$\gamma_{\mu\beta}(C_n^2) = \begin{cases} \frac{n}{5}, when\ n = 5k, k \in \mathbb{N}, k \geq 3 \\ n, \qquad for\ all\ other\ n \end{cases}$$

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