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Research Paper

A Case Study on Outpatient Waiting Time for Treatment with Multiple Servers Queuing Model at Public Eye Hospital in Bangladesh

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ABSTRACT: *In public hospital outdoor patient services all over the world have become an important component of health care. Queuing is a major challenge for healthcare service particularly in developing countries, queuing theory helps decision making to improve waiting problem is not commonly used by managers in developing countries in contrast to their counterparts in the developed world. In this paper we introduce data analysis of observation to reduce outdoor patients waiting time without cost consideration at public hospital.*
KEYWORDS: *Operation research; Queuing Theory; Queue Model; Multiple Server Model; Public Hospital; BIOEH, Sensitivity Analysis, Ophthalmology Department(OD).*

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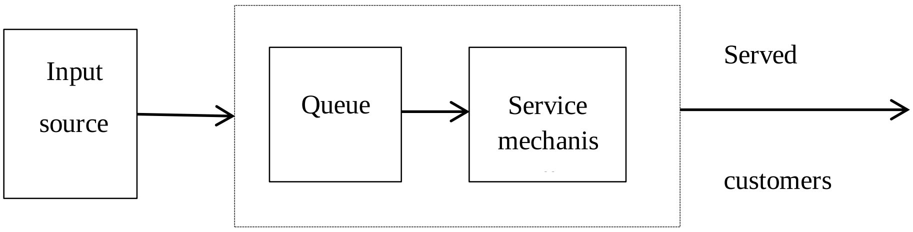
I. INTRODUCTION

Operation research is a rational method of problem-solving and decision making and Queuing theory is a section of operations research in mathematics that models the function of waiting in lines. Queuing theory, also known as the theory of congestion, is a section of operation research that research the familiarity between the want on a service system and the delays bear by the purchaser of the system. The Impetus for the progress of queuing theory was the burgeoning telephone industry in the early 1900s and the history of Queue goes back to primitive man. Below Isan earlier queue which is described in the Bible.A.K. (AgnerKarup) Erlang published his first paper on Queuing theory in 1909. The study of queues deals with quantifying the phenomenon of waiting in lines using ambassador step of performance, such as average queue length, average waiting time in queue, and average facility utilization. Queue model is structured so that queue lengths and wait times can forecast. A queuing system is a generic model that comprises three elements: a user source, a queue and a service facility that contains one and more (possibly an infinite number of) identical servers in parallel. Each user of the queuing system passes through the queue where he may remain for a period of time (positive, possibly zero) and then is processed by a single server because of the parallel arrangement of the servers. Once a purchaser has left the server, after obtaining the service, the purchaser is considered to have left the queuing system as well. We all wait in queues to buy a movie ticket, make bank deposit pay for groceries, mail a package, obtain food in a cafeteria, in hospitals, in the supermarket, etc.

II. BASIC STRUCTURE OF QUEUING MODEL

The basic process assumed by the most queuing models is the following, customers requiring service are generated over time by an input source. These customers enter the queuing system and join a queue. At certain times, a member of the queue is selected for service by some rule known as the queue discipline. The required service is then performed for the customer by the service mechanics, after which the customer leaves the queuing system. This process is depicted in the following Figure1Many alternative assumptions can be made about the various elements of the queuing process.

Figure1: The basic queuing process



III. APPLICATION OF QUEUING MODEL TO PATIENT FLOW BASED ON MULTI SERVER MODEL

This is common for health care managers to project workload for physical infrastructure and manpower planning. Some information such arrival rate, service rate, number of server and probability utilization can be taken from this application.

Data Collection and analysis in BIOEH: We considered the Outdoor department of BIOEH in Cumilla. This department is one of the modern units in Bangladesh and consists of several major areas: triage, resuscitation room, immediate care unit, space for minor emergencies, room for minor surgeries, critical observer. The data used in the case study includes detailed information's over the April (1− 30),2019.

Service process:
1st step: patient arrivals.
2nd step: patient is in waiting room.
3rd step: Registration
4th step: Patient in waiting line
5th step: Doctor's room.
6th step: If patient is in good condition then exit otherwise again consulting room. Building on the patient flow we will use the queuing theory to estimate the average waiting time. The results are important for the management of the outdoor department in order to make optimal decisions, to organize in optimal way the work flow.
Here we use first seven days' data. Average number of patients 279per day divided by 6.5 hours per day arrival rate resulting 43patients per hour. We obtained λ = 43, thus we will consider in the following that the patients arrive at a rate of 43/hour and stay in a single queue. Using the created database, using same rationality as above, the calculated annual average service rate is: μ = 45 patients per hour. It is known that the system is in steady state if the relation is fulfilled:

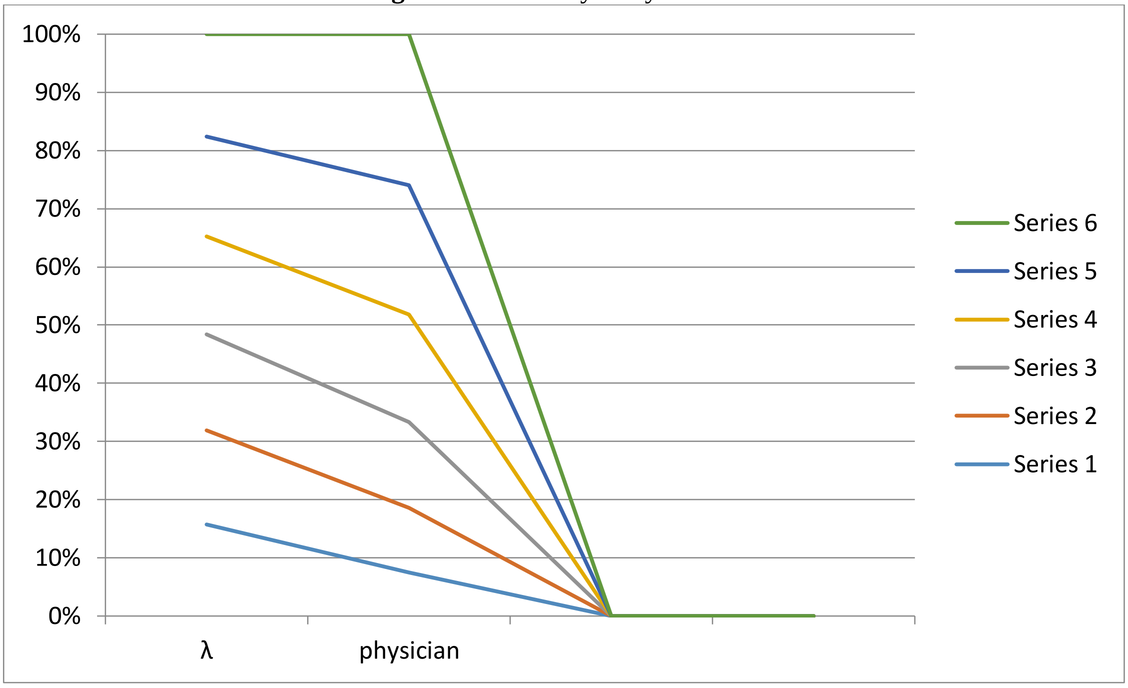
$$\frac{\lambda}{n\mu} < 1$$

Where *n* presents the number of human resources, the number of physician in the outdoor department. In the case, the number of arrivals per hour λ will determine the minimum number of physicians' *n* as can be seen in Table1:

Table 1: Minimum number of physician

λ	45	46	47	48	49	50
n calculated	2	3	4	5	6	7

Figure 2: Sensitivity analysis



We use M / M / 2 queuing model to estimate different specifications of the queue, different characteristics of Eye section. In our case study we have: $\lambda = 43$; $n = 2$; $\mu = 45$; $\alpha = 43/45$

In the case of M / M / 2 model to calculate the probability, we use the condition, that the overall sum of probabilities must be 1.

We can write:

$$p_0 + p_1 + (p_2 + p_3 + \cdots)=1$$

We substitute the probabilities, using n=2 and we get:

$$p_0 + p_0\frac{\lambda}{\mu} + p_0((\frac{\lambda}{\mu})^2\frac{1}{2!2^0}+(\frac{\lambda}{\mu})^3\frac{1}{2!2}\ldots) = 1$$

$$p_0 + p_0\frac{\lambda}{\mu} + p_0(\frac{\lambda}{\mu})^2\frac{1}{2!}+p_0(\frac{\lambda}{\mu})^3\frac{1}{3.2^2}\ldots) = 1$$

The geometric series is convergent and introducing the sum of the series we have,

$$p_0 + p_0\frac{\lambda}{\mu} + p_0(\frac{\lambda}{\mu})^2\frac{1}{2!} + p_0(\frac{\lambda}{\mu})^3\frac{1}{2^2} =1$$

The probability that no patient is in the OD is:

$$p_0 = \frac{1}{1+\frac{\lambda}{\mu}+(\frac{\lambda}{\mu})^2\frac{1}{2!}\ldots} = 25$$

In the case of outdoor department, the probability that no patient in the outdoor department, no queue is:

$$p_0 = 25$$

Calculating the sum of the series we obtain:

$$L_q = p_0\frac{1}{2!}(\frac{\lambda}{\mu})^2$$

$= 11.52$

Using the little's law, the mean waiting time in the queue can be obtained from:

$$W_q = \frac{L_q}{\lambda}$$

$= 0.27$

$= 16.2$ minute

Average treatment time,

$$W_t = \frac{1}{\mu}$$

$= 0.22$ hour

$= 13.2$ minute

Thus the total waiting time in the system,

$$W = W_q + W_t$$

$= 29.4$ minute

The overall number of patients in the OD is on average per week L = W λ
= 1264.2

Data collection and analysis in Comilla Medical College Hospital Ophthalmology Department:

Ophthalmology department is one of the most important department in any hospital .The outdoor access of the patients is realized through OD which belongs to the hospitals .The OD is designed to continue the treatment begun in the street ,at the accident site ,or to begin the treatment if the patient has arrived with a non-medical vehicle ,to diagnose the patient in cooperation with the specialists of the hospital and to observe the patient until the hospital admission or until his release ,in case the patient's condition does not require hospital treatment .For this study we considered the OD of Comilla Medical College Ophthalmology Department .The data used in the case study includes detailed information's over the May (1 − 30),2019.

Service process:

At first patient personnel interaction occurs in the triage area, where a triage doctor evaluates the patient, determines the seriousness of the patient's health condition and assigns corresponding department. The assumption that service is provided on a first-come, first-served basis in the most commonly encountered rule. The ED does not serve on the basis, patient do not all represent the same risk, level of triage; those with the highest risk, the most seriously ill, are treated first. Building on the patient flow we will use the queuing theory to estimate the average waiting time. The results are important for the management of the Emergency department in order to make optimal decisions, to organize in optimal way the work flow. Here we use first seven days' data. Average number of patients 20 per day divided by 5.5 hours per day arrival rate resulting 4 patients per hour. We obtained λ = 4, thus we will consider in the following that the patients arrive at a rate of 4/hour and stay in a single queue. Using the created database, using same rationality as above, the calculated annual average service rate is: μ = 4 patients per hour. It is known that the system is in steady state if the relation is fulfilled:

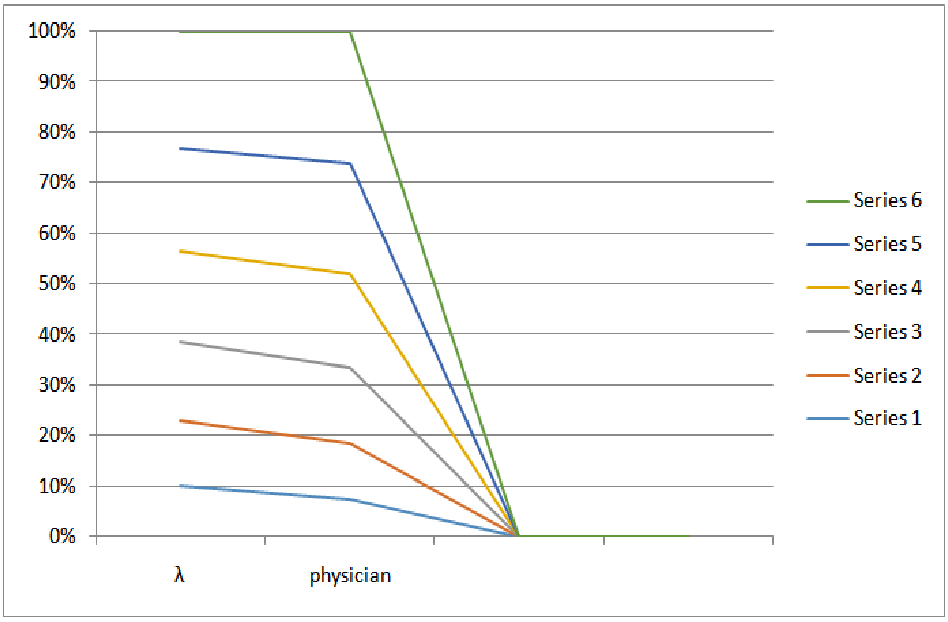
$$\frac{\lambda}{n\mu} < 1$$

Where n presents the number of human resources, the number of physician in the outdoor department. In the case, the number of arrivals per hour λ will determine the minimum number of physicians' n as can be seen in table 2.

Table 2: Minimum number of physician

λ	4	5	6	7	8	9
n calculated	2	3	4	5	6	7

Figure 3: Sensitivity analysis



We use M / M / 2 queuing model to estimate different specifications of the queue, different characteristics of Eye section. In our case study we have: λ = 4; n = 2; μ = 4; α =4/4

In the case of M / M / 2 model to calculate the probability, we use the condition, that the overall sum of probabilities must be 1. we can write:

$p_0 + p_1 + (p_2 + p_3 + \cdots)=1$

We substitute the probabilities, using n=2 and we get:

$p_0 + p_0 \frac{\lambda}{\mu} + p_0((\frac{\lambda}{\mu})^2 \frac{1}{2!2^0}+(\frac{\lambda}{\mu})^3 \frac{1}{2!2}\ldots) = 1$

$p_0 + p_0 \frac{\lambda}{\mu} + p_0(\frac{\lambda}{\mu})^2 \frac{1}{2!}+p_0(\frac{\lambda}{\mu})^3 \frac{1}{3.2^2}\ldots) = 1$

The geometric series is convergent and introducing the sum of the series we have:

$p_0 + p_0 \frac{\lambda}{\mu} + p_0(\frac{\lambda}{\mu})^2 \frac{1}{2!} + p_0(\frac{\lambda}{\mu})^3 \frac{1}{2^2} =1$

The probability that no patient is in the OD is:

$p_0 = \frac{1}{1+\frac{\lambda}{\mu}+(\frac{\lambda}{\mu})^2\frac{1}{2!}\ldots} = 0.4$

In the case of outdoor department, the probability that no patient in the outdoor department, no queue is:

$p_0 = 0.4$

Calculating the sum of the series we obtain:

$L_q = p_0 \frac{1}{2!}(\frac{\lambda}{\mu})^2$

$= 0.2$

Using the little's law, the mean waiting time in the queue can be obtained from:

$W_q = \frac{L_q}{\lambda}$

= 0.05 hour

= 3 minute

Average treatment time,

$W_t = \frac{1}{\mu}$

= 0.25 hour

= 15 minute

Thus the total waiting time in the system,

W = $W_q + W_t$

= 18 minute

The overall number of patients in the OD is on average per week L = W λ

= 72

IV. FINDINGS

According to this calculation, (i) In BIOEH, the average waiting time of each patient in this system is 29.4 minute and the average number of patients 1264.2. The average number of patients or the patient flow of this hospital is very high but the number of specialists of this hospital which is not sufficient. So this hospital must be increase the number of specialists. (ii) In Comilla Medical College Hospital Ophthalmology department, the average number of patients is very low but huge number of specialists. Finally, it has to be said that, to increase the patient flow of this hospital this hospital could be digitalized.

V. CONCLUSION

This paper introduced that patients are generally dissatisfied with long waiting times and experience a negative effect as a result. It is further established that Queuing theory and modeling is an effective tool that can be used to make decisions on physician and staff needs for optimal performance of the hospital. This research provides suggestions to the hospital to construct the appointment system, take attention to patient flow and make scheduling of the retention to increase the effective and efficient outdoor department performance. This is a primary study that detached each variable separately. This analysis performed to ensure that the waiting time targets not met the minimum service standard of the hospital without consideration of cost. For further research, anyone extends this work with huge data and cost-effective to reduce outdoor patient waiting time.

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