



Research Paper

The covid-19 pandemic crisis on the volatility of the S&P 500: An application of the Markov Switching Autoregressive model

Carlos Alberto Gonçalves da Silva

Visiting Researcher at the Postgraduate Program in Economics Sciences (PPGCE) - State University of Rio de Janeiro (UERJ), Rio de Janeiro, Brazil.

Scholarship from the Research Support Foundation of the State of Rio de Janeiro (FAPERJ), Brazil.

Abstract: This article makes an econometric analysis using the Markov Switching Autoregressive (MS-AR) model, with the objective of showing the dynamics presented by the main US stock market index, the S&P 500, because this index measures the performance of most capitalized companies in the United States market. The analysis covers the period from January 2005 to November 2020, when the subprime crisis occurred and the COVID-19 crisis began. In particular, two regimes (regime 1-low volatility and regime 2-high volatility) were used in the model so that the parameters of the S&P 500 index behave differently during economic crises with the representative regimes. The S&P 500 remained in regime 1 (low volatility) for five periods, totaling 110 months. In regime 2 (high volatility - 2008 and 2020 crises), it remained for about 50 months, that is, 39 months in the 2008 crisis (including the global financial crisis-2009) and 11 months in the COVID-19 crisis. In addition, regime 1 is more persistent, that is, the probability of staying in that regime in a later period is 93,61% and a change to regime 2 of 6,39%. In regime 2, the probability of maintaining this regime in the period $t + 1$ is 92,52%, while the probability of changing to regime 1 is 7,42%.

Keywords: Markov Switching Autoregressive model, Covid-19 Pandemic, Probability of transition

Received 29 Mar, 2021; Revised: 10 Apr, 2021; Accepted 12 Apr, 2021 © The author(s) 2021.

Published with open access at www.questjournals.org

I. INTRODUCTION

The econometric works on the estimation of regressions subject to regime changes that follow a Markov chain were developed by Quandt (1972), Goldfeld and Quandt (1973). Hamilton (1989) made important advances in the method developed by Goldfeld and Quandt (1973), by specifying that changes in regimes follow an auto-regressive process. In this sense, he developed a non-linear and smoothed estimation algorithm to find the high and low regimes of the economic series, seeking to maximize the likelihood function in relation to the parameters estimated in the model. This methodology allowed statistical inferences to be made about the different regimes not observed in the series. The model endogenously estimates the dates of the structural changes in the series. Hamilton (1989) applied the method to investigate the nonlinear behavior of the growth of the United States economy and the results showed that the model can be used as an important tool for measuring business cycles.

Hamilton and Susmel (1994) use a model with changes, with respect to volatility. According to the authors, the regime change model, applied to the returns of the American stock market, fits the data better than the ARCH models without regime change.

Ang and Bekaert (2002) applied using a non-linear model to interest rates in the USA, Germany and the United Kingdom. Thus, the authors showed that interest rate regimes correspond reasonably well with US economic cycles, being extremely important to study the effects of monetary policy shocks on the economy.

Ismail and Isa (2006) used regime change testing in their study to detect non-linear characteristics in the exchange rates of three Asian countries. They found that the null hypothesis of linearity is rejected and there is evidence of structural breaks in the exchange rate series.

Júnior and Zuanazzi (2014) tested the hypothesis of non-linearity of the sensitivity of the return on assets of companies from Rio Grande do Sul under different Markovian risk regimes: periods of crisis and stability. They considered three assets of Rio Grande do Sul companies tradable on the São Paulo Stock Exchange (Bovespa). The results showed that the non-linear model (MS-CAPM) is the most suitable. In

addition, evidence that assets are more susceptible to macroeconomic changes in times of crisis than in periods of stability.

Mahjoub and Chaskmi (2019) applied the Markov Switching model with two regimes, to identify periods of speculative bubble formation and explosion in the Iranian capital market. Regimen 1 is bubble growth and the explosion stage and regime 2 identifies bubble loss. The result of the research shows that the stock index of the Iranian capital market in the analyzed period

Panda et al. (2017) examine the changing behavior of the dynamic Markov regime between the spot and the futures market in relation to interest rates in India. The study uses daily data on volumes, weighted average price, weighted average yield for the spot market and total values, open interest, settlement price from January 21, 2014 to October 30, 2014. All data come from Clearing Corporation of India Ltd. (CCIL) and the National Stock Exchange (NSE). The authors used regime change regression to capture the behavior of changes, as well as the estimated probability and estimated duration of each regime.

Peira and Soledad (2002) implemented a regime change framework to study speculative attacks against EMS currencies during 1979–1993. To identify speculative episodes, we model exchange rates, reserves and interest rates as time series subject to discrete regime changes between two possible states: "quiet" and "speculative". We allow the odds of switching between states to be a function of fundamentals and expectations. The regime change framework improves the ability to identify speculative attacks vis-à-vis the speculative pressure indices used in the literature. The results also indicate that fundamentals (mainly budget deficits) and expectations drive the likelihood of moving to a speculative state.

Ozdemir (2020) in his study is to assess the feed price driven dynamics of the U.S. wholesale beef prices in which regime switches are induced by transitions between Markov regimes. By allowing the transition probabilities to vary according to some main grain feed prices, we examine if the regime transition probabilities vary over time under two different states of the growth rate of beef prices as "low-mean growth" and "high-mean growth" price regimes. The results show that when the prices are in high-mean growth regime, the probability that it will remain in this regime is greater than that it will switch to low-mean regime. This findings also indicate that livestock feed prices provides some predicted power to the model of beef price regime switching process and supports livestock feed prices contributing to whether the beef price levels remains in low/high-mean regime. By employing Markov switching dynamic regression model, we also find that all types of the feed prices have a significant effect on the beef prices in low-growth regime, but only the prices of hay and sorghum significantly affect the beef prices in the high-growth regime.

Xaba et al. (2019) used a Markov-switching dynamic regression (MS-DR) model to estimate appropriate models for BRICS countries. The preliminary analysis was done using data from 01/1997 to 01/2017 and to study the movement of 5 stock market returns series. The study further determined if stock market returns exhibit nonlinear relationship or not. The purpose of the study is to measure the switch in returns between two regimes for the five stock market returns, and, secondly, to measure the duration of each regime for all the stock market returns under examination. The results proved the MS-DR model to be useful, with the best fit, to evaluate the characteristics of BRICS countries.

Choi and Hammoudeh (2010) use the Markov Switching model with two volatility regimes for the strategic commodity prices of Brent oil, WTI oil, copper, gold and silver, and the S&P 500 index, but with varying high-to-low volatility ratios. The dynamic conditional correlations (DCCs) indicate increasing correlations among all the commodities since the 2003 Iraq war but decreasing correlations with the S&P 500 index. The commodities also show different volatility persistence responses to financial and geopolitical crises, while the S&P 500 index responds to both financial and geopolitical crises.

Moolman (2004) found that Linear models are incapable of capturing business cycle asymmetries. This has recently spurred interest in non-linear models such as the Markov switching regime (MS) technique of modelling business cycles. The MS model can distinguish business cycle recession and expansion phases, and is sufficiently flexible to allow different relationships to apply over these phases. In this study, the South African business cycle is modelled using a MS model. This technique can be used to simultaneously estimate the data generating process of real GDP growth and classify each observation into one of two regimes (i.e. low-growth and high-growth regimes).

Several authors developed works using Markov Switching models, including Kim (1994), Norden and Schaller (1995), Oliveira and Pereira (2018), Krolzig and Toro (2004), Assoe (1998), Safaei and Mostafaei (2012), Norden and Schaller (1993) and Diebold et al. (1994).

This article analyzes the impacts of the two crises on the stock markets (2008 and Covid-19), highlighting the S&P 500 stock market index using the Markov Switching Autoregressive model.

II. METHODOLOGY AND DATA

Markov Switching Autoregressive Model

Hamilton (1989) proposed MS that is based on the assumption that the development of X_t can be explained by states (or regimes), where a two regime Markov-switching regression model can be expressed as:

$$\text{Regime 1: } Y_t = \mu_1 + \phi Y_{t-1} + \varepsilon_t$$

$$\text{Regime 2: } Y_t = \mu_2 + \phi Y_{t-1} + \varepsilon_t$$

where Y_t is the dependent variable,

μ_1 and μ_2 are the intercepts in each state (regime),

ϕ is the autoregressive coefficient and ε_t is the error at time t.

In the case where the state (regime) shifts are known, the two regime Markov-switching model can be expressed as:

$$Y_t = S_t \mu_1 + (1 - S_t) \mu_2 + \phi Y_{t-1} + \varepsilon_t$$

where S_t represents the regime and is equal to 1 if the process is in regime 1 and 2 if it is in regime 2.

However, in most cases it is not possible to observe in which regime S_t the process is currently in and therefore unknown. In Markov-switching regression models the regime S_t follows a Markov chain. A model with k regime-dependent intercepts, can be expressed as:

$$Y_t = S_t \mu_{s_t} + \phi Y_{t-1} + \varepsilon_t$$

Where $\mu_{s_t} = \mu_1, \mu_2, \dots, \mu_k$ for $S_t = 1, 2, \dots, k$ regimes.

The transition of probabilities between the regimes is carried out by a first order Markov process as follows:

$$\rho_{ij} = \Pr(S_t = j | S_{t-1} = i)$$

On what ρ_{ij} refers to the probability of being on the regime j given that the process is in the regime

i , where $\sum_{i=1}^N \rho_{ij} = 1$ for all $i, j \in (1, 2, \dots, N)$.

The transition probabilities in a square matrix of order N , known as the transition matrix and denoted by P , have the following form:

$$P = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}$$

where

$$\rho_{11} = P[s_t = 1; s_{t+1} = 1]$$

$$\rho_{12} = P[s_t = 2; s_{t+1} = 1]$$

$$\rho_{21} = P[s_t = 1; s_{t+1} = 2]$$

$$\rho_{22} = P[s_t = 2; s_{t+1} = 2]$$

$$\rho_{11} + \rho_{12} = 1 \quad \text{e} \quad \rho_{21} + \rho_{22} = 1$$

Thus, it is assumed that the transition matrix is irreducible and unconditional (if one of the values of the transition matrix is equal to the unit and all other eigenvalues are within the unit circle). With these conditions, there is a stationary probability distribution of the regimes (Krolzig, 1997). Unconditional probabilities can be determined as follows:

$$\rho_1 = (1 - \rho_{11}) / (2 - \rho_{11} - \rho_{22})$$

$$\rho_2 = (1 - \rho_{22}) / (2 - \rho_{11} - \rho_{22})$$

The probability of being in regime 1 in equilibrium is obtained by ρ_1 and the probability of being in regime 2 is determined by ρ_2 .

In estimating the model, the joint distribution y_t and S_t relative to past information is used:

$$f(y_t, S_t | Y_{t-1}) = f(y_t | S_t, Y_{t-1}) f(S_t | Y_{t-1})$$

Where Y_{t-1} represents all information included in the history of the time dependent variable $t-1$ e

$f(y_t | S_t, Y_{t-1})$ is the conditional normal density function for the regime $S_t = j$.

The maximum likelihood estimator is used to determine the parameters of the MS-DR. Therefore, the probability function of the model log with two regimes is expressed as follows:

$$\ln L = \sum_{t=1}^T \ln \left\{ \sum_{j=1}^2 f(y_t | S_t, Y_{t-1}) \Pr(S_t = j | Y_{t-1}) \right\}$$

Where the term $\Pr(S_t = j | Y_{t-1})$ is the probability of being in each regime. Given away $\Pr(S_{t-1} = i | Y_{t-1})$, $i = 1, 2$ at the beginning of time t, the probabilities of being in each regime are obtained as follows:

$$\Pr(S_t = j | Y_{t-1}) = \sum_{i=1}^2 \Pr(S_t = j | S_{t-1} = i) \Pr(S_{t-1} = i | Y_{t-1}),$$

where $\Pr(S_t = j | S_{t-1} = i)$, $j = 1, 2$; $i = 1, 2$ are transition probabilities of the elements of matrix P, considered constant. The probability of being in one regime or another, can be changed through macroeconomic performance and information obtained from the stock market.

Being Y_t observed at the end of the period of period t, the probabilities are updated using the following equation:

$$\Pr(S_t = j | Y_t) = \frac{f(y_t | S_t = j, Y_{t-1}) \Pr(S_t = j | Y_{t-1})}{\sum_{j=1}^2 f(y_t | S_t = j, Y_{t-1}) \Pr(S_t = j | Y_{t-1})}$$

where $f(y_t | S_t = j, Y_{t-1})$ s the probability density function of a distribution for the regime $S_t = j$.

In the view of Doornik (2013) the Markov-switching models can be MS-AR (Markov-switching autoregressive) and MS-DR (Markov-switching dynamic regression). The first is characterized by a more gradual adjustment, appropriate to the most stable series, whose autoregressive component is formed by the difference between the lagged endogenous variable and the average estimated for the endogenous variable in the S_{t-1} regime; and the second adjusts immediately to the new regime, with a more accentuated transition, since the autoregressive component covers only the endogenous variable.

In the present article, the series data are monthly, which chose to use the MS-AR model as an estimation method to identify regime changes, the number of periods, the duration and the probability of transition from one regime to another. The model is inadequate with a high order of autoregressors.

The MS-AR model can be specified as:

$$y_t - \mu(S_t) = \rho [y_{t-1} - \mu(S_{t-1})] + \varepsilon_t, \varepsilon_t \sim \text{IIN} [0, \sigma^2]$$

Finally, from the transition matrix it determines the expected duration of each regime. The closer the probability is to one, the longer it takes to switch from another regime. Thus the expected duration can be expressed as:

$$\text{Expected duration } (D_i) = \frac{1}{1 - \rho_{ij}}$$

The duration time in each of the two regimes can be determined as:

$$D_1 = 1/(1 - \rho_{11}) \qquad D_2 = 1/(1 - \rho_{22})$$

Linearity Test (BDS)

Once it is detected that the distribution is not normal, it is necessary to test the model for linearity. This test was developed by Brock, Dechert, and Scheinkman (1987) used to test if the random variables that compose a series are independent and identically distributed (IID), that is, it can verify several situations in which the variables are not IID, such as non-stationarity, nonlinearity and deterministic chaos. The test is based on the concept of spatial correlation of chaos theory and according to the authors the BDS statistic is formulated through the Equation:

$$W_m^n(\varepsilon) = \frac{\sqrt{N} (C_m^n(\varepsilon) - (C_1^n(\varepsilon))^m)}{\sigma_m(\varepsilon)}$$

Where $W_m^n(\varepsilon)$ it converges to a normal distribution $N(0, 1)$ as n tends to infinity.

Thus, hypothesis tests are:

H_0 : the series follows an iid (independent and identically distributed) process.

H_1 : the series does not follow an iid (independent and identically distributed) process.

Data

The data used in this study refer to the monthly S&P500 index, covering the period from January 2005 to November 2020, in a total of 191 monthly observations. The data were obtained from the Yahoo finance website.

III. EMPIRICAL RESULTS

Preliminary Analysis

The daily returns were calculated using the formula: $r_t = \ln(P_t) - \ln(P_{t-1})$. This P_t represents the number of points at closing on day t and P_{t-1} the number of points at closing on the previous day (t-1). Figures 1 and 2 show the behavior of the S&P 500 series of quotations and monthly returns in the period considered.

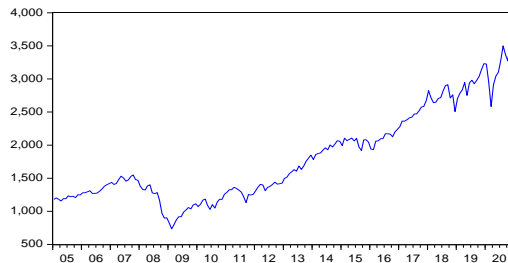


Figure 1. Monthly quotes for the S&P 500 index

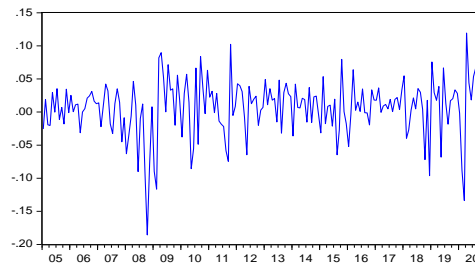


Figure 2. Monthly returns for the S&P 500 index

In the visual inspection of Figure 2 the analysis period, there is a marked volatility in returns. Thus, it was necessary to test the normality and stationarity of the series of returns from the S&P 500 market for the application of the MS-AR model.

Some basic descriptive statistics are presented in Table 1. It can be observed that the monthly returns of the S&P 500 present a leptocurtic distribution due to the excess of kurtosis (5,2790) in relation to the normal distribution (3.0), that is, it has heavier tail. It is also verified that the series is negatively asymmetrical which would indicate that stock market lows are more likely than market highs. The analysis of the results shows that both the mean (0.0057) and the median (0.0115) presented values close to zero. The variation between the minimum value (-0,1856) and the maximum value (0.1194) shown by the series can be explained due to some significant oscillations in the index returns. The low value of the standard deviation (0.0431) indicates that, in general, the high variations in the series occurred in a few occasions, that is, in periods of positive and negative peaks. The statistics of Jarque - Bera indicated the rejection of the normality of the distribution of the series, with p-value equal to zero.

Table 1. Statistical summary of S&P 500 returns

| Statistics | S&P 500 |
|--------------------|----------|
| Mean | 0,0057 |
| Median | 0,0115 |
| Maximun | 0,1194 |
| Minimum | -0,1856 |
| Standard Deviation | 0,0431 |
| Asymmetry | -0,8650 |
| Kurtosis | 5,2790 |
| Jarque-Bera | 65.1533 |
| p-value (prob.) | (0,0000) |
| Observations | 191 |

The Dickey-Fuller (1981), Phillips-Perron (1988) and Kwiatkowski, Phillips, Schmidt, Shin (1992) tests with constant and trend, identified that the S&P 500 series of returns are stationary and do not contain unit roots, as presented in Table 2.

Table 2. Stationary test for the S&P 500 returns series.

| Variable | ADF | Critical value (5%) | PP | Critical value (5%) | KPSS | Critical value (5%) |
|----------|----------|---------------------|----------|---------------------|--------|---------------------|
| S&P 500 | -12,4474 | -3,4336 | -12,4456 | -3,4336 | 0,0433 | 0,1460 |

Before the estimation of the Markov Switching Autoregressive model (MS-AR), a nonlinearity test may be necessary to describe the characteristics of the historical series of the returns S&P 500 index. Thus, in Table 3 it shows that the results presented indicate the effects of nonlinearity, that is, the probability is less than 5% at the level of significance, implying a rejection of the null hypothesis that the series of returns of the stock is linearly dependents.

Table-3. Test to the time independence of S&P 500

| Dimension | BDS Statistics | Statistics Z | Probability |
|-----------|----------------|--------------|-------------|
| 2 | 0,0349 | 5,0631 | 0,0000 |
| 3 | 0,0680 | 6,1750 | 0,0000 |
| 4 | 0,0883 | 6,7018 | 0,0000 |
| 5 | 0,0983 | 7,1279 | 0,0000 |
| 6 | 0,1020 | 7,6314 | 0,0000 |

Source: Prepared by the author based on the research.

In the process of modeling and choosing autoregressive models-AR (p), should test different models and check the choice of the most suitable based on the lowest values of the Akaike (AIC) and Schwarz (SIC) information criteria, as well as the Durbin-Watson (DW) statistic that also detects the presence of autocorrelation in the residues of a regression analysis, that is, when DW is approximately equal to 2.0, it indicates that there is no autocorrelation in the residues.

Thus, after several simulations and with the criteria mentioned above, the best estimate was the AR (1), for the S&P 500 stock market, due to the lower values observed for the AIC and SIC criteria, as well as the coefficient of Durbin-Watson (DW) closest to 2 (table 4).

Table 4 - Selection of the order of the autoregressive component AR (p)

| Autoregressive Model | AIC | SIC |
|----------------------|---------|---------|
| AR(1)* | -3,4254 | -3,3914 |
| AR(2) | -3,4190 | -3,3850 |
| AR(3) | -3,4228 | -3,3887 |

Source: Prepared by the author based on the research.

(*) best adjusted model

Markov Switching Autoregressive model (MS-AR)

Table 4 shows the model estimates using the maximum likelihood method, using the OxMetrics 6.0 software (PcGive14). The adjusted model refers to the MS (2)-AR(1), the mean and variance change according to the regime. All parameters obtained are significant. The regime (1) expresses a positive average of the S&P 500 returns together with a low volatility. In regime (2), it shows a negative average result and high volatility in S&P500 returns. In regime 1, the estimated average monthly return is 1,169% with a variance of 0,02154. The regime 2 identifies a negative average monthly return of -0,195% with a variance of 0,05923.

Portmanteau indicate that there is no presence of autocorrelation of residues. The results of the ARCH-LM tests suggest the acceptance of the model homoscedasticity hypothesis. As for the normality tests Jarque-Bera does not reject the hypothesis of normality. Thus, the model presents a positive diagnosis and an adequate adjustment demonstrated in the results of the various tests carried out in the present study.

In the transition and persistence matrix of the regimes, it appears that the current regime 1 is more persistent, that is, the probability of remaining in this regime in a later period is approximately 93,61%, and that of changing to regime 2 is on the order of 6,39%. In regime 2 the probability of continuing in this regime in the period $t + 1$ is 92,52%, while the probability of switching to regime 1 is 7,48%. Thus, for the period from January 2005 to November 2020, the expected duration of the current regime 1 is 22 months. In regime 2, the estimated duration is 16 months. The unconditional probability in periods of low volatility is 57,89% and 42,11% in periods of high volatility.

Table 4. Estimation of the MS(2)-AR(1) model.

| Regime 1 (low volatility) | | | Regime 2 (high volatility) | | |
|--------------------------------|-------------|-------------------------|----------------------------|-------------|-------------|
| Parameter | Coefficient | | Parameter | Coefficient | |
| AR(1) | -0,20310 | (0,1166)*** | AR(1) | 0,10550 | (0,0564)*** |
| $\mu (s_1)$ | 0,01169 | (0,0021)* | $\mu (s_2)$ | -0,00195 | (0,0008)** |
| σ^2 | 0,02154 | (0,0022)* | σ^2 | 0,05923 | (0,0062)* |
| ρ_{11} | 0,93610 | (0,0375)* | ρ_{12} | 0,92520 | (0,0641)* |
| Descriptive statistics | | | | | |
| Log-likelihood | 356.3210 | | | | |
| Linearity test (χ^2)(4) | 55,4920 | (0,0000) ⁽¹⁾ | | | |
| Normality test (χ^2) | 3,9054 | (0,1419) ⁽¹⁾ | | | |

| | | |
|---|---------------------------|-------------------------|
| ARCH test (1-1) | 2,0808 | (0,1279) ⁽¹⁾ |
| Pormanteau test - χ^2 (36 lags) | 32,7610 | (0,6235) ⁽¹⁾ |
| Transition probability matrix | | |
| | Regime 1,t | Regime 2,t |
| Regime 1, t+1 | 0,9361 | 0,0748 |
| Regime 2, t+1 | 0,0639 | 0,9252 |
| Average duration period of regimes | | |
| | Unconditional probability | Duration period |
| Regime(1) | 0,5789 | 22 |
| Regime(2) | 0,4211 | 16 |

Notes: *, **, *** denote statistical significance at the 1%, 5%, 10%, respectively.

Standard errors are in parentheses.

p value (1).

Source: Prepared by the author based on the research.

Figure 4 shows the behavior of the series of indices, returns, smoothed and predicted probabilities for the S&P 500 state 1 and 2 regimes. The upper panel presents the series of S&P 500 returns, and the middle and lower panels trace the smoothed probabilities for the market in regime 1 (low volatility) and regime 2 (high volatility), respectively.

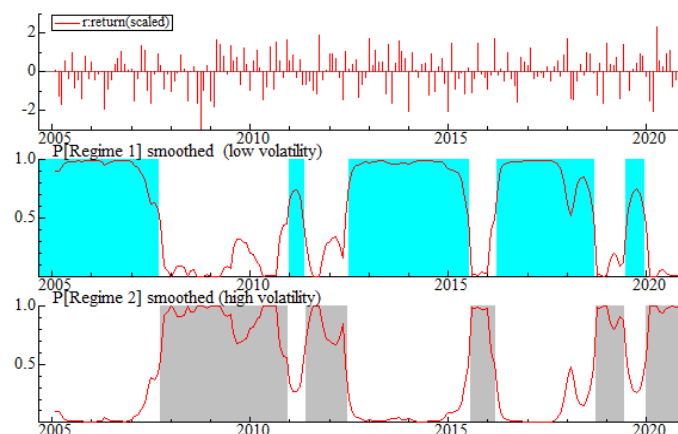


Figure 4. Smoothed probabilities of regimes 1 and 2 obtained from the MS(2)-AR(1) model for S&P 500 returns in the period from January 2005 to November 2020.

From the estimated probabilities, the specific dates of the low volatility (1) and high volatility (2) regimes can be obtained, shown in Table 5. The S&P 500 remained under the low volatility regime for five periods, totaling 110 months. In the regime of high volatility (crises of 2008 and 2020), the S&P 500 remained for about 50 months, that is, 39 months in the crisis of 2008 and 11 months in the crisis covid-19 of 2020 (January to November of 2020).

Table 5 - Specific dates of the regimes: MS(2)-AR(1) model

| Regime 1 (low volatility) | | | Regime 2 (high volatility) | | |
|---------------------------|--------|-------------|----------------------------|--------|-------------|
| Period | Months | Probability | Period | Months | Probability |
| 2005(2) - 2007(9) | 32 | 0,925 | 2007(10) - 2010(12) | 39 | 0,885 |
| 2011(1) - 2011(5) | 5 | 0,670 | 2011(6) - 2012(6) | 13 | 0,792 |
| 2012(7) - 2015(7) | 37 | 0,940 | 2015(8) - 2016(3) | 8 | 0,875 |
| 2016(4) - 2018(9) | 30 | 0,883 | 2018(10) - 2019(6) | 9 | 0,916 |
| 2019(7) - 2019(12) | 6 | 0,657 | 2020(1) - 2020(11) | 11 | 0,946 |

Source: Prepared by the author based on the research.

The slowdown in the American economy during the Great Recession following the 2007-2008 financial crisis (or "subprime mortgage crisis") that occurred at the end of the period that began in the 1980s. The 2008 financial crisis occurred due to a housing bubble in the United States, caused by the increase in property values, which was not accompanied by an increase in the population's income. Several banks started to offer more credits, expanding real estate credit and attracting consumers, which caused the appreciation of the properties. Until with high demand, the interest rate went up, knocking down real estate prices. As many of these loans were high risk, many people were unable to repay them and several banks were left without capital. On September 15, 2008, one of the most traditional American banks, Lehman Brothers, filed for bankruptcy.

This, followed by a huge drop in world stock exchanges, marks the beginning of one of the most severe economic crises the world has ever known.

The global financial crisis, more precisely the one that started in 2008 in the United States real estate market and that spread around the world in the following years, mainly affecting Europe, was a speculative crisis, in which debts and securities based on speculation suddenly lost their value. Europe crisis, especially in the eurozone, is nothing more than an extension of the 2008 United States housing crisis. And the reason for that is simple: globalization. What happened is that subprimes were traded worldwide, involving investors mainly from developed countries, with emphasis on the European Union. With the bankruptcy of the market, these investors and everything that depended on them were also affected.

To avoid bank failures, many governments have spent a lot to bail them out and avoid an even more severe recession, which has increased public debt and deficit in these countries, increasing the risk of debt defaults by many governments. Some of these, in particular, were in a more serious situation, such as Portugal, Ireland, Italy, Greece and Spain. The problem was that this generated more economic stagnation, as the consumer market became less active and profits decreased, worsening the situation.

The United States, likewise, also suffered from rising debt, which forced the country to even raise the public debt ceiling. The crisis has largely affected the most dependent developed and underdeveloped countries. The so-called “emerging economies”, such as Brazil, Russia and China, felt these effects to a lesser extent, largely due to their high reserve funds and the investments made with these funds. In addition, these countries have managed to increase employment and the performance of their broad consumer markets, thus boosting their domestic economies.

In the second period of crisis, beginning in February 2020, S&P 500 had a negative impact due to the covid-19 pandemic, which has been generating strong turbulence in world markets and isolation policies to contain the pandemic progress, reflecting on the economy the effects of the shutdown of several economic activities (commerce, industry, aviation and tourism).

In figure 5 of the S&P 500 stock market index scores, the bearish period begins on February 19, 2020, reaching March 23, 2020 followed by the reversal and bullish period recorded until September 2, 2020 at which the index exceeded its score for the period before the start of the covid-19 pandemic. Subsequently, a brief period of decline begins until September 23, 2020, and recovered with significant growth until November 30, 2020.



Figure 5. Scores for the S&P 500 index

IV. CONCLUSION

The objective of the study was to analyze the changes in S&P 500 returns between January 2005 and November 2020, using the Markov-Switching Autoregressive (MS-AR) model. In the adjusted model, the mean and variance are modified according to the regime. The regime (1) expresses a positive average of the S&P 500 returns together with low volatility. In regime (2), it shows a negative average result and high volatility in S&P 500 returns. In regime 1, the estimated monthly average return is 1,169% with a variance of 0,02154. Regime 2 identifies a negative average monthly return of -0,195% with a variance of 0,05923.

In early February 2020, the S&P 500 had a negative impact due to the covid-19 pandemic, which has been generating strong turbulence in world markets and isolation policies to contain the pandemic's progress, reflecting in the economy the effects of the paralysis of several economic activities (commerce, industry, aviation and tourism). Although the downward trend of the stock exchanges is a pattern observed worldwide due to the effects of the covid-19 pandemic, it can justify the sharp percentage of the fall of the S&P 500.

In the matrix of transition and persistence of the regimes, it appears that the current regime 1 is more persistent, that is, the probability of remaining in this regime in a later period is approximately 93,61%, and that of moving to regime 2 is on the order of 6,39%. In regime 2, the probability of continuing this regime in the

period $t + 1$ is 92,52%, while the probability of changing to regime 1 is 7,48%. Thus, for the period from January 2005 to November 2020, the expected duration of the current regime 1 is 22 months. In regime 2, the estimated duration is 16 months.

ACKNOWLEDGMENTS

I am grateful to the Postgraduate Program in Economic Sciences (PPGCE) of the State University of Rio de Janeiro (UERJ) for participating as a Visiting Researcher in the FINRISK Research Group approved by Depesq-SR2 / CNPq, dedicated to quantitative finance and risk analysis .

Finally, I would like to thank the Research Support Foundation of the State of Rio de Janeiro (FAPERJ) for granting the research grant that made this article possible.

REFERENCES

- [1]. Ang, A., Bekaert, G. (2002). International Asset Allocation with Regime Shifts, *Review of Financial Studies*, 15, p. 1137- 1187.
- [2]. Assoe, K. G. (1998). Regime Switching in emerging stock market returns. *Multinational Finance Journal*, v.2, n.2, 101-132.
- [3]. Brock, W.A., Dechert, W.D., Scheinkman, J. (1987). A Test for Independence Based on the Correlation Dimension. Department of Economics, University of Wisconsin, SSRI Working Paper, 8702.
- [4]. Choi, K., Hammoudeh, S. (2010). Volatility behavior of oil, industrial commodity and stock markets in a regime-switching environment. *Energy Policy*, v.38, n.8, p.4388-4399.
- [5]. Dickey, D. A., Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive Time series with unit root . *Econometrica*, 49(4), 1057-1072.
- [6]. Diebold, F. X.; Lee, J. H.; Weinbach, G. C. (1994). Regime switching with time-varying transition probabilities. Oxford University Press, 283-302..
- [7]. Doornik, J. A. (2013). A markov switching model with component structure for US GNP. *Economics Letters*, v.118, n.2, 265- 268.
- [8]. Goldfeld, S. M., Quandt, R. E. (1973). A Markov model for switching regressions. *Journal of Econometrics*, 1, 3-16.
- [9]. Hamilton, J. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica* 57, 357-384.
- [10]. Hamilton, J. D., Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of econometrics*, 64(1-2), 307-333.
- [11]. Ismail, M. T., Isa, Z. (2006). Modelling exchange rates using regime switching models. *Sains Malaysiana*, 35(2), 55-62.
- [12]. Jarque, C., Bera, A. (1987). Test for normality of observations and regression residuals. *International Statistical Review*. 55(2), 163-172.
- [13]. Junior, M. V. W., Zuanazzi, P. T. (2014). A sensibilidade de ativos em diferentes ambientes de risco: Uma análise para empresas gaúcha. *Ensaio FEE*, v. 35, n.1, 231-248.
- [14]. Kim, C. (1994). Dynamic linear models with Markov-switching. *Journal of Econometrics*, 60, 1-22..
- [15]. Krolzig, H. M. (1997). Markov-Switching Vector Autoregressions: Modelling, Statistical Inference, and Application to Business Cycle Analysis, Berlin, Springer.
- [16]. Krolzig, H. M.; Toro, J. (2004). Multiperiod Forecasting in Stock Markets: A Paradox Solved. *Decision Support Systems*, Volume 37, 4, 531-542.
- [17]. Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54(1), 159-178.
- [18]. Mahjoub, M. R., Chaskmi, S. A. N. (2019). Identification the Periods of Formation and Bursting of Speculative Bubbles in Iranian Stock Market Using Quantitative Models. *Advances in mathematical finance & applications*, 4 (4), 129-140.
- [19]. Moolman, E. (2004). A Markov switching regime model of the South African business cycle. *Economic Modelling*. v. 21, 631-646.
- [20]. Norden, S. V.; Schaller, H. Regime Switching in Stock Market Returns. *Econometrics*, 357-384, 1995.
- [21]. Norden, S. V.; Schaller, H. The predictability of stock market regime: evidence from the Toronto stock exchange. *The Review of Economics and Statistics*, v.75, n.3, 505-510, 1993.
- [22]. Oliveira A. B.; Pereira, P. L. V. Asset allocation with markovian regime switching: Efficient frontier and tangent portfolio with regime switching. *Brazilian Review of Econometrics*, v.38, n.1, 97-127, 2018.
- [23]. Ozdemir, D. (2020). Cyclical causalities between the U.S. wholesale beef and feed prices: A Markov-switching approach. *Economics and Business Letters*, 9(2), 135-145.
- [24]. Panda, P., Deo, M., Chittineni, J. (2017). Dynamic regime switching behaviour between cash and futures market: A case of interest rates in India. *Theoretical and Applied Economics*. v. 24, n. 4(613), 169-190.
- [25]. Pera, M., Soledad, M. (2002). A regime-switching approach to the study of speculative attacks: A focus on EMS crises. *Empirical Economics*, 27, 299-334.
- [26]. Phillips, P. C. B., Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika*, 75(3), 335-346.
- [27]. Quandt, R. E. (1972). A new approach to estimating switching regressions. *Journal of the American Statistical Association*. v.67(338), 306-310.
- [28]. Safaei M.; Mostafaei H. Point Forecast Markov Switching Model for U.S. Dollar/ Euro Exchange Rate. *Sains Malaysiana* 41(4), 481-488, 2012.
- [29]. Xaba, D., Muroke, N. D., Rapoo, I. (2019). Modeling Stock Market Returns of BRICS with a Markov-Switching Dynamic Regression Model. *Journal of Economics and Behavioral Studies*. vol. 11, n. 3, 10-22.