Quest Journals Journal of Research in Environmental and Earth Science Volume 3~ Issue 5 (2017) pp: 20-33 ISSN(Online) : 2348-2532 www.questjournals.org

Research Paper

Mathematical Modeling of Solid Waste Management Processes A Case of Solid Waste Management In Awka City, Anambra State, Nigeria

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Received 18 October, 2016; **A**ccepted 17 May, 2017 © The author(s) 2017. **P**ublished with open access at **www.questjournals.org**

ABSTRACT: Virtually all waste managers in developing countries do not have any well defined method of collecting and keeping data on solid wastes (or any other form of waste) in their localities. Consequently, the required basic data on their waste management systems are lacking; the existing ones are not organized and cannot be relied upon for proper engineering analysis and adequate policy making. This claim is supported by both published reports and findings resulting from this study. The study aimed at developing mathematical models of waste generation and disposal processes as tools for use by waste managers. The developed models were applied in evaluating the performance of the Anambra State Waste Management Authority using a consecutive thirty one days data collected on solid waste management in Awka, the Anambra State Capital. Results show that the models are ideal and can be used when making economic considerations and in scheduling waste disposal vehicles. A phoning programme is recommended as a vital component in the use of the models in waste management. The phoning programme could save Anambra State Waste Management Authority some costs it incurs in managing solid waste in the state; distributing the said costs indirectly among the informant callers.

Keywords: accumulation, disposal, dumpsite, evacuators, generation, Queuing theory, solid waste

Nomenclature

- *R* Number of evacuator/disposal truck runs in a stated location
- *W_a* Expected waiting time of waste at a given dumpsite (= server mean service time)
- *W^q* Expected waiting time per given quantity of waste in the queue
- *W^s* Expected waiting time per given quantity of waste in the system
- *η* Disposal system evacuation efficiency .i.e. percentage of waste removed from the system in a given period
- *s* Number of parallel servers facility (disposal vehicles/trucks)
- *ρ* λ/sμ = ratio of arrival (dump) rate to service (evacuation) rate of waste in the system in a given period. It is a measure of traffic intensity (accumulation rate) of waste in the system
- $P_q(t)$ Steady state probability of having q units in the system
- P_0 Probability of zero quantity of waste in the system
- $P_{s,o}$ robability of server being idle (zero disposal service in the system)
- *Pw* Probability that waste in the system must wait for service
- P_{λ} Probability that new waste arrives in the system V_h Volume of waste container Volume of waste container
- Avg Average or mean
- mod Mode or highest frequency
- pd per day
- cb chain-up bins
-
- cbpd chain-up bins per day
- pp per period
- Σ Sum of, add all, or total up
- ASWAMA Anambra State Waste Management Authority (or Agency)
	- SWM Solid waste management
	- MSW Municipal solid management
- USW Urban solid waste

N.B: If any of the above symbols is a subscript of another it implies its meanings with respect to the item to which it is attached.

Each subscript of the lower letters *i, j,* and *z* refer to a set of serial numbers 1, 2, 3, ... to the n^{th} , k^{th} and mth terms respectively, say.

A bar "-" on top of a symbol refer the symbol to the mean value of the variable the symbol represents

Any other symbol not defined in this column are defined in the body of the text.

I. INTRODUCTION

Several researchers have identified solid waste management as an uphill task for many communities of the globe. Many factors have also been identified by some these researchers, [2],[3],[4],[5],[6],[7],[8] and [9], among others as constraints in providing effective waste management in the concerned localities. In an effort to minimize/eliminate these constraints, various models have developed. Mathematical methods of analyzing data which will assist in their policy making and economic considerations will always be readily accepted by waste managers that sincerely seek for solutions to the challenges posed by solid waste produced in their states.[1].

Meanwhile, for one to make a meaningful contribution or discussion about a system, he/she must have had some substantial knowledge about the system and its operations. Hence, the researchers commenced this study by making several pre-visits to different waste managers offices and dump sites in Awka, Onitsha, Nnewi and Ekwulobia Urban Centres of Anambra State to enable them appreciate the structure and nature of waste management practices in the state. Accumulation is a natural phenomenon and another form of queue. Waste, like any other thing in life increases (accumulates) in volume at some rate in a given system when there exists inability to dispose them at the same rate at which they are generated. Accumulation of waste in our environment makes the environment untidy and leads to many societal problems, which includes environmental, economic and public health problems.

Waste accumulation structure is dynamic in discipline, with the waste dumped discretely at the sites both in the day and in the night hours The structure of waste management system in Anambra State is depicted in Figs 1 and 2. The entire structure can be approximated to what is obtainable in a queuing system.

Fig 1 shows that solid waste generated in different residential homes and other sources are discarded at the various roadside waste dumpsites (as storage containers) in the area. The dumped waste are later collected by waste evacuating vehicles (as serving/disposing elements) and sent to the final dump site (as sink element) located at some points at the outskirts of the urban centres.

Waste Input Sources		Waste Handling System Elements	
Generating Elements		Serving Elements	Sink Flaments
Waste Generators	Storage Elements	Waste Evacuators	Waste Treatment
Homes	Waste Bins/containers	Compactors	Plants/Dump Sites
Offices	Compactor bins	Trailers	Open Dumps
Hotels World	π _{an} Chain-up bins	Whete Tippers	Inconerators
Construction Sites		Back Loaders	Landfills
Big Establishments			

Figure 1: Basic waste handling processes in a waste management system

As Fig 2 illustrates, waste bins kept at the various dump stations in each of the zones constituting the study area are considered as customers in a single queue and randomly served by the server/evacuator trucks who go to any of them that is free and waiting for service. A server truck could visit more than one waste dump station in a single trip if waste at the first station were not enough to fill up the vehicle's container. Although waste evacuation takes place almost on daily bases, most of the dumpsites were still not served for days leading to refuse piling up at those sites and causing some environmental and social problems.

Figure 2: Flow diagram of a typical Multiple Servers-Multiple Waste Depot System

II. MATERIALS AND METHODS

In the analysis, solid waste generation and evacuation in Awka, Anambra State, was viewed in the thinking that waste containers kept at different locations in Awka formed a queue with many servers (ASWAMA disposal vehicles) visiting them and rendering disposal services. This assumption should be right, since in reality a customer can either go to a service center to receive some service or have a worker(s) from the servicing firm come to him/her to render the service. Once the waste in a dumpsite was evacuated (whether totally or partially), the dumpsite or waste container(s) stationed therein was taken as having been served and departed. New waste dumped at the served dumpsite makes the container(s) at the dumpsite a new customer(s) to be served. By this consideration, the queue length is unknown, infinite and unrestricted. Number of servers that operated in the study area varied from 0 to 15 disposal trucks of different capacities per day. Also average number of chain-up bin loads of waste evacuated daily varied from 0 to 63. Variation in the number of servers was caused, among other reasons, by administrative bottlenecks and incessant breakdown of most of the disposal vehicles.

A consecutive thirty one (31) days data on solid waste generation and evacuation in three roadside dump sites (Ogbalingba, Eke Awka Market and Ifite Market) in Awka Metropolitan city of Anambra State were used to show the applications of the models developed in the study. Actually, different disposal trucks of various capacities serviced the three dumpsites under review, but it is taken in this exercise that only one disposal truck (server) worked in the three dumpsites within the thirty one (31) day study period such that the data obtained in TABLES 2, 4 and 5 hold true. Data in the said tables contain the estimated quantities of waste found at the three dumpsites during the morning and evening checks of the study period, which act as the input (independent) variables from which other parameters, as the output (dependant) variables, derive their values. Waste dumped between 6 am - 6 pm at the sites were considered as day arrivals, while the ones deposited between 6 pm - 6 am were termed night arrivals. As these refuse arrived, they built up and filled the containers. Collated data were first organized and descriptive statistics used to compute the combined average waste bin loads generated (i.e. waste production/dump rates) and the quantities discharged (i.e. waste evacuation/disposal service rates) in each of the three dumpsites in the month of December, 2014. Also computed were the approximate waste disposal system utilization in the same period. Summary of the analysis are contained in TABLES 2 to 7. Microsoft Excel was very helpful in achieving this fit.

3.1 Waste Queuing and Evacuation Models Development

3.1.1 Assumptions

- \checkmark The number of waste containers (bins) in use is known but in terms of the quantities of waste to be generated it is unknown, infinite and unrestricted. That is to say, there are many batched quantities (measured in terms of bin sizes) to be evacuated (served) in a given period (hourly, daily, weekly, etc)
- \checkmark The service/unloading time of bins is controllable
- The waste bins are arbitrarily but strategically distributed (placed) at various locations within the research area.
- \checkmark Quantity of solid waste found at roadside dumpsites at any given time is the total waste generated by the people living/operating within the area.
- \checkmark Waste existing at various public used roadside dumpsites are in queue
- Waste generation and disposals are stochastic, dynamic and man-machine-material interaction processes of the system
- \checkmark The system is non-fully automated
- \checkmark Arrival rate of waste at the dump sites is stochastic, but the evacuation is executed according to management plans.
- \checkmark State (quantity) of waste before and after evacuation at a given dumpsite in a given period(s) is always recorded.
- \checkmark Cost of waste evacuation is dependent on the quantity of waste disposed (transported) in a stated period, say, daily, weekly, monthly or annually.
- \checkmark Waste not brought to a public dumpsite is negligible/not considered at all.
- Unit of measurement is the unit volume (full load) of the waste container kept at a dump site under consideration
- \checkmark Size and number of waste bins (containers) and bin locations (dumpsites) may vary, but such data is recorded and treated from the point of such a variation
- \checkmark For purposes of the present study, unless where otherwise stated, loading of an evacuator at a waste bin/dump site is measured by the fraction of full loaded chain-up bins carried by the truck per its trip from the site.
- \checkmark Any indisposed waste at a given waste bin site, after a truck's visit to the site on a day is considered a new joiner to the line of bins waiting for service during the truck's next visit to the site.
- For a recording that starts at evening check of day $i = 1$, $c_{ii} = 0$ and $d_{ii} > 0$; if recording starts at morning check: $d_{ii} = 0$ and $c_{ii} > 0$.

 To make the development of the models in this module better understood, a simulation table consisting of *k* inputs x_{ij} (for *k* number of dumpsites), $j = 1, 2, \dots, k$; $i = 1, 2, \dots, n$, and a number of the desired responses y_{ij} , was constructed in a Microsoft Excel spreadsheet (see Table 1).

					Dumpsite 1								\ddots			Dumpsite k				All Dumpsite					
T _i		Inputs													Responses, Inputs										
me		x_{i1}							Responses, y_I					\ddots	x_{i1} y_I				Σ y _i (Responses)						
\dot{i}													P_t							P_t					P_t
(da	L	q	q	$q_{\left(d\right)}$	λ	μ	L	$\cal L$	W	W	W		(t		L	q	q	$q_{(d+)}$		(t)	$q_{\left(d\right)}$	λ	μ		(t
ys)	\overline{a}	\boldsymbol{d}	\boldsymbol{N}	$+N$)	q	\boldsymbol{q}	\boldsymbol{R}	\boldsymbol{S}	\boldsymbol{a}	\boldsymbol{q}	s	η		\cdots	\overline{a}	\boldsymbol{d}	$\cal N$	N)			$+N$)	\overline{a}	\overline{a}	\ddotsc	
1														\ddotsc											
2														\cdots											
٠														\ddots											
\boldsymbol{n}														\cdots											
Σ																									
$=$																									
\boldsymbol{M}																									
ea																									
\boldsymbol{n}																									

Table 1: Simulation table for queue model development, demonstration and validation

 In estimating the quantities of waste generated and evacuated in a given dumpsite, the following relations could be established:

$$
q_j = Q_{p,j} - Q_j \tag{1}
$$

Rewriting (1) to show the number of evacuator runs and the mean disposable quantity: $q_j = Q_{p,j} - r_j D_j$

Where the quantity evacuated varies, the average of *D* quantity of waste evacuated from the given dumpsite in a period *t* is given by the equation:

$$
\overline{D}_j = r_j^{-1} Q_j \tag{2b}
$$
\n
$$
q_j = Q_{p,j} - r_j \overline{D}_j \tag{2c}
$$

So, For all the dumpsites in the area under consideration:

$$
Q_{\rm p,j} = q_{\rm j} + r_{\rm j} D_j \tag{3a}
$$

$$
Q = Q_{\rm rem} + Q_{\rm eva} \tag{3c}
$$

Or

$$
Q = Q_{rem} + Q_{eva}
$$
\n
$$
\Sigma Q_{p,ij} = \Sigma q_{ij} + \Sigma (xD)_{ij}
$$
\n(3b)

For all the dumpsites, it was assumed that check starts in the morning of day $i = 1$, with $L_{w,(i-1)j} \geq 0$, $q_{N,(i-1)j} \geq 0$, $\mu_{q,ij} = 0$, $q_{d,ij} = 0$, $q_{N,ij} = 0$ and $\mu_{q,(i-1)j} = (r D)_{(i-1)j}$. Thus, in accordance with (3):

$$
q_{M,ij} = (Q - rD)_{(i-1)j} + q_{N,(i-1)j} = q_{M,oj}
$$
\n(4a)

Or rewriting (4a) in terms of queue parameters

$$
L_{q,oj} = q_{M,ij} = L_{w,(i-1)j} + q_{N,(i-1)j}
$$
\n(4b)

At the evening check of the same 1st day we also assume that $q_{M,ij} \ge 0$, $L_{w,ij} \ge q_{M,0j}$, $q_{N,ij} = 0$ and $\mu_{q,ij} \ge 0$. By this we obtain that:

$$
q_{d,ij} = (L_{w,ij} + \mu_{q,ij}) - q_{M,oj} \tag{4c}
$$

At the morning check of the next day, $i+1$, with $\mu_{q,ij} \ge 0$, $L_{w,ij} \ge q_{d,ij} \ge 0$, $q_{N,ij} \ge 0$:

$$
q_{n}(i+1)j = (L_{W,jj} + q_{N,jj})
$$
\n(5a)

At the evening check of day *i*+1 still, with no evacuation made and $q_{N,ii} \ge 0$. Thus:

$$
q_{d,(i+1)j} = (L_W + \mu_q)_{(i+1)j} - q_{M,(i+1)j} \tag{5b}
$$

From the foregoing, therefore, we conclude that for a given day *i*, the total quantity of waste (number of bin loads) deposited at dump site *j*: ن س

$$
q_{(d+N)j} = (q_d + q_N)_{ij} \tag{5c}
$$

A waste container is considered full if only:

$$
V_b = \sum_{i=1}^{m} q_{(d+N)} \ge 1 \quad \text{(where } 1 \le m \le n\text{)}\tag{6a}
$$

And number of fully and/or over loaded bins:

$$
N_{Bf} = \sum_{i=1}^{n} \left(\frac{q_{(d+N)}}{v_b} \right) \ge 0 \tag{6b}
$$

For *k* number of dumpsites, daily quantity of waste arriving (deposited) in a period *i* to *n* days is obtained thus: $\lambda_q = (L_q + q_d)_{ij}$ $(7a)$

$$
L_{s,n} = L_{q,o} + (\sum_{i=1}^{n-1} q_{(d+N)} - \sum_{i=1}^{n-1} \mu + q_d)_{ij}
$$
(9b)

For *k* number of dumpsites, the average number of waste bins in the system:
\n
$$
L_s = \frac{1}{nk} \sum_{j=1}^k (L_{q,o} + \sum_{i=1}^{n-1} q_{(d+N)} - \sum_{i=1}^{n-1} \mu + q_d)_{ij}
$$
\n(9c)

And the number of loaded bins waiting for service in the system on the
$$
n^{\text{th}}
$$
 day:
\n
$$
L_{w,n} = L_{q,o} + (\sum_{i=1}^{n-1} q_{(d+N)} - \sum_{i=1}^{n} \mu_q + q_d)_{ij}
$$
\n(9d)

From the general queuing theory, we obtain:

$$
W_q = L_q \lambda^{-1}
$$
\n(10a)
\n
$$
W_s = L_s \lambda^{-1}
$$
\n(10b)
\n(10b)
\n(10c)

$$
W_{\alpha} = \sum_{i=1}^{n-1} N_{(\mu=0)} \tag{10c}
$$

A waste management system's daily evacuation efficiency (portion of waste evacuated) is defined by the relation:

$$
\eta_{ij} = \frac{\mu_{q,ij}}{\lambda_{q,i,j}} \ge 100\%
$$
\n(11)

While the overall system's efficiency is defined by the relation:

$$
\eta = \frac{2i_{i=1} \, 2j_{i=1} \, \mu_{q,ij}}{\left(\sum L_{q,oj} + \sum_{i=1}^{n} \sum_{j=1}^{k} q_{(d+N)}\right)_{ij}}\tag{11b}
$$

The waste disposal system utilization:

$$
\rho = \frac{\lambda}{s\mu} \tag{12}
$$

Idle time of server in the system is defined by the conditions that:

$$
\mu = 0
$$
\n
$$
q_{(d+N)} \ge 0
$$
\n
$$
t_{\text{os}} = t_{\mu=0}
$$
\n
$$
N_{\mu=0} = 1 \text{ (say)}
$$
\n(13)

Thus, total number of idle times of disposal truck(s) serving *k* number of dumpsites in a given period,:
 $N_{s,o} = \sum_{i=1}^{n} \sum_{j=1}^{k} N_{(u=0),ij}$ (14)

Therefore, probability that a server is idle:

$$
(P_o)_s = (N_s)_o \times n^{-1} \tag{15}
$$

Probability that waste container(s) in the system period is full and/or overflowing:

$$
P_{\text{B}f} = \sum_{i=1}^{n} \sum_{j=1}^{k} \left(\frac{N_{\text{B}f}}{n} \right)_{ij} \text{ {where } N\text{B}f: N\text{B}f = (N\text{B}f + N\text{B}of) }
$$
 (16)

It is assumed that disposal trucks (servers) visit dumpsites either in the morning or evening hours of a day and that during the visits, the waste bins should be totally emptied such that:

$$
\begin{aligned}\n& N_{\mu > 0 \ge N_b \\
& q_{(d+N)} = 0 \\
& N_{\lambda=0} = 1 \quad \text{(say)}\n\end{aligned} \tag{17}
$$

By this assumption, the probability of no waste in dumpsite *j* at morning check of a given period:

$$
P_{o,d,j} = \left(\frac{N_{\lambda=0}}{n}\right)_{d,ij} \le 0.5 \tag{18a}
$$

Probability of no waste in dumpsite *j* at evening check of a given period:

$$
P_{o_{iN,j}} = \left(\frac{N_{\lambda=0}}{n}\right)_{N,ij} \le 0.5 \tag{18b}
$$

Therefore, probability of no waste in dumpsite *j* is:

$$
P_{o_{i,j}} = \left(\frac{N_{\lambda=0}}{2n}\right)_{d,ij} + \left(\frac{N_{\lambda=0}}{2n}\right)_{N,ij} \le 1
$$
\n(18c)

For the entire system, the probability of no waste in *k* number of dumpsites is:

 $(18d)$

Probability that new stock of waste arrives at dumpsite *j* in a given period:

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$$
P_{\lambda} = \frac{\sum_{i=1}^{n} q_{(d+N)}}{(L_{q,oj} + \sum_{i=1}^{n} q_{(d+N)})}
$$
(19)

It can be seen clearly from the above equations that the performance measures are functions of two basic queuing parameters - waste arrival rate (the average rate of waste container fill) and the container service rate (the average rate of waste evacuation). Values computed for these parameters give how well the management service mechanism handles volumes of waste generated in the given system.

III. MODELS APPLICATION, RESULTS AND DISCUSSIONS 4.1 Waste dump and disposal data presentation and discussions

The input data in TABLES 2 and 4 to 6 enable derivation of more information (responses from the input variables) for evaluating the performance of the waste management system. The three dumpsites were juxtaposed in columns set as shown in TABLE 1 but have been presented here in separate tables for want of space. As can be seen from TABLE 2, even though the overall waste evacuation efficiency (80.6%) at Ogbalingba dumpsite is commendable, the system still showed not to be stable as $\lambda > \mu$.

Waste accumulated at Ogbalingba dumpsite at an average rate of 4.654 cbpd, while the server operated at an average rate of 0.645 cbpd. Data on this relationship are contained in TABLE 3 and plotted in Figs 3a and 3b for visual appreciation.

Table 3: Determination of best system utilization at Ogbalingba dumpsite

				p 7.215 3.6075 2.405 1.8038 1.443 1.2025 1.0307 0.9019 0.8017 0.7215	

A trendline equation generated on plot of Fig 3a shows that the function has a power distribution/relationship: *ρ* $= 7.215s^{-1}$, with $R^2 = 1$. Figure 3b shows that going by the present service rate at the dumpsite, it would be very rear not to see dumped waste at the site.

Again in TABLE 4, the overall waste evacuation efficiency (90.8) at Eke Awka dumpsite is quite impressive. However, the system still showed not to be stable. Waste generation/dump rate at the site was estimated as an average rate of 8.308 cbpd, while the waste containers were served at an average rate of 2.968 cbpd. By the same reasoning as in discussing the waste management at Ogbalingba dumpsite, data plot on Eke Awka Market gave the relation: $\rho = 2.7978s^{-1}$ and $R^2 = 1$ and requires a service rate, $\mu = 8.9$ cbpd to stabilize the subsystem.

15	8.7	1.6		0.0	1.6	10.3	ō		10.3	10.3	72. ٥	20.3	72.0	0.0	0.0 4	0.55
19	10. 3	0.0		3.6	3.6	10.3	8		2.3	13.9	24. o	24.0	32.4	77.7	0.0 \blacktriangle	0.59
20	.9	0.7		4.5	5.2	6.6	ō		6.6	11.1	48. ۰	21.5	80.7	0,0	0.0	0.62
	11.										24.				0.0	
21		1.2		3.0	4.2	12.3	Ð		3.3	15.3	o 48.	21.7	29.9	73.2	5 0.0	0.66
22	6.3	0.6		2.8	3.4	6.9	ó		6.9	9.7	o 72.	21.9	67.5	O, O	0.0	0.69
23	9.7	1.9		0.9	2.8	11.6	ō		11.6	12.5	۰ 24.	20.1	77.6	0.0		0.74
24	12. 5	0.6		4.0	4.6	13.1	10		3.1	17.1	o	22.9	31.3	76.3	0.0 5	0.79
25	7.1	0.5		2.7	3.2	7.6	٥		7.6	10.3	48 ō.	22.4	65.1	0.0	O, O	0.82
26	10. з	2.2		0.2	2.4	12.5	s		4.5	12.7	24. o	19.5	24.4	64.0	0.0 5	0.86
27	4.7	0.2		2.4	2.6	4.9	۰		4.9	7.3	48. o	23.0	71.5	0.0	0.0 2	0.88
											24.				0.0	
28	7.3	0.0		3.2	3.2	7.3	8		0.0	10.5	o 45	24.0	34.5	109.6	٠ 0.0	0.91
29	3.2	1.5		2.5	4.0	4.7	o	1	4.7	7.2	o 24.	16.3	73.5	0.0	2 0.0	0.93
30	7.2	4.4		0.0	4.4	11.6	6		5.6	11.6	o 48	14.9	24.0	51.7	0.0	0.98
31	5.6	0.8		3.6	4.4	6.4	ō		6.4	10.0	o	21.0	75.0	0.0		00.1
		33.	\sim	65. ٠	98.	257.	COLLECTION	\sim						66.9		

Tables 5, 6 and 7 are explained in the same reasoning as in TABLES 2 and 4.

TABLE 6 contains summary of the analysis made for the overall system performance. The table depicts the combined effects of TABLES 2, 4 and 5. Also shown in the table are the overall system's server daily evacuation efficiency (η_d) and the transient state probabilities of quantities of waste in the three dumpsites.

Table 6: Summary of daily waste management data on Ogbalingba, Eke Awka Market and Ifite Market dumpsites in Awka Capital

City of Anambra State, Nigeria collected from 1st - 31st December, 2015.

Table 7 is derived from TABLES 2, 4, 5 and 6. The table shows a summary report on the mean values obtained for the various dependent variables by applying the relevant developed models. A close look at the table reveals that almost all the variables in the models developed from the study are well represented.

			Dumpsite Obtainable				
		Ogbalingba	Eke Awka Market	Ifite Market		Computed	
						Values for All Dumpsites	
Parameter/			Mean values per demonstration run		Over-	Mean	Unit of
Symbol	Source/Ref. Eqn.		$(t = i \text{ to } n)$		all		Measure

Table 7: Awka waste management system performance evaluation

Table 31: Data on the new combined system's performance for the three sample dumpsites

N_b	$\overline{}$	3	4	\overline{c}	9	3	bins
\boldsymbol{n}		31	31	31	31	31	days
L_q	4b, 5a	4.38	7.24	4.05	15.7	5.22	cb
q_d	$(4c)$, $(5b)$, TABLES 2, 4, 5 & 6	0.27	1.07	0.28	1.6	0.54	cb
$(N_{\lambda=0})_d$	TABLES 2, 4, 5 & 6	3	5 ⁵	$\overline{4}$	12	$\mathcal{L}_{\mathcal{A}}$	\sim
q_N	$(4c)$, $(5b)$, Tables 2, 4, 5 & 6	0.41	2.12	0.38	2.9	0.97	cb
$(N_{\lambda=0})_N$	TABLES 2, 4, 5 & 6	7	3	Ω	10	÷	$\overline{}$
$q_{(d+N)}$	(5c)	0.68	3.18	0.65	4.5	1.51	cb
λ	$(7a)$, $(7b)$	4.65	8.30	4.33	17.3	5.76	cbpd
μ	(51) , TABLES 2, 4, 5 & 6	0.65	2.97	0.58	4.2	1.40	cbpd
$N_{s,o}$	(14)	23	18	25	13	4.33	\sim
L_{w}	(9d)	0.0	5.4	3.7	9.1	3.03	cb
$L_{\rm s}$	$(9b)$, $(9c)$	4.3	6.4	4.0	14.7	4.90	cb
W_a	(10c)	61.9	42.6	71.2	175.7	58.58	hrs
W_q	(10a)	22.5	20.9	22.3	65.7	21.90	hrs
W_s	(10 _b)	63.1	40.5	73.7	177.3	59.10	hrs
η	(11a), (11b)	13.86	35.74	13.42	24.26	24.26	$\%$
ρ	(12)	7.215	2.798	7.450	4.122	\sim	Erlang
P_{α}	(18c)	0.161	0.129	0.065	0.355	÷.	÷.
$(P_o)_d$	(18a)	0.048	0.081	0.065	0.065	$\overline{}$	$\overline{}$
$(P_o)_{N}$	(18b)	0.113	0.048	0.000	0.054	$\overline{}$	$\overline{}$
$(P_s)_o = P_w$	(15)	0.742	0.581	0.806	0.419	÷	\overline{a}
P_{λ}	(19)	0.851	0.974	0.894	0.991	\overline{a}	-

Summary of the waste management system's performance data of TABLE 7 gives the *λ* and *μ* for each of the three dumpsites. For the entire system (i.e. the mean of means), $\lambda = 17.3$ cbpd and $\mu = 4.2$ cbpd, showing that there is instability in the system. For the system to be considered stable, the waste service (disposal/removal/ evacuation) rate must be greater than the waste arrival (dump/generation/production) rate, i.e. $\mu > \lambda$. The mean values obtained for these parameters were used in coding a single line multi-server queuing system based on (12) to estimate the optimum number of servers (*s*) that should make the system stable. TABLE 8 refers.

Table 8: Evaluation of the queue system performance when λ and μ are constant and *s* is varied

	2.0595	1.373	1.0298	\mid 0.8238 0.6865 0.5884 0.5149 0.4577		0.4119
P_0 0.0107	\vert 0.0145 \vert 0.0157 \vert 0.0161 \vert 0.0162 \vert 0.0162 \vert 0.0163 \vert 0.0163 \vert 0.0163 \vert					0.0107

From the data in TABLE 8, it could be observed that as *s* increases the system becomes more stable. However from $s = 1$ to $s = 4$ the system still remains unstable with $\rho > 1$. From $s \ge 5$ servers, $\rho < 1$ and the system becomes more stable . Implying that $\mu > \lambda$. But employing more number of servers (disposal trucks) keeps the facilities underutilized. Five (5) of such trucks working in the entire system or by increasing the present service rate by 5 times will make $\rho \approx 0.8238$ and suggests to be the best option among those provided in TABLE 8.

TABLE 9 contains a new set of data generated based on the new scenario: $\lambda = 17.3$ cbpd and $\mu = 21$ cbpd. A plot of the server system utilization factor against the number of servers (Figure 51) shows clearly that *ρ* decreases as *s* increases. A trendline equation generated from this plot shows that *ρ* relates with *s* as: *ρ* = $0.8238s^{-1}$ and gives the R^2 value of 1. Also of interest is the possibility of not having huge piles of waste as we see them in the three dumpsites studied in the present course when these five trucks work at the dumpsites. This possibility of having no waste in the system is illustrated in Figure 4b. The plot shows a positive possibility of such a condition as the number of servers increases. The five trucks of choice give almost the same probability \approx 0.016 as 6 to 9 servers.

A careful perusal of the values in TABLES 2 through 7 reveals the most of reasons why waste containers in Awka municipality and other areas in Anambra State are always fully loaded, and in some cases, overflowing with waste; appearing to the uninformed passersby in the street as if ASWAMA is not doing much work. But the study has shown from both the subsystems and the overall systems efficiencies that the agency is really performing, and that its service facilities are in fact, being over utilized. The over utilization could be the major reason for the incessant breakdowns of these facilities.

4.2 Flow chart for systematic application of the waste queuing models

Flow chart for application of the waste queuing models developed in this study in Excel spreadsheet is depicted in Figure 5. Values for two basic queuing parameters - λ and μ - for $i = 1$ to *n* and $j = 1$ to *k* are required as input data which Excel uses to generate values for other parameters based on coded models of interest.

Figure 5: Flow chart for application of the study waste queuing models

IV. CONCLUSION

The following conclusions are drawn from this study:

- all the Models based on the functional relationships existing among waste production and evacuation rates parameters in solid waste management have been developed and presented in this paper by applying queuing theory in the system..
- $\ddot{}$ The models can be used as monitoring/assessment tools for evaluating the performances and estimating the financial transactions made by a waste manager over time.
- \blacksquare The study provides a systematic method of collecting, analyzing and keeping quantitative data on quantity of solid waste generated and the quantity disposed to a landfill, transfer station or final dumpsite in a locality.
- $\ddot{\bullet}$ Application of the models in the study showed that the three dumpsites used as case study had different waste generation and evacuation/disposal lead times, which can be used in making far reaching waste management decisions.
- Results also show that improvement exists in the system as the waste evacuation lead times gets closer to or equals the waste generation lead times.
- It is hoped that application of these models will in no small measure assist the ASWAMA and other solid ₩., waste managers in making economic decisions when scheduling their disposal trucks for waste evacuation.

V. RECOMMENDATION

During the period the field study was conducted, it was observed that waste evacuators routing was left at the discretion of the vehicles drivers and the members of their crew, who drive about town collection waste randomly. This random search/operation increases the costs of waste evacuation. A well conducted solid waste inventory management system should be a close loop system and should also lend its supports to sustainable integrated solid waste management and development. To achieve this fit, ASWAMA should open up customers

phone calls centre/programme where the public can feed the agency with information about the state of their environment, especially the waste bin locations that need evacuation atten-tion of the agency. This will give the public the opportunity of participating in solid waste management in the state and also save ASWAMA the costs of employing some workers to do this, among other benefits. This method was used during the field study to monitor both the waste bin sites and the evacuator trucks that went to the bin sites in any day. A sample of the information flow structure is depicted in Figure 6. This programme should be linked to a computer based data collection and database management system.

Figure 6: An information flow structure for solid waste management

ACKNOWLEDGEMENTS

We wish to acknowledge Mrs. Christy Ubaka (the D.G., Anambra State Branch of the Federal Environmental Protection Agency), Mrs. Njideka Oraedum (the Managing Director of ASWAMA) and her Special Assistant, Mr. Samuel Ike and all other ASWAMA staff for their assist in one way or the other during the period of our data gathering in the study. We personally observed the concerns of all these people in seeking for how to find a lasting solution to the problems created by solid wastes in the state. We hope to see this solution come some day in the near future.

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