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Research Paper

Representation of the Improper Fractions in Greek Primary School Textbooks

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ABSTRACT:*This study presents the research conducted on the Greek textbooks of elementary mathematics about the structure of books on fractions and more specifically on the concept of improper fractions. That is, which type of representations are used in elementary school textbooks for the above meaning and with what frequency. At the same time, comments are made on the adequacy and type of representations and their correlation with the difficulties faced by students, based on research, on the concept. The results show a limited extent of the activities of primary school textbooks dealing with improper fractions, the absence of the definition of the concept and the incomplete presence of multiple representations with a predominance of symbolic representation.*

KEYWORDS: *Fractions, Improper, Primary, Textbooks, Representations*

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I. INTRODUCTION

Fractions are essential for learning more advanced math, such as algebra [5] not to mention their importance for academic achievement, career development and life functioning [16]. Despite their significant role for mathematical literacy, little emphasis has been placed on them resulting in cognitive misunderstandings (such as that fractions are always smaller than one [14],[12]) and difficulties for students.

The present study is part of a larger study consisting of three parts with the foundational goal of creating proposals that will help reduce the difficulties Greek students face in fractions.

The first part aims to explore the difficulties faced by primary and secondary school students on explicit numbers and on the concepts of fractional serialization as a representation on the number line, as well as on the concepts of unit division and improper fractions.

The second part of the research aims to identify the causes and reasons that students in Greek education face these difficulties and what factors affect the presence or absence of these difficulties. Emphasis is placed on teachers' perceptions of explicit numbers, the structure of mathematics textbooks, the Interdisciplinary Curriculum Framework and Curricula, as well as teaching aids and approaches proposed by various researchers internationally.

The last part includes the presentation of proposals for addressing the reasons that cause students' difficulties in the above concepts in the form of applied proposals and didactic interventions.

The present study is a part of the second research part that concerns the research of the structure of the Greek primary school textbooks on the expressions. More specifically, it examines the fields of representation that relate to the concepts of improper fractions, as well as the frequency with which these representations appear in elementary school math textbooks.

Regarding the use of the terms used in the present study, when reference is made to symbolic representations, it means the representation of the rational number in its algebraic form, that is, a number of the form $\frac{a}{b}$. Also, the term “diagrammatic representations” means the representation of a rational number with images and geometric shapes. When reference is made to “discrete quantities”, one means a set of separate-discrete objects, and finally the reference to verbal representations refers to numerals of the form "one eighth".

II. THEORETICAL BACKGROUND

As mentioned above, the first part of the wider research included identifying and grouping the difficulties that students face on the concept of fraction and more specifically on the concepts of placing fractions on the number line, dividing the unit into equal parts and improper fractions.

Based on these studies [2] conducted to identify the difficulties of the above concepts in students and the three levels of education (primary, secondary and higher education) it appeared that all three levels present significant difficulties in the concept of improper fractions, as most students find it difficult to schematically represent an improper fraction or even to recognize it from a diagram by writing it in its symbolic form. Thus, for example, when they are presented with the Figure 1 and asked to write which fraction the shape represents, most students respond 3/4**.**

Figure 1: Diagrammatic representation of 3/2

The above difficulties were confirmed in another one research study by [3] that was conducted to investigate the stability of the results in relation to the above research. This research confirmed the significant difficulties in the concepts of improper fractions.

All these difficulties that have been confirmed by various other research have been attributed to various factors. According to [9], most textbooks today include a variety of representations designed to promote a conceptual understanding of the meaning of fractions. The central position of the various fields of representation in the teaching and learning of mathematics is supported by other researchers such as [6], [14], [16]. Also, [11] in his research attributes the difficulties that exist in the perception of fractions and proportions possibly to the inappropriate method of their teaching in the classroom. The same view is supported by [15] adding that the failure in teaching the concept of fraction is due to the complexity of the concept and the traditional approach to fractions, which is formal and mechanical from the beginning [13]. We observe, therefore, that different researchers have each attributed to different factors these difficulties that students face in explicit numbers. However, they all agree on the degree of difficulty in understanding fractions and more specifically the improper fractions. The fact that students tend to treat all fractions as smaller than one is considered as one of the main reasons for the observed difficulty in conceiving the improper fractions [12]. Our research team, believing that these difficulties of students are due to a combination of factors that includes, among other things, the findings of the above research, attempts through a long-term research attempt that combines all the possible causes reported by various researchers as well as new ones that may emerge, to highlight these factors with the ultimate goal of formulating and proposing substantial solutions to address these difficulties of students with interventions in the Greek educational system.

III. STUDY OF THE SCHOOL TEXTBOOKS

3.1 Purpose of the Study

The present study focuses on the structure of the Greek primary school textbooks on the expressions. More specifically, it intends to study the forms of representations as well as the frequency presented in mathematics textbooks for the concept of improper fractions while recording the evolutionary course of these concepts as students move from one class to another. The findings will be studied to determine whether some factors in the structure of primary school textbooks contribute to the creation of students' difficulties in the explicit. Specifically, theobjectivesincludethefollowing:

- To identify and examine the representations in the concepts of improper fractions.
- To make a possible correlation of the representations with the difficulties that the students face on these concepts.
- To suggest ways to improve the teaching of these concepts in case there is a correlation between the structure of the books and the difficulties of the students.

3.2 Research Methodology

Research Tools

To achieve the objectives of the research study, the Greek textbooks of mathematics and the six grades of primary school were studied (from the first grade of primary school to the sixth grade of primary school). The group of 38 books in total studied includes student books, workbooks, and teacher books.

Data Analysis

In the context of this study, we applied a descriptive analysis of the data collected from the mathematics textbooks. The mathematics textbooks of 6 grades were analyzed according to the aim of the study. They included both student's book and workbook. We also, analyzed teacher's book to pinpoint if there are any references about the improper fractions.

IV. RESULTS

The concept of improper fraction, although not referred to as "improper" in elementary school textbooks, is nevertheless presented and defined as the fraction greater than the fractional unit or the fraction whose numerator is greater than its denominator.

Figure 2: The first appearance of an improper fraction with a symbolic representation

The first improper fractions appear in the third grade of elementary school in their symbolic form in chapter 34, p. 86 of the student's book in the context of teaching decimal fractions (Figure 2). More specifically, students are asked to calculate lengths with fractions. The goal of these activities, according to the teacher's book, is for students to find and write a decimal fraction as the sum of a whole number and a decimal fraction less than one. However, the occurrence of improper fractions occurs without any reference to the fact that they are fractions that are larger than the fractional unit. It is therefore up to the teacher to choose whether or not to clarify the presence of improper fractions.

In the same chapter, chapter 34 p. 87, students are given a large square that is used as a unit of area. In this activity the decimal fractions are presented as subdivisions of the area of the large unit square. Through a series of successive steps this activity results in the improper fraction which students are asked to represent the decimal fractions with squares and perform the conversions and calculations only on a symbolic level, without the corresponding diagrammatic representation (Figure 3). This lack of diagrammatic representation is possible if it leads students to mechanistic processes, without giving them the opportunity to become aware of the improper fraction and how it is represented in various representational systems. Exercises like the above are given in the workbook 3rd issue, pp. 24 and 25 of the same chapter (Figure 4a).

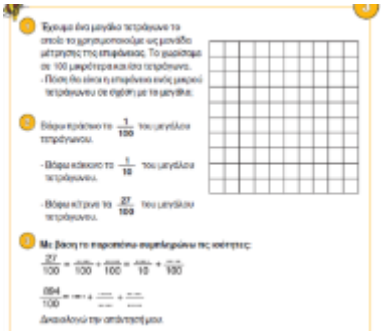

Figure 3: Lack of diagrammatic representation for fraction 894/100

In the same class, in chapter 35, which deals with decimal fractions and decimal numbers, improper fractions in their symbolic form reappear, which students are asked to convert into decimal numbers with the help of a calculator (Figure 4b). And at this point there is no conscious presentation of the improper fractions. Also, the result is not commented that if for fractions less than one the integer part of the decimal number is zero, while if the fraction is improper the integer part of the decimal number is different from zero. Although the aim of the chapter is not the above separation, however, this commentary would give a first impetus to students to observe the fractions in depth and begin to distinguish them in larger and smaller units. This way of thinking can take students away from mechanistic processes, while at the same time preparing them for the introduction of the concept into various fields of representation.At the end of the same chapter is the conclusion, which emphasizes the decimal part of the decimal number and presents the improper fraction as a form of representation of the decimal number.

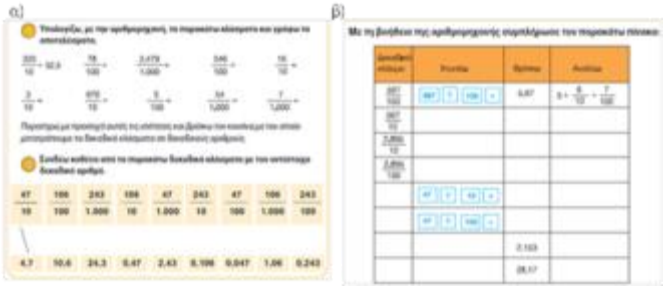

Figure 4: a) Exercise for converting fractions to decimal numbers in the student's book. b) Exercise for converting fractions to decimal numbers in the workbook

After chapters 34 and 35, the improper fractions reappear to chapter 57 "Fractions and Decimals" of student's book. There they are presented for the first time in various forms of representation, such as symbolic, diagrammatic and discrete quantities (Figure 5). Moreover, in an exercise in the workbook of the same chapter, students are asked to separate the given fractions by comparing them with the unit (Figure 6), without having

mentioned in a previous chapter the norm of this separation. More specifically, the book of the teacher of the third grade on page 152 for the exercise of picture 6 states: "The case with the 3/5 that is bigger than the unit may make it difficult for the students, because they are used to forming fractions smaller than the unit".

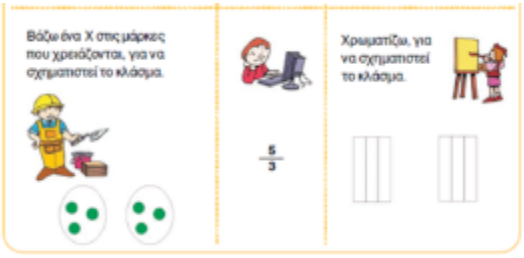

Figure5: Symbolic, diagrammatic, and discrete representations of improper fractions

Summarizing the findings from the textbooks of the 3rd class, we observe that the concept of improper fractions and its representations are not included in the objectives of the book and there is only one exercise in all books of this class with the various forms of representation of improper fractions.

In the 4thgrade of primary school, emphasis is placed on decimal numbers, which occupy 34% of the mathematics chapters, and in this context, some improper fractions appear in their symbolic form, without any mention, simply to serve the teaching of decimal numbers and decimal fractions. More specifically, improper fractions under the conditions mentioned before, appear on pages 63, 68 of the student's book and on pages 7, 18, 28, 35 of the 2nd issue of the workbook. However, on page 18 of the same issue, which is a repetitive chapter, there is only one exercise that gives the verbal representation of 12 tenths and asks the students to represent it diagrammatically and symbolically (Figure 7). The aim of the exercise, according to the teacher's book, is to connect decimal, decimal fractions, and mixed numbers with a virtual representation.

Figure6: Indirect definition of improper fractions in student's workbook

Figure7: Connecting decimal, decimal fractions and mixed numbers with a virtual representation

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Summarizing the findings from the textbooks of the 4th grade, we observe that the concept of improper fractions and its representations are not included in the objectives of the book and there is only one exercise in all the books of this class with the various forms of representation of improper fractions (Figure 7).

In the 5th grade of elementary school, improper fractions appear in Chapter 7, p. 26 of the student's book as an exercise in extending decimal numbers and decimal fractions. In this exercise (Figure 8) students are asked to represent 12/10 and 140/100 diagrammatically, symbolically and as a fraction on the geometric model of the number line. According to the teacher's book there is no goal for improper fractions.

The exercise of Figure 9below is presented in the chapter 16 "Fractional Units" p. 47 as an introductory activity. Students are asked to form 8/3 and 11/6 with the help of the given shapes. However, there is no relevant goal in the teacher's book.

Moreover, in the workbook of the same chapter p. 11 (first issue) is given an exercise in which students are asked to fill in the missing fraction in the first case and the fraction in the second case to form two whole units.

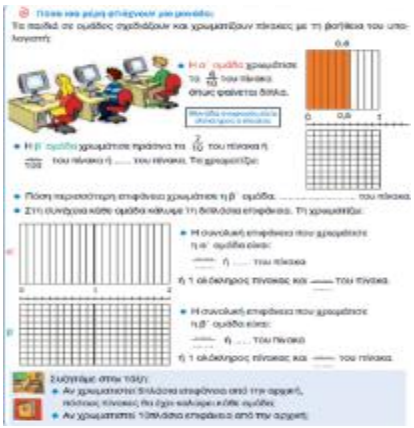

Figure8: Multiple representation of improper fractions

Also, in chapter 19 "Number Management Strategies" p. 53 of the student's book the students are given the diagrammatic representation of the improper fraction 5/4 and they are asked to write its symbolic form in half and twice the quantity (Figure 10). According to the teacher's book, the main goal of the chapter is for students to recognize numbers with different symbolic forms and to manage them, understanding the amount that these numbers express each time without the use of techniques, while the most specific goals are, among others, students to convert a mixed fraction to a simple one and to be able to make calculations with mixed fractions using the properties of operations. In other words, the emphasis is on the form of the mixed number and not on the form of the improper fraction. However, for the specific exercise in Figure 12, the teacher's book states: "Children can find other recipes and work with their own (variation)". They explain how they worked: with painting, with fractions (improper), with estimation. This is the first time the word "improper" is appeared, but no further instructions are given for the management of the term "improper" by teachers and students.

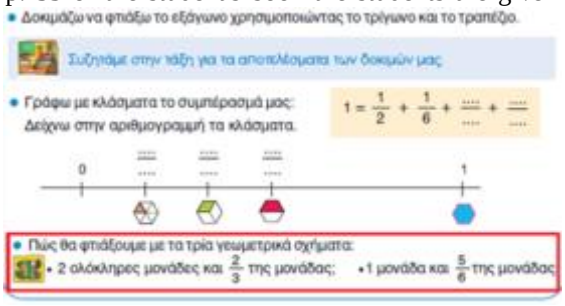

Figure9: Improper fractions through diagrammatic and symbolic representation

A similar exercise to that of Figure 10 is found in chapter 20 "Number Management" of the workbook p. 18 of second issue (Figure 11) with the difference that twice the quantity students must represent diagrammatically themselves, as it is not given like the other examples. It is the first exercise and the only one in the Greek mathematics textbooks in which students are asked to diagrammatically represent an improper fraction. In this chapter (chapter 20) as a prerequisite for students' knowledge the teacher's book on p. 104 mentions "[students] converting improper fractions into mixed numbers and vice versa" which is the aim of the previous chapter (chapter 19).

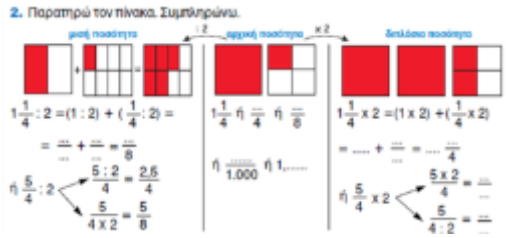

Figure10: Improperfractionswithdiagrammaticandsymbolic

Chapter 19 "Number Management Strategies" ends with the conclusion of Figure 12, according to which a quantity can be expressed in different ways and the example of the number "one and a half" is given, followed by its diagrammatic and symbolic representation.

In addition to the above examples, on pages 27, 40, 58, 59, 74, 75, 79, 89 of the student's book and on pages 21, 23, 27 of the first issue of the workbook, on pages 22, 35 of 2nd issue of the workbook and on pages 15, 18 of the 3rd issues of the workbook appear minimal improper fractions in their symbolic form, without any mention, simply to serve the teaching of the respective mathematical concept.

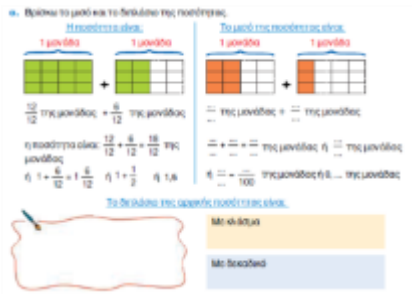

Figure11: Improperfractionsindiagrammaticandsymbolic representation

Summarizing the findings from the textbooks of the 5th grade, we observe that the concept of improper fractions appears for the first time defined as "improper" but only in the teacher's book, without again being an autonomous chapter. In total there are four exercises in the books of this class that present the improper fractions in different fields of representation.

In the 6th grade of elementary school, in chapter,3 emphasis is placed on decimal numbers and in this context, some improper fractions appear in their symbolic form, without any mention, simply to serve the teaching of decimal numbers and decimal fractions.

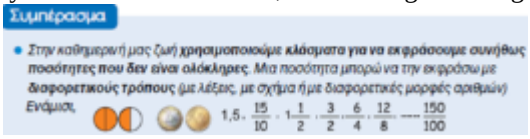

Figure12: Conclusion on the use of fractions with example 1.5

In chapter 19 "Fractions of the Same Name and Heteronyms" p. 45 of the student's book we have the first definition of improper fractions which is defined as a fraction whose numerator is greater than its denominator.

In the same chapter in the workbook p. 7 (2nd issue) an exercise is given in which the students from the diagrammatic representation must find the symbolic representation (Figure 13), while the improper fractions are also given in problems.

Figure13: Exercise for the transfer of improper fractions from the diagrammatic representation to the symbolic one

In chapter 20 "The Fraction as an Exact Quotient of Division" p. 47 of the student's book is given an activity that asks to place improper fractions in the number line by making the numbers decimal (Figure 14a), as one of the objectives of this chapter is to note its position fraction on the number line based on its decimal value. A similar activity is given in the workbook of the same chapter (Figure 14b).

In addition to the above examples, on pages 51, 56, 120 of the student's book, on page 11 of the 1st issue of the workbook and on pages 10, 11 of the 2nd issue of the workbook the improper fractions appear in various activities, since were taught and defined in previous chapters but only in their symbolic form.

Summarizing the findings from the 6th grade textbooks, we observe that the concept of improper fractions appears for the first time defined as a fraction whose numerator is greater than its denominator. There are a total of four exercises in the books of this class in which multiple representations are used for the concept under consideration.

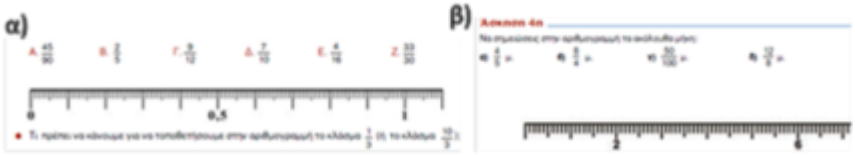

Figure14: Placing improper fractions on the number line.

From the above analysis it appears that the improper fractions appear in their majority in the elementary school textbooks in their symbolic form, while there are only nine activities in the 38 elementary school textbooks which present the improper fractions in other fields of representation. More specifically, the discrete units appear in only one activity in the 3rd grade, the position of the improper fractions in the geometric model of the number line appears in three activities in the 6th grade, and the other activities present the scope of the diagrammatic representation.

The significant difficulties that students face in understanding improper fractions can therefore be said to be due to the way this concept is represented in textbooks, as well as to the lack of fields of representation in textbooks. Besides, previous research studies have shown that the more often a student encounters a form of representation, the more familiar they become and the better they learn it [8],[10]. It would be good, therefore, to introduce more dynamically the concept of improper fraction in Greek elementary school textbooks and to make wider use of multiple representations, as this concept is related to the ability to place numbers on the number line, with the construction of fractional numbers that pave the way for the development of a sense of coherence and continuity of numbers [7], as well as the ability to construct and solve a project problem [4].

V. CONCLUSIONS

In the present study, all the chapters of the Greek textbooks of elementary mathematics that included the concept of the improper fraction were presented. Also, we presented the way in which this concept evolves within the textbooks of all six grades of elementary school, the representations with which the concept is introduced, as well as their frequency.

According to the research findings, students come in contact for the first time with the concept of improper fraction in 3rd grade through an activity of multiple representations (discrete units, diagram, and symbolic form) which is the only one in the books of 3rd grade. The same happens in the 4th grade. We generally observe that the concept of improper fractions and its representations are not included in the objectives of these two classes and therefore there is no autonomous chapter for this concept.

In the 5th grade, the activities presented by the improper fractions through multiple representations increase to four with the teacher's book mentioning the word "improper", but without giving more instructions. The conscious introduction of the students in this sense takes place in the 6th grade with the existence of a definition and the presence of four activities that manage different fields of representation, namely the diagrammatic, the symbolic and the number line.

In addition to the above activities, the improper fractions appear in their symbolic form in some exercises of the student book and the workbook from the 3rd grade to the 6th grade, mainly in the chapters where the decimal numbers are taught.

In conclusion, no chapters are devoted to the teaching of the improper fractions in the Greek primary school textbooks. Only in the 6th grade is this concept taught as part of the chapter on homonymous and heteronymous fractions. Therefore, they may highlight possible correlations between the structure of the books and the difficulties that students face in understanding the concept of improper fractions as shown by various studies. Besides, if a topic is not covered in the textbook, it is unlikely that it will be taught in the classroom [1].

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