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Research Paper

Mechanical Treatment of Elderly Parkinson Patient: Parallel Vs. Series Dual Vibration Absorbers

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ABSTRACT: *Diseases in the central nervous system lead to severe movement disorder as Parkinson's tremor. Symptoms of Parkinson's disease can mostly start to appear in an average age of 59 years. Mechanical treatment is suggested to help elderly people who exhibit spatial and temporal movement variability. Two degree-of-freedom (DOF) parallel and series vibration absorbers are designed to reduce the flexion motion for patients suffering from resting tremor. Numerical studies are based on a three DOF biodynamic modeling of human hand in horizontal plane. The modeled system considers the flexion-extension planar motion of the shoulder, elbow and wrist joints. Tremor is modeled as sinusoidal function driven at two resonance frequencies with white Gaussian noise added to the signal. Equations of motion are derived and solved using Cholesky decomposition for the uncontrolled and using Runge-Kutta method for the controlled dynamically coupled systems. The two DOF parallel absorbers show better results which cause 54.39–69.81%, 16.40–40.93% and 12.0–42.76% reduction at the shoulder, elbow and wrist joints. The parallel dual absorber was safe against failure by yielding and fatigue. However, the series dual absorber having lower performance was critical to failure.*

Keywords: *Cholesky, absorber, Parkinson, white noise*

I. INTRODUCTION

Tremor associated with natural processes is referred as physiological tremor and is present in all humans [1, 2]. Physiological tremor can exist due to the effect of ballistocardiogram, i.e., the mechanical activity of the heart producing passive vibration of the body tissues [3]. It presents high frequency 8–12 Hz [4] with low amplitude. On the other hand, tremor can be associated to neurological disorders and is referred as pathological tremor, like cerebellar, essential and Parkinsonian tremor [5]. Pathological tremor can be due to distortions and amplifications of neural oscillatory activity in the central nervous system [6] and dysfunction of neuromuscular system. Involuntary tremor is characterized by repetitive and stereotyped movements.

The English doctor James Parkinson published in 1817 the first well defined description of Parkinson disease (PD) in his article titled 'An Essay on the Shaking Palsy' [7]. PD is a multi-system neurodegenerative disorder caused by the lack in the level of dopamine within the brain and affects the brain's control of the muscles. It leads to shaking (tremor), increased muscle tone (stiffness), slowed movements and balance problems. Parkinsonian tremor is usually a resting tremor which appears when muscles are relaxed and limbs are supported against gravity [2]. Rest tremor is characterized by adduction-abduction or flexion-extension motion with frequency 3–7 Hz [8].

Some studies denote that the reduced brain volume of older verse young adults is correlated with the motor deficits. Older adults have less consistent actions in comparison with young adults [9–11]. The spatial and temporal movement variability may arise from peripheral changes in the neuromuscular system [12]. Older adults exhibit also deficits in coordination of bimanual and multi joint movements. Moving their shoulder and elbow joints simultaneously becomes slower and less smooth as opposed to performing single joint actions [13]. PD incidence increases with aging. It is difficult to specify the exact number of young age affected patients, even though they are rarely affected by this illness.

Human tremor is a public health problem that can lead to social and physical deterioration. Initial symptoms may initiate early in life, but tremor progresses with time and becomes significant to physician when the patient is elderly. The disease can be detected for patients above 40 years with an estimated 5–10% of the all Parkinson patients. Symptoms are mostly diagnosed in the age of 60–70 with an average age of 59 years. As

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shown in "Fig. 1", the incidence of PD increases sharply above the age of 60 years. Through the age of 85, the incidence rate consistently increases [15]. The effect is greater on man than the women with proportions which vary with age. Billions of dollars are invested in the last decade on medical and social aspects in many countries, to diagnose and treat such tremor. In the United State, about 630,000 people had diagnosed PD in 2010. The national economic burden of PD exceeds $14.4 billion in 2010 (approximately $22,800 per patient) [16].

Initial symptoms of PD are usually visualized as hands trembling with the resting tremor. Dopaminergic drugs are medications which aim to temporarily restoring dopamine level in the substantia nigra and striatum. Thereby reduces motor symptoms and signs of PD without curing its progression.

Levodopa is the most effective dopaminergic medication for elderly patients which is converted in the brain into dopamine. It is given in combination to carbidopa medication to reduce the side effects. The given dose of drugs increases with symptoms of PD, but high doses can produce involuntary tremor at peak medication levels and lead to serious side effects. After several years, patients experience motor fluctuation where motor symptoms are more difficult and they becomes non responsive to medication. In addition, about 25% of Parkinsonian patients can lose their life’s quality since they don’t respond to drugs or neurosurgery treatments [17, 18].

Vibration absorbers can be used as a mechanical treatment for tremor reduction of elderly Parkinson patients. Absorbers can transmit the tremor into vibration of its proof mass. Mechanical oscillations are considered as the main source tremor, in which joints and muscle movements satisfy the laws of physics. Oscillations can be translated into the movements of masses and springs due to the nature of the complex joint-muscle-tendon system [19]. Therefore, a dynamic model of human hand can describe its biodynamic response at musculoskeletal level and used for numerical testing of the absorber’s performance.

Method of biodynamic response is recently being used to describe the motion of human hand and solve problems related to tremor at musculoskeletal level. Hashemi et al. [20] modeled human hand as two degree-of-freedom (DOF) system in horizontal plane with flexion-extension planar motion at shoulder and elbow frictionless joints. Hand segments are described as uniform rigid rods with masses concentrated at centroid and inertia. The system reflects the resting tremor of Parkinson patients under the effect of elbow muscle activation. Single DOF tuned vibration absorber (TVA) was designed and attached to the forearm. It was able to reduce tremor amplitude numerically and in the fabricated system’s model. Igusa [21] studied the multiple mass dampers tuned within frequency range. He found that it is more robust than tuned mass damper with same total mass. Brennan [22] demonstrated the used of parallel multiple TVA, tuned at slightly different frequencies, which results in an improved broadband device. Zuo [23] proposed a series combination between multiple TVA. This configuration was more efficient that single DOF TVA. He provided ready-to-use design charts for the use of two TVAs in series. Gebai et al. [24] have designed a single passive DVA that was able to suppress the steady state response of the Parkinson’s tremor of the hand excited at its fundamental frequency. Moreover, they used a single DVA that was able to help elderly patients suffering from neurodegenerative disorder by reducing the homogenous as well as the steady state response of the involuntary tremor at the hand joints [25].They also suggested a dual parallel DVA to reduce the pathological tremor of Parkinson patients excited at dual harmonic resonance frequencies [26, 27]. The study reveals that the dual DVA was very effective in reducing pathological tremor.

In this study, a three DOF hand model system is used to reflect the biodynamic response of elderly patients suffering from PD. Flexion-extension planar motion is considered at shoulder, elbow and wrist frictionless joints in horizontal plane. Movements are produced due to the activation of shoulder and elbow muscles modeled to have sinusoidal behavior with white Gaussian noise added to the signal. A multi-DOF absorber is suggested to attenuate the motion of the multi-DOF vibrating hand system without severe side effect. The dual dynamic vibration absorber (DVA) is studied in terms of its capability in reducing the amplitude of fle-

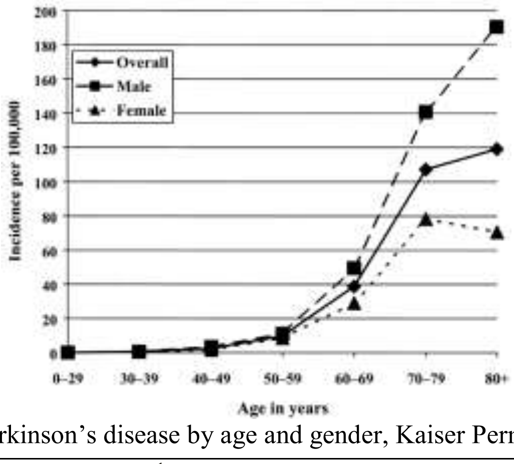


Figure 1. Incidence of Parkinson’s disease by age and gender, Kaiser Permanente, 1994–1995 [14].

xion motion of Parkinsonian elderly patient when attached to the forearm. A comparison is done between the performance of the parallel and series connected proof masses in the dual DVA. Equations of motion are derived for the non-Lagrangian dissipative system. The response of the dynamically coupled is solved in time domain using Cholesky decomposition for the uncontrolled system and using fourth order Runge-Kutta method. Its behavior is analyzed numerically for the controlled and uncontrolled hand model in time and frequency domains. The absorbers are tested mathematically against failure by yielding and fatigue to check its expected capability in withstanding the involuntary tremor if it is used for experimental study.

II. BIODYNAMIC HAND MODEL

2.1 Primary Hand System

In order to solve problems related to the resting tremor at the hand of elderly patients, the upper arm and ulna and radius of forearm are modeled as truncated cones and the palm as rectangular plates. Sizing these segments is based on data provided experimentally for the right hand as shown in "Table 1" [28]. Based on the geometry and parameters collected, the mass moment of inertia is calculated at the centroid of the upper arm, forearm and the palm, respectively as:

$$I_1 = 0.0228 kg.m^2, I_2 = 0.0082\, kg.m^2,\ I_3 = 0.0012 kg.m^2 \quad (1)$$

Hand segments are modeled to be linked by one DOF frictionless joints. Shoulder, elbow and wrist joints are described as pivot, hinge and saddle joints "Fig. 2". Movements are produced by shoulder, elbow and wrist single joint muscles and biceps brachii double joint muscle. Muscles are modeled mechanically as spring-damper system as shown in "Table 2" and assumed to be regulated independently.

Table 1. Parameters of hand segments.

Right Hand	Length (cm)		Mass (kg)		Density (kg/m^3)	Centroid (m)	
Upper arm	l_1	36.4	m_1	2.070	2.070	a_1	0.427 l_1
Forearm	l_2	29.9	m_2	1.160	1.160	a_2	0.417 l_2
Palm	l_4	20.3	m_3	0.540	0.540	a_3	0.361 l_3

The lengths l_1, l_2, l_3, masses m_1, m_2, m_3 and positions at centroid a_1, a_2, a_3 correspond to the upper arm, the forearm and the palm, respectively.

The dynamic modeling shown in "Fig. 3" represent the three DOF flexion motion hand joints and the vibration absorber attached at the forearm in horizontal plane. The system is at stable equilibrium when the flexion motions at shoulder, elbow and wrist joints are respectively: $\theta_1 = 45°, \theta_2 = 90° and\ \theta_3 = 0°$ [20]. To avoid the use of Jacobian transformation matrix, equations of motion are derived at the instant where $\theta_1 = 0°, \theta_2 = 90° and\ \theta_3 = 0°$ with zero initial angular velocities.

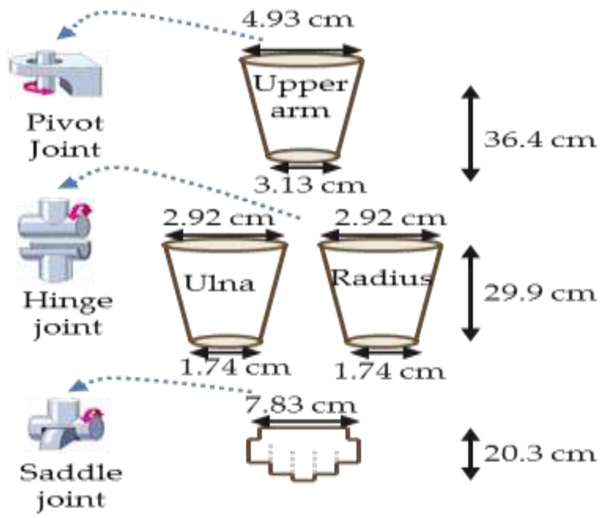


Figure 2. Geometry of hand segments and their corresponding joints.

Coriolis' theorem is used to determine the kinematics of the system at points representing the location of concentrated masses. The equations are derived for the non-Lagrangian dissipative system in the form of:

$$[M]\{\ddot{\theta}\}+[C]\{\dot{\theta}\}+[K]\{\theta\}=\{f\} \tag{2}$$

$\theta, \dot{\theta}, \ddot{\theta}$ are the angular displacement, velocity and acceleration vectors at hand joints. $\theta=\{\theta_1 \quad \cdots \quad \theta_n\}^T$ is the angular flexion angle for the n DOF system.

For the uncontrolled hand system, $\theta_1, \theta_2, \theta_3$ corresponds to the flexion angle at shoulder, elbow and wrist joints respectively as shown in "Fig. 3".

Table 2. Stiffness and damping coefficients of the muscles.

Muscle	Shoulder	Elbow	Biceps	Wrist
$k\,(N.m/rd)$	180	70	40	10
$c\,(N.m.s/rd)$	0.002 k_1	0.002 k_2	0.002 k_3	0.002 k_4

k_1, k_2, k_3 are the stiffness coefficients of the shoulder, elbow, biceps brachii, wrist joint muscles.

Mass Matrix:

$$M=\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$M_{11}=\left(I_1+m_1a_1^2\right)+\left(I_2+m_2a_2^2\right)+m_2l_1^2+m_d\left(l_1^2+l_d^2\right)+m_3\left(l_1^2+l_2^2+a_3^2+2l_2a_3\right) \tag{3}$$

$$M_{12}=\left(I_2+m_2a_2^2\right)+m_d l_d^2+m_3\left(l_2^2+a_3^2+2l_2a_3\right),\quad M_{13}=m_3\left(a_3^2+l_2a_3\right)$$

$$M_{23}=M_{13},\ M_{31}=M_{13},\ M_{32}=M_{23},\ M_{33}=I_3+m_3a_3^2$$

m_d is the mass of the device holding the absorber l_d is the distance from elbow joint to controlled device along the forearm.

$$K=\begin{bmatrix} k_1+k_3 & k_3 & 0 \\ k_3 & k_2+k_3 & 0 \\ 0 & 0 & k_4 \end{bmatrix} \tag{4}$$

$$C=\begin{bmatrix} c_1+c_3 & c_3 & 0 \\ c_3 & c_2+c_3 & 0 \\ 0 & 0 & c_4 \end{bmatrix} \tag{5}$$

k_1, k_2, k_3, k_4 and c_1, c_2, c_3, c_4 are respectively the stiffness and damping coefficients of the shoulder, elbow, biceps brachii, wrist joint muscles

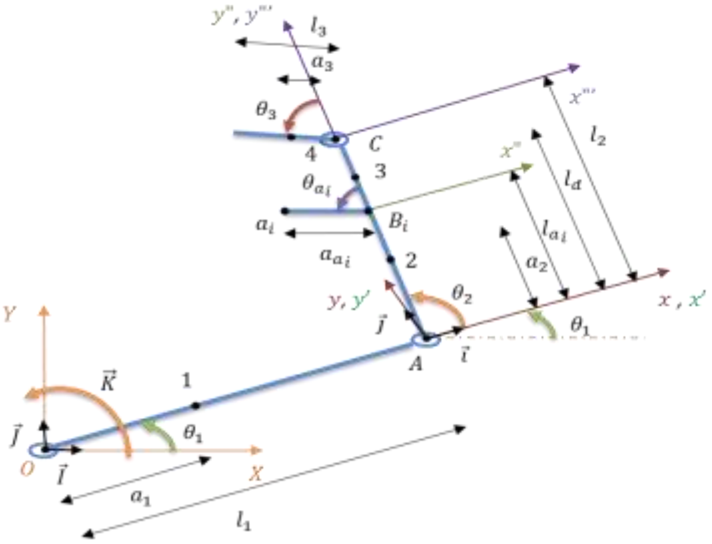


Figure 3. Dynamic modeling of the human hand.

$$f=\{f_1 \quad f_2 \quad 0\}^T \tag{6}$$

f_1, f_2 are the input moments due to shoulder and elbow muscles activation, respectively. Both are driven (ω) at the first two smallest frequencies of the primary system (ω_n) as sinusoidal function with white noise effect representing the resting tremor:

$$f_k = F_{k_1}\cos(\omega_1 t)+F_{k_2}\cos(\omega_2 t),\ k=\{1,2\} \tag{7}$$

$$F_{k_1}=F_{k_2}=1\,N.m,\ \omega_1=\omega_{n_1}\ and\ \omega_2=\omega_{n_2} \tag{8}$$

Using the characteristic equation of the free undamped system, the natural frequencies of the uncontrolled system can be determined:

$$\omega_{n_1}=3.569Hz,\ \omega_{n_2}=5.300Hz\ and\ \omega_{n_3}=12.566Hz \tag{9}$$

2.2 Uncontrolled System's Response

Modal analysis can be used for systems with positive definite diagonal mass matrix to determine the steady state response and added to the homogenous response by satisfying the initial conditions to form the total closed form response in time domain. Dynamically coupled systems needs sophisticated means to handle and factorize the inverse symmetric non-diagonal mass matrix. It is preferred to avoid computation of the square root of such matrix. Cholesky factorization is favorable in this case [29]. Cholesky decomposition method finds a lower triangular matrix L for factorizing a positive definite mass matrix:

$$[M]=LL^T \tag{10}$$

$i=\{1,\cdots,n\}$ and $j=\{i+1,\cdots,n\}$, where *n* is the DOF:

$$\begin{cases} L_{ii}=\sqrt{\left(M_{ii}-\sum_{k=1}^{i=1}L_{ik}^2\right)} \\ L_{ji}==\dfrac{M_{ji}-\sum_{k=1}^{i=1}L_{jk}L_{ik}}{L_{ii}} \\ L_{ij}=0 \qquad i>j \end{cases} \tag{11}$$

For the free undamped system of (2), let $\theta(t)=\left(L^T\right)^{-1}q(t)$, then:

$$I_n\ddot{q}+\tilde{K}q=0 \tag{12}$$

$\tilde{K}=L^{-1}K\left(L^T\right)^{-1}$ is the symmetric mass normalized stiffness matrix. Substituting $q(t)=ve^{j\omega t}$ in (12) yields the symmetric eigenvalue problem:

$$\tilde{K}v=\lambda v\ and\ \lambda_j=\omega_j^2, \qquad \text{for } j=\{1,\cdots,n\} \tag{13}$$

λ, v are the eigenvectors and the eigenvalue of $\tilde{K}$.

Modal analysis using Cholesky can be used if and only if the considered system has proportional damping coefficient matrices so that:

$$C=\gamma M+\beta K,\ \text{ i.e. } \tilde{C}I_n\tilde{K}=\tilde{K}I_n\tilde{C} \tag{14}$$

λ, β are real constants. I_n is the $n\times n$ identity matrix. $\tilde{C}=L^{-1}C\left(L^T\right)^{-1}$ is the mass normalized damping coefficient matrix.

Equation (14) is satisfied for the uncontrolled system, so its equations of motion in (2) can be decoupled.

Let $q(t)=\Pr(t)$, then (2) yields the modal equation:

$$I_n\ddot{r}+P^T\tilde{C}P\dot{r}+P^T\tilde{K}Pr=\bar{f} \tag{15}$$

$r=\{r_1 \quad r_2 \quad \cdots \quad r_n\}^T$ is the modal coordinates of θ and $P=[v_1 \quad v_2 \quad \cdots \quad v_n]$ is the orthonormal eigenvectors matrix of $\tilde{K}$.

Hence, the modal transformation matrix $S=\left(L^T\right)^{-1}P$ is obtained and the modal force (moment) $\bar{f}=S^Tf$ can be calculated, $\bar{f}=\{\bar{f}_2 \quad \bar{f}_2 \quad \cdots \quad \bar{f}_n\}^T$. Note that:

$$P^T \tilde{K} P = diag\left(\omega_j^2\right) \; and \; \mathrm{P}^\mathrm{T} \tilde{C} P = diag\left(2\zeta_j \omega_{n_j}\right) \tag{16}$$

The n-decoupled equations of motion of (2) are in the form of:

$$\ddot{r}_j + 2\zeta_j \omega_{n_j} \dot{r}_j + \omega_j^2 r = \bar{f}_j \tag{17}$$

For hand vibrating as an under-damped system $\zeta_i > 1$, solving (17) gives directly the response of the system in the modal coordinates as:

$$r_j(t) = R_j \cos\left(\omega t - \varphi_j\right) \tag{18}$$

$$R_j = \frac{\bar{f}_j}{\sqrt{\left(\omega_{n_j}^2 - \omega^2\right)^2 + \left(2\zeta_j \omega_{n_j} \omega\right)^2}} \; and \; \varphi_j = \tan^{-1}\left(\frac{2\zeta_j \omega_{n_j} \omega}{\omega_{n_j}^2 - \omega^2}\right) \tag{19}$$

R, φ are the magnitude and the phase angle of the response in modal coordinates.

Finally, the response in the physical coordinates is calculated using the transformation matrix S :

$$\theta(t) = Sr(t) \tag{20}$$

The response of the multi-input force (moment) system is the summation of responses due to the forces (moments) having different driving frequencies.

III. TREMOR REDUCTION

3.1 Vibration Absorber

A double vibration absorber is suggested to reduce the amplitude of Parkinsonian tremor at the hand of elderly patients which is modeled to vibrate under two driving frequencies. The dual DVA contains two proof masses, each mass serves in reducing the vibration at a specified undesired tremor frequency and transmitting the vibration of the system of interest into oscillation of the corresponding proof mass. Each single absorber: ‘absorber 1’ and ‘absorber 2’ of the dual absorber is designed as stainless steel alloy cantilevered beam with copper proof mass attach along its length having the configuration shown in "Fig. 4". The dual DVA is considered with m_d = 52g controller device. The device must be designed to have slider along a circular path to prevent the beams torsional motion. It allows the flexion-extension angular displacements only. Each designed dual DVA is studied when placed at:

$$l_d = l_a = l_2 - 14.3cm \tag{21}$$

l_d, l_a are the distance along the forearm from elbow joint to controller device and absorber’s joint, respectively. It is shifted 14.3cm away from wrist joint to avoid interference of both the dual absorber with the oscillating palm.

The effective mass of the designed single absorbers is:

$$m_{a_i} = m_0 + 0.23 m_{beam}, \;\; i = \{1, 2\} \tag{22}$$

$m_{a_1}, m_{a_2}, m_0, m_{beam}$ are the effective masses of ‘absorber 1’, ‘absorber 2’ and their corresponding mass of the copper block and the cantilevered beam, respectively.

Two absorbers are tested for reducing tremor amplitude: the parallel and series dual DVA having the dimensions shown in "Table 3". Dunkerley’s semi-empirical formulation [30] is used to approximate the lower band natural frequency of each single absorber system in order to determine the position of the copper mass along the beam at tuning condition $\left(\omega_{a_i} = \omega_i\right)$. Dunkerley’s equation:

$$\frac{1}{\omega_{a_i}^2} \approx \frac{1}{\omega_{beam}^2} + \frac{1}{\omega_{m_0}^2} \tag{23}$$

ω_a, ω_{beam} are the fundamental frequency of the absorber’s system and the Euler Bernoulli cantilevered beam. ω_{m_0} is the natural frequency due to the static deflection of the copper proof mass on the beam.

Then, (23) yields the control equation used to evaluate distance from absorber’s joint to the proof mass along the beam (a_{ai}) for each single absorber in the dual DVA, the control equation is:

$$\frac{1}{\omega_{a_i}^2} \approx \frac{m_{beam} L_{beam}^3}{3.5160^2 \times E_{beam} I_{beam}} + \frac{m_0 a_d^2 \left(3L_{beam} - a_{ai}\right)}{6E_{beam} I_{beam}} \tag{24}$$

The elastic modulus and the density of the steel beam are $E_{beam} = 189.6GPa$ and $\rho_{beam} = 7800\, kg/m^3$. The density of the copper mass is $\rho_{m_0} = 8900\, kg/m^3$. I_{beam} is the cross-sectional moment of inertia of the beam.

Each parallel and series dual DVA is formed of two single absorbers ('absorber 1' and 'absorber 2'). Absorber 1' is tuned to the fundamental frequency of the primary system $(\omega_{a_1} = \omega_1)$ at the shoulder's response to shoulder muscle activation. 'Absorber 2' is tuned to the second natural frequency $(\omega_{a_2} = \omega_2)$ at the palm's response due to elbow muscle activation. Substitution in (24) yields the position of the copper mass at 'absorber1' as $a_{a1} = 6.772cm$ and its position at 'absorber 2' as $a_{a2} = 5.772cm$. Both single absorbers are hold under one common controller device having m_d . The effective mass of 'absorber 1' is calculated to be $m_{a_1} = 122.1g$ and that of 'absorber 2' is $m_{a_2} = 121.9g$.

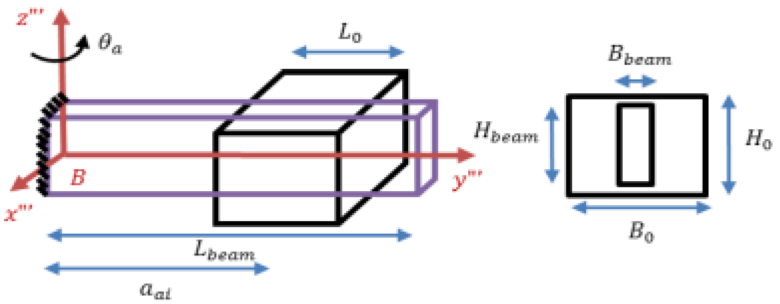


Figure 4. Modeled single degree of freedom vibration absorber.

Table 3. Dimensions of the single absorbers.

Dimensions		**L** (*cm*)	**H** (*cm*)	**B** (*cm*)
Absorber 1	Beam	8	1.9	0.03
	Attached mass	2.2	2.5	2.5
Absorber 2	Beam	6.3	1.9	0.03
	Attached mass	2.2	2.5	2.5

L, H and B are the length, height and base of the absorber's system shown in "Fig. 4".

Absorber's mass moments of inertia are not considered due to their negligible values. The total mass of each absorber (effective mass and the controller device) is designed by considering the acceptable mass ratio range as the absorber's constraint:

$$0.05 < \mu_a < 0.1, \qquad \mu_a = \frac{m_{a_i} + m_d}{m_p} \tag{25}$$

m_p is the total mass of the primary system, such that:

$$m_p = m_1 + m_2 + m_3 \tag{26}$$

3.1.1 Parallel Dual DVA

It is formed of two single DVAs connected in parallel. The two single DVAs have the same position of absorber's joint (21) and separated by a small distance perpendicular to the longitudinal axis of the forearm in order to avoid interference between the absorbers. Same strategy is used to derive the equations of motion for the primary mass with the parallel dual DVA attached at the forearm in the form of (2) at the instant where 'absorber 1' and 'absorber 2' initial conditions are respectively $\theta_{a_1} = 0^\circ$ and $\theta_{a_2} = 0^\circ$ with zero initial velocity, where:

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} \end{bmatrix} \tag{27}$$

$$M_{11}=\left(I_1+m_1a_1^2\right)+\left(I_2+m_2a_2^2\right)+m_2l_1^2+m_d\left(l_1^2+l_d^2\right)+m_3\left(l_1^2+l_2^2+a_3^2+2l_2a_3\right)$$
$$+m_{a_1}\left(l_1^2+l_{a_1}^2+a_{a1}^2+2l_{a_1}a_{a1}\right)+m_{a_2}\left(l_1^2+l_{a_2}^2+a_{a2}^2+2l_{a_2}a_{a2}\right)$$
$$M_{12}=\left(I_2+m_2a_2^2\right)+m_dl_d^2+m_3\left(l_2^2+a_3^2+2l_2a_3\right)+m_{a_1}\left(l_{a_1}^2+a_{a1}^2+2l_{a_1}a_{a1}\right)+m_{a_2}\left(l_{a_2}^2+a_{a2}^2+2l_{a_2}a_{a2}\right)$$
$$M_{13}=m_3\left(a_3^2+l_2a_3\right),\ M_{14}=m_{a_1}\left(a_{a1}^2+l_{a_1}a_{a1}\right)$$
$$M_{15}=m_{a_2}\left(a_{a2}^2+l_{a_2}a_{a2}\right),\ M_{21}=M_{12},\ M_{22}=M_{12}$$
$$M_{23}=M_{13},\ M_{24}=M_{14},\ M_{25}=M_{15},\ M_{31}=M_{13}$$
$$M_{32}=M_{23},\ M_{33}=I_3+m_3a_3^2,\ M_{34}=0,\ M_{35}=0$$
$$M_{41}=M_{14},\ M_{42}=M_{24},\ M_{43}=M_{34},\ M_{44}=m_{a1}a_{a1}^2$$
$$M_{45}=0,\ M_{51}=M_{15},\ M_{52}=M_{25},\ M_{53}=M_{35}$$
$$M_{54}=M_{45},\ M_{55}=m_{a_2}a_{a2}^2$$

$$K=\begin{bmatrix} k_1+k_3 & k_3 & 0 & 0 & 0 \\ k_3 & k_2+k_3 & 0 & 0 & 0 \\ 0 & 0 & k_4 & 0 & 0 \\ 0 & 0 & 0 & k_{a_1} & 0 \\ 0 & 0 & 0 & 0 & k_{a_2} \end{bmatrix} \tag{28}$$

$$C=\begin{bmatrix} c_1+c_3 & c_3 & 0 & 0 & 0 \\ c_3 & c_2+c_3 & 0 & 0 & 0 \\ 0 & 0 & c_4 & 0 & 0 \\ 0 & 0 & 0 & c_{a_1} & 0 \\ 0 & 0 & 0 & 0 & c_{a_2} \end{bmatrix} \tag{29}$$

$$f=\{f_1 \quad f_2 \quad 0 \quad 0 \quad 0\}^T \tag{30}$$

k_{a_1}, k_{a_2} and c_{a_1}, c_{a_2} are respectively the stiffness and damping coefficients of 'absorber 1' and 'absorber 2' of the parallel dual DVA. The very small damping coefficient values can be obtained from beam's material without adding any damper. Their designed values at tuning conditions are:

$$k_{a_1}=0.339\,N.m/rd \ \ and \ \ k_{a_2}=0.290\,N.m/rd \tag{31}$$
$$c_{a_1}=0.005k_{a_1} \ \ and \ \ c_{a_2}=0.005k_{a_2} \ \text{in} \ N.m.s/rd \tag{32}$$

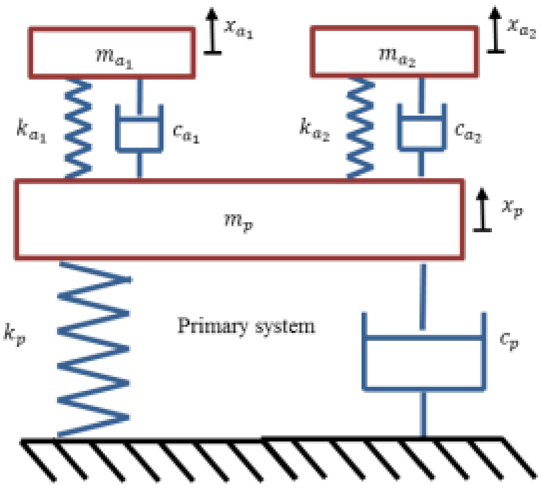


Figure 5. Two degree of freedom parallel vibration absorber.

The obtained natural frequencies of this five DOF system are:

$$\omega_{n_1}=3.260Hz, \omega_{n_2}=3.755Hz, \omega_{n_3}=4.955Hz, \omega_{n_4}=12.705Hz \tag{33}$$

The controlled hand system with the parallel dual DVA can have its equivalent dynamic molding after converting parameters corresponding to angular motion into that of linear motion. "Fig. 5" shows model of the parallel two DOF system having linear motion and attached to a modeled primary system.

3.1.2 Series Dual DVA

It is formed of two single DVAs connected in series. ‘Absorber 1’ single DVA of the series dual DVA is attached to the forearm at position of (21) and its ‘absorber 2’ is attached to the copper mass of the ‘absorber 1’ to have a series connection between both copper masses. The series connection between these two single absorbers produces a long absorber with maximum length of 8+6.3=14.3*cm*. The equation of motion for the primary hand system with the series dual DVA attached at the forearm is derived in the form of (2) at the instant where ‘absorber 1’ and ‘absorber 2’ initial conditions are respectively $\theta_{a_1} = 0^\circ$ and $\theta_{a_2} = 0^\circ$ with zero initial velocity, where:

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} \end{bmatrix}$$

$$\begin{gathered} M_{11} = \left(I_1 + m_1 a_1^2\right) + \left(I_2 + m_2 a_2^2\right) + m_2 l_1^2 + m_d\left(l_1^2 + l_d^2\right) \\ + m_3\left(l_1^2 + l_2^2 + a_3^2 + 2l_2 a_3\right) + m_{a_1}\left(l_1^2 + l_{a_1}^2 + a_{a1}^2 + 2l_{a_1} a_{a1}\right) + m_{a2}\left(l_1^2 + \left(l_{a_1} + a_{a_1} + a_{a2}\right)^2\right) \\ M_{12} = \left(I_2 + m_2 a_2^2\right) + m_d l_d^2 + m_3\left(l_2^2 + a_3^2 + 2l_2 a_3\right) + m_{a_1}\left(l_{a_1}^2 + a_{a1}^2 + 2l_{a_1} a_{a1}\right) \\ + m_{a_2}\left(l_{a_1} + a_{a1} + a_{a2}\right)^2,\ M_{13} = m_3\left(a_3^2 + l_2 a_3\right) \\ M_{14} = m_{a_1}\left(a_{a1}^2 + l_{a_1} a_{a1}\right) + m_{a_2}\left(l_{a_1} + a_{a1} + a_{a2}\right)\left(a_{a1} + a_{a2}\right) \\ M_{15} = m_{a_2}\left(l_{a_1} a_{a2} + a_{a1} a_{a2} + a_{a2}^2\right) \\ M_{21} = M_{12},\ M_{22} = M_{12}, M_{23} = M_{13}, M_{24} = M_{14}, M_{25} = M_{15} \\ M_{31} = M_{13}, M_{32} = M_{23},\ M_{33} = I_4 + m_4 a_4^2, M_{34} = 0, M_{35} = 0 \\ M_{41} = M_{14},\ M_{42} = M_{24},\ M_{43} = 0,\ M_{44} = m_{a_1} a_{a1}^2 + m_{a_2}\left(a_{a1} + a_{a2}\right)^2 \\ M_{45} = m_{a2}\left(a_{a1} a_{a2} + a_{a2}^2\right),\ M_{51} = M_{15},\ M_{52} = M_{25}\ M_{53} = 0, M_{54} = M_{45},\ M_{55} = m_{a_2} a_{a2}^2 \end{gathered} \tag{34}$$

$$K = \begin{bmatrix} k_1 + k_3 & k_3 & 0 & 0 & 0 \\ k_3 & k_2 + k_3 & 0 & 0 & 0 \\ 0 & 0 & k_4 & 0 & 0 \\ 0 & 0 & 0 & k_{a_1} & 0 \\ 0 & 0 & 0 & 0 & k_{a_2} \end{bmatrix} \tag{35}$$

$$C = \begin{bmatrix} c_1 + c_3 & c_3 & 0 & 0 & 0 \\ c_3 & c_2 + c_3 & 0 & 0 & 0 \\ 0 & 0 & c_4 & 0 & 0 \\ 0 & 0 & 0 & c_{a_1} & 0 \\ 0 & 0 & 0 & 0 & c_{a_2} \end{bmatrix} \tag{36}$$

$$f = \{f_1 \quad f_2 \quad 0 \quad 0 \quad 0\}^T \tag{37}$$

k_{a_1}, k_{a_2} and c_{a_1}, c_{a_2} are respectively the stiffness and damping coefficients of ‘absorber 1’ and ‘absorber 2’ of the series dual DVA. To keep the absorbers safe from large vibration magnitude induced due to the series connection, the damping coefficient is needed to be increased to reduce absorber’s fluctuation. Beams material is suggested to be coated by a damping material that can increase beam’s damping ratio. Their designed stiffness and damping coefficient values are:

$$k_{a_1} = 1.169\,N.m/rd \ \ and\ \ k_{a_2} = 0.346\,N.m/rd \tag{38}$$

$$c_{a_1} = 0.02k_{a_1} \;\; and \;\; c_{a_2} = 0.007k_{a_2} \text{ in } N.m.s/rd \tag{39}$$

The obtained natural frequencies of this five DOF system are:

$$\omega_{n_1} = 2.937Hz, \omega_{n_2} = 3.577Hz, \omega_{n_3} = 5.228Hz, \omega_{n_4} = 12.785Hz \tag{40}$$

The controlled hand system with the series dual DVA can have its equivalent dynamic molding after converting parameters corresponding to angular motion into that of linear motion. "Fig. 6" shows model of the parallel two DOF system having linear motion and attached to a modeled primary system.

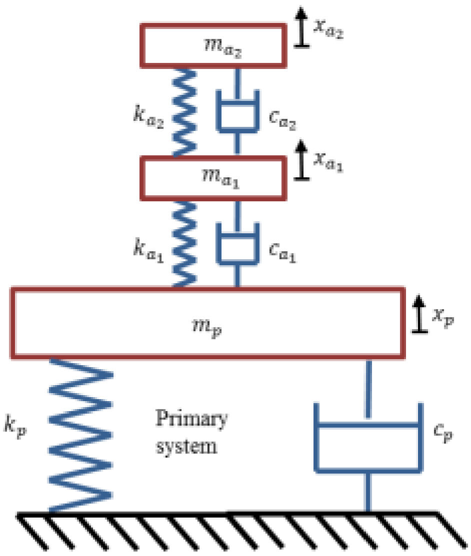


Figure 6. Two degree of freedom series vibration absorber.

3.2 Vibration Absorber

The response in time domain for the parallel and series dual DVA can't be determined from the derived modal analysis using Cholesky that was used in the uncontrolled system. It is due to the fact that the controlled equations of motion are not able to be decoupled in the modal coordinates, since the condition of (14) is not satisfied. The fourth order Runge-Kutta iterative method is used to determine the total response in time domain for the multi-DOF dynamically coupled systems with non-proportional damping coefficient matrices.

Equation (2) can be written in the form:

$$\ddot{\theta}(t) = [M]^{-1}\left(f(t) - [C]\dot{\theta}(t) - [K]\theta(t)\right) \tag{41}$$

The time is $t_i = ih$, where the time step $h = 0.001s$ is used to approach the exact solutions.

Let $X = \begin{Bmatrix} \theta(t) \\ \dot{\theta}(t) \end{Bmatrix}$ for $\theta = \{\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_{a1} \;\; \theta_{a2}\}^T$. Substitute in (41) to obtain:

$$g(X,t) = \begin{bmatrix} [0] & I_n \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} + \begin{Bmatrix} 0 \\ [M]^{-1} f(t) \end{Bmatrix} \tag{42}$$

Fourth order Runge-Kutta:

$$\begin{gathered} X_{i+1} = X_i + \frac{1}{6}[Kr_1 + 2Kr_2 + 2Kr_3 + Kr_4] \\ Kr_1 = hg(X_i, t_i) \\ Kr_2 = hg\left(X_i + \frac{1}{2}Kr_1, t_i + \frac{1}{2}h\right) \\ Kr_3 = hg\left(X_i + \frac{1}{2}Kr_2, t_i + \frac{1}{2}h\right) \\ Kr_4 = hg(X_i + Kr_3, t_{i+1}) \end{gathered} \tag{43}$$

IV. ANALYSIS OF NUMERICAL RESULTS

4.1 Behavior in Frequency Domain

The frequency domain response is used to show the behavior of the system over a range of driving frequencies with white noise vibration. The response of the uncontrolled primary system at shoulder, elbow and wrist joints will be compared to their response due to the addition of the dual parallel and series DVA. The response in frequency domain is derived using the complex transfer function of the system. Using (2), for the sinusoidal input moments, the transfer function of the system is:

$$H(\omega)=\left\{\left(-\omega^2[M]+[K]\right)+j\omega[C]\right\}^{-1} \tag{44}$$

Then, the magnitude of the response is obtained as:

$$\Theta=\sum_{k=1}^{2}\sum_{m=1}^{2}\alpha_k F_{k_m} \tag{45}$$

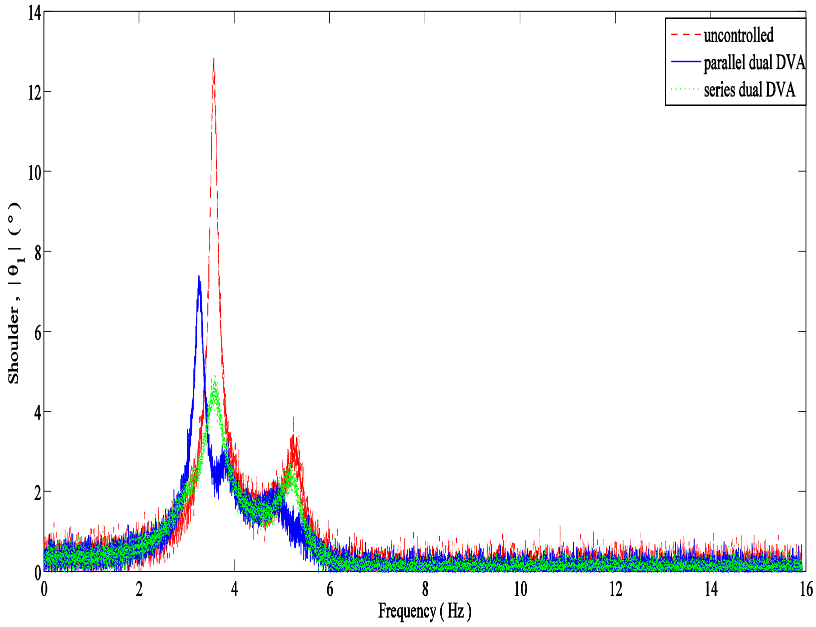
α is the Receptance transfer function, such that $H(\omega)=\{\alpha_1 \quad \cdots \quad \alpha_n\}$.

"Fig. 7" shows the behavior in frequency domain. Both parallel and series dual absorbers are very effective at shoulder joint "Fig. 7a", but the parallel DVA was better in reducing the amplitudes at peaks. In "Fig. 7b", the series DVA was better than the parallel DVA at the first peak of elbow response. At the palm, the parallel DVA was more efficient than the series DVA at the first two natural frequencies of the uncontrolled system.

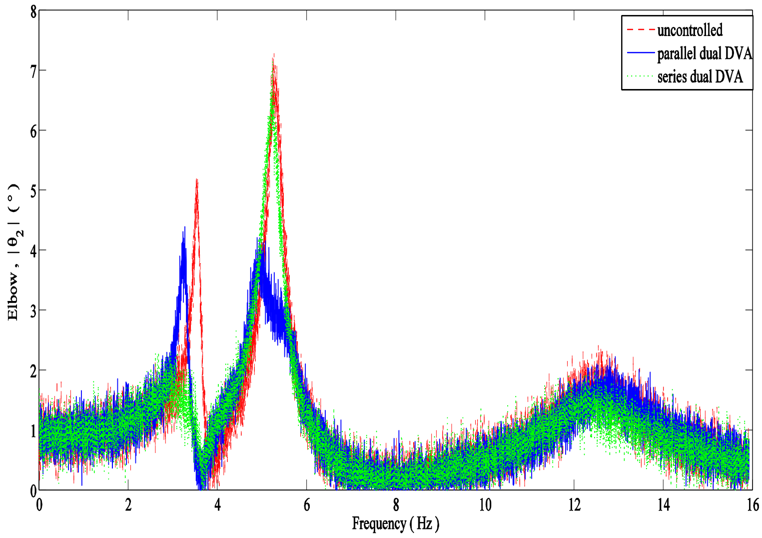
4.2 Behavior in Time Domain

The total responses at hand joints (particular and steady state response) are determined numerically in time domain when the multi-DOF system is operating at the first two resonance frequencies with white noise vibration.

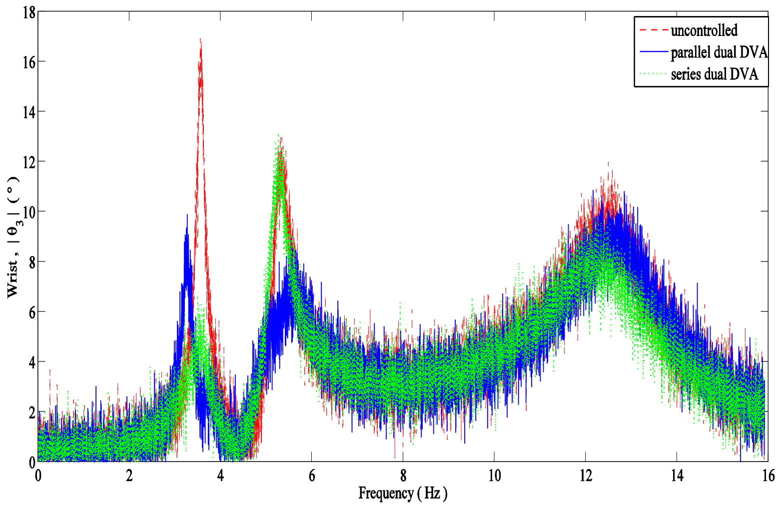
"Fig. 8a–c" Shows the total response at shoulder, elbow and wrist joints in time domain. Absorbers are usually designed to reduce the steady state vibrational motion. The designed absorbers were effective at the homogenous response due to the oscillations starting from the considered initial angular displacements. The effect of the homogenous response disappeared after approximately 4 seconds.



(a)



(b)



(c)

Figure 7. Frequency domain response at: (a) shoulder joint; (b) elbow joint; (c) wrist joint.

The percentage of reduction in tremor magnitude at the joints is calculated according to the following equation:

$$\%\,\text{Reduction} = \frac{\Theta_{\text{uncont.}} - \Theta_{\text{cont.}}}{\Theta_{\text{uncont.}}} \times 100 \tag{46}$$

$\Theta_{\text{uncont.}}, \Theta_{\text{cont.}}$ are the average uncontrolled and controlled flexion angle amplitude at the joints.

"Fig. 8" also shows the steady state vibrational motion at hand joints which appears is after the first 7 seconds. Both parallel and series dual DVA caused reduction in tremor amplitude at the steady state uncontrolled response. The average range in the percentage of reduction in the amplitude is calculated using (46) and summarized in "Table 4". It shows that the absorbers were very effective in tremor attenuation. However, the parallel dual DVA was more efficient in suppressing the shoulder, elbow and wrist joint amplitude. At the palm, the parallel dual absorber reduces 54.39–69.81% of the amplitude while the series dual absorber reduced 12.0–42.76%.

Table 4. Average of percentage reduction in tremor amplitude at steady state.

% Reduction	**Shoulder joint**	**Elbow joint**	**Wrist joint**
Conventional model	77.98–80.62%	60.34–69.95%	54.39–69.81%
Elastic-damper	57.70–74.42%	16.40–40.93%	12.0–42.76%

4.3 Behavior in Time Domain

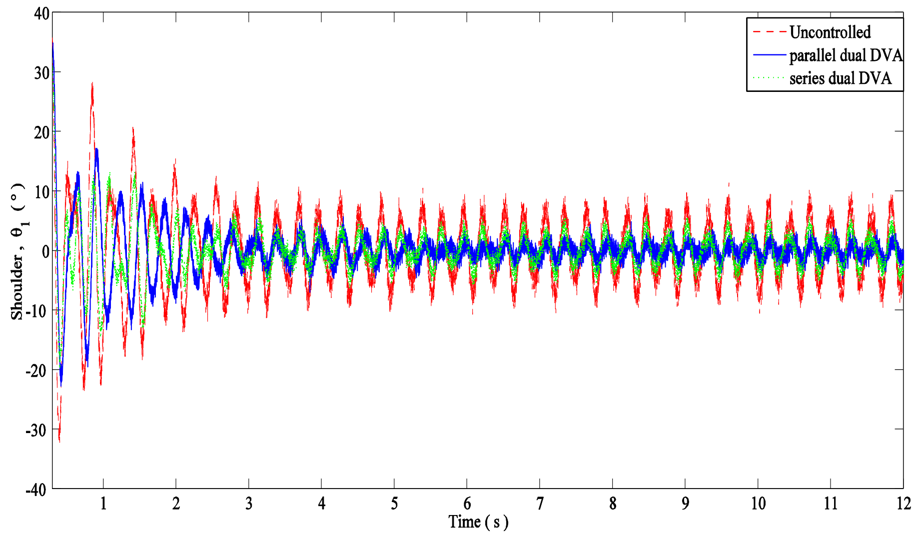
Stainless Steel alloy type 301 having an ultimate tensile $S_{ut} = 1379MPa$ and yielding stresses $S_y = 1138MPa$, is selected as beam's material for all the single absorbers forming the parallel and series DVA with 90% reliability. "Fig. 9" shows the behavior of 'absorber 1' and 'absorber 2' of the dual parallel and series DVA'. Absorber's beam is fluctuating between the maximum and minimum angular displacement with an average values shown in "Table 5" at the steady state.

The Safety factors against fatigue and yielding for the fluctuating type of stresses are calculated as followed:

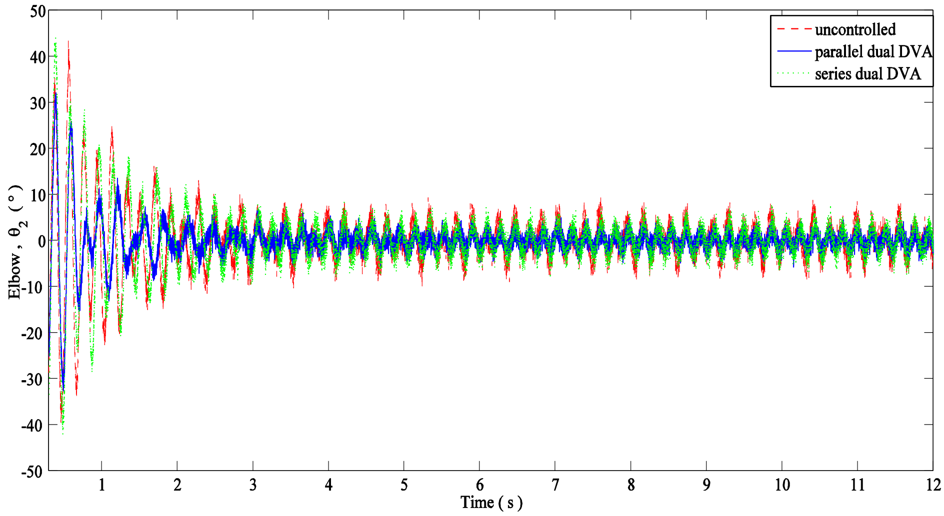
$$\sigma = k_f \frac{k_a \Theta \frac{B_0}{2}}{I_{beam}} \tag{47}$$

$$S'_e = 0.5 S_{ut} \quad (S_{ut} < 1400MPa) \tag{48}$$

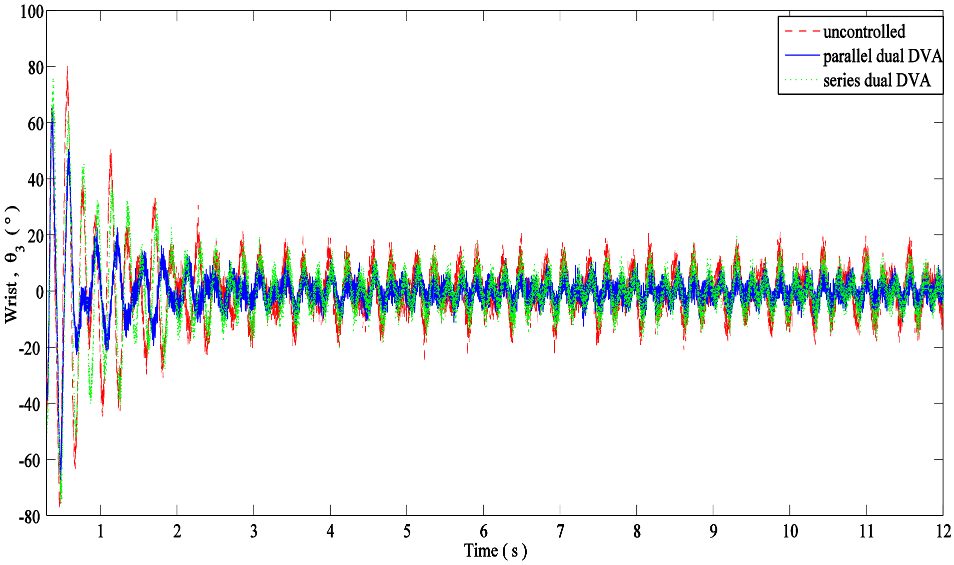
$$S_e = C_{load} C_{surf} C_{size} C_{temp.} C_{reliab.} S'_e \tag{49}$$



(a)



(b)



(c)

Figure 8. Time domain total response at: (a) shoulder joint; (b) elbow joint; (c) wrist joint.

$$N_f = \frac{S_e S_{ut}}{\sigma'_{alt.} S_{ut} + \sigma'_{mean} S_e} \quad (50)$$

$$N_y = \frac{S_y}{\sigma_{max}} \quad (51)$$

S'_e, S_e are the uncorrected and corrected endurance limit of beams material. $C_{load}, C_{surf}, C_{size}, C_{temp.}, C_{reliab.}$ are the load, size, surface, temperature and reliability correction factors for the endurance limit of the rotating beam experiment test. $\sigma'_{alt.}, \sigma'_{mean}$ are the alternating and mean bending stresses of the cantilevered beams. σ_{max} is the maximum bending stress. N_f, N_y are the fatigue and yielding safety factors.

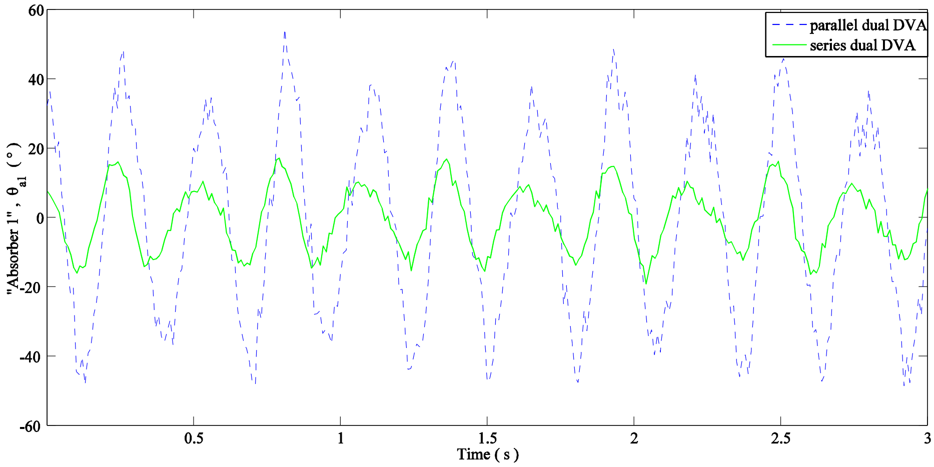
Table 5. Average of maximum and minimum flexion angle of the single absorbers.

Flexion Angle	Parallel dual DVA		Series dual DVA	
	Absorber 1	Absorber 2	Absorber 1	Absorber 2
Maximum	43.05 °	28.01 °	15.88 °	23.87 °
Minimum	31.99 °	18.55 °	7.76 °	8.10 °

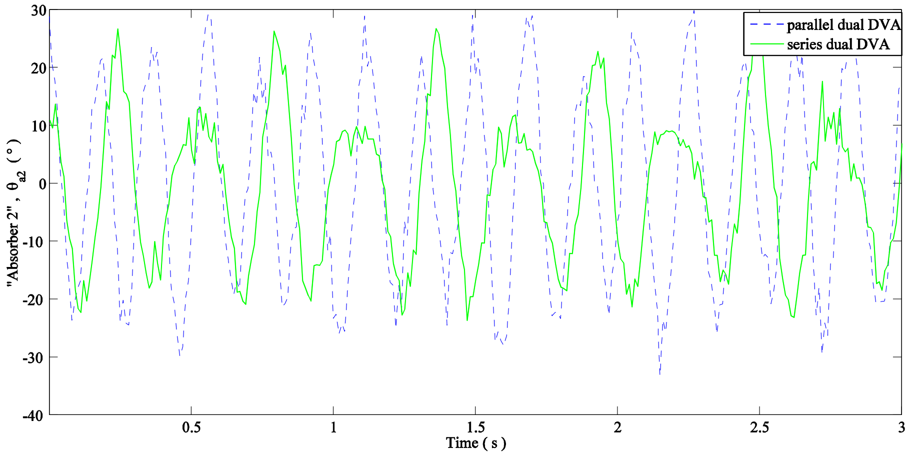
The corrected endurance limit of beams material is calculated to be $S_e = 410.67 MPa$. Then, the safety factors against failure of the beams of ‘absorber 1’ and ‘absorber 2’ of the dual parallel and series DVAs are shown in "Table 6". It shows that the parallel dual DVA was safe against failure by yielding and fatigue, but the series dual DVA was expected to yield and operate for a finite life time.

Table 6. Safety factors against failure by fatigue and yielding for the dual DVAs.

Flexion Angle	Parallel dual DVA		Series dual DVA	
	Absorber 1	Absorber 2	Absorber 1	Absorber 2
N_f	1.18	1.45	0.75	1.69
N_y	1.27	2.28	1.00	2.24



(a)



(b)

Figure 9. Time domain steady state response at: (a) ‘absorber 1’ joint; (b) ‘absorber 2’ joint.

V. DISCUSSION

In a previous study, Hashemi et al. [20] provided a two DOF biodynamic hand system in horizontal plane by modeling the hand of a Parkinson patient as an upper arm and forearm uniform rigid rods. Motion at palm joint was not considered and the forearm rod includes the real forearm plus the palm length. The primary system’s natural frequency was 2.27 Hz and 6.59 Hz reflecting the resting tremor of the hand. The analysis was done to compare the tremor amplitude of the primary system before and after adding the absorber at the forearm of the hand. The system was excited at elbow joint with single excitation frequency at 2.24 Hz neighborhood the first resonance frequency of the modeled hand. The absorbers natural frequency was 2.755 Hz which is higher than the excitation frequency. The optimum position of the absorber at the forearm was found at 160 mm away from the forearm end resulting in the best performance, i.e. the absorber was attached along the thumbs. The response in time domain for the uncontrolled and the controlled systems were derived using modal analysis. The tremor flexion motion amplitude of the excited system was reduced due to attaching the absorber from 3° to 0.5° at shoulder joint and from 4° to 2.2° at elbow joint. The modeled hand was fabricated for a comparison with the numerical study. Experimental and theoretical results were qualitatively similar. The absorber reduced more than 80° of tremor amplitude of the experimental model.

Rahnavard et al. [31] used the same two DOF hand model as that of Hashemi. However, they designed an optimum single DOF dynamic absorber to suppress rest tremor at the joints. The H_2 optimization criterion is used to reduce the total vibration energy of the system for overall frequencies. The absorber was able to reduce more than 98% and 80% of the flexion motion at elbow joint at the first and second natural frequencies of the primary system in the frequency domain. Tremor is modeled to have random nature instead of sinusoidal one. Two types of random inputs were considered with an amplitude rms equal to 10.25 N.m. The first type of random input is selected with 2.8 Hz frequency mean and the second type has 8 Hz frequency mean, both have

20% variance in the mean. In the first type, the absorber was able to reduce 60% and 39% of flexion motion at shoulder and elbow joints and in the second type 33% and 50% reduction is revealed in time domain.

In our study, a new three DOF system is designed by considering flexion motion of the palm. The system was modeled to vibrate using two excitation resonance frequencies with white noise input moment. The dual parallel and series DVA are suggested to reduce the double excited system at shoulder and elbow joints. The absorbers natural frequency was exactly equal to the resonance frequencies of the primary system and attached to the forearm of the hand. Both absorbers reduced the fluctuating tremor average amplitudes. The parallel dual absorber was better than the series type. It reduces 54.39–69.81% of tremor at the palm. The life time of each designed absorber is considered.

VI. CONCLUSION

Human hand is modeled dynamically as three DOF systems to reflect the behavior of patients having pathological tremor at musculoskeletal level. Elderly people suffering from Parkinson tremor can lose their life’s quality due to the severe side effects resulting from the medications used to decrease the symptoms of this disease. Mechanical treatment is suggested to reduce the amplitude of the flexion motion of the upper limbs from patients having resting tremor. The modeled hand system is driven by two resonance frequencies in range of resting tremor produced due to shoulder and elbow muscles activation. Tremor is modeled as sinusoidal functions with white Gaussian noise added to the signal. Two DOF absorbers are modeled to reduce this tremor. A comparison is done between the performance of the parallel and series dual DVAs in reducing tremor amplitude at steady state. Equations of motion of the three DOF primary hand system and the five DOF controlled systems are derived for the non-Lagrangian modeled systems. The response uncontrolled dynamically coupled hand system is solved using Cholesky decomposition. For the controlled system, equations of motion are solved using fourth ordered Runge-Kutta method with good accuracy. Each dual DVA has two single absorber’s (‘absorber 1’ and ‘absorber 2’) modeled as stainless steel cantilevered beams with copper mass attached along its length. The parallel dual DVA’s beams are tuned and attached directly to the forearm. ‘Absorber 1’ of the series dual DVA is connected to the forearm while their second beam is connected to the copper mass of ‘absorber 2’. Both absorbers reduced the fluctuating tremor average amplitudes. The parallel dual absorber was better than the series type. It reduces 54.39–69.81% of tremor at the palm. The absorbers are expected to operate for long time without yielding. However, the beam of ‘absorber 1’ of the series dual DVA is feeling both copper masses attached to the dual absorber, which causes its critical vibrations.

Experimental validation of the mechanical vibration absorber can be done to study its potential as a mechanical treatment replacing medications which produce serious side effects over time. However, the designed absorber is a passive controller which is able to reduce the designed voluntary motion.

REFERENCES

[1]. S. Smaga, Tremor – Problem – Oriented Diagnosis, Am Fam Physician, 68, 2003, 1545-1552.

[2]. V. Borges, and H. B. Ferraz, Tremors, Rev Neurosci, 14(1), 2006, 043-047.

[3]. R. Bhidayasiri, Differential diagnosis of common tremor syndromes. Postgrad Med J, 81(962), 2005, 756-762.

[4]. K. E. Lyons, and R. Pahwa, Handbook of essential tremor and other tremor disorders (CRC Press, 2005).

[5]. M. F. S. Almeida, G. L. Cavalheiro, A. A. Pereira, and A. O. Andrade, Investigation of age-related changes in physiological kinetic tremor, Ann Biomed Eng, 38(11), 2010, 3423-3439.

[6]. J. H. McAuley, and C. D. Marsden, Physiological and pathological tremors and rhythmic central motor control, Brain, 123(8), 2000, 1545-1567.

[7]. J. Parkinson, (1997). An Essay on the Shaking Palsy (Sherwood, Neely, and Jones, London, 1817); MH Polymeropoulos, et al. Science, 276, 2045-2047.

[8]. S. Morrison, G. Kerr, and P. Silburn, Bilateral tremor relations in Parkinson's disease: Effects of mechanical coupling and medication, Parkinsonism Relat D, 14(4), 2008, 298-308.

[9]. J. L. Contreras-Vidal, H. Teulings, and G. Stelmach, Elderly subjects are impaired in spatial coordination in fine motor control, Acta Psychol, 100(1), 1998, 25-35.

[10]. J. D. Cooke, S. H. Brown, and D. A. Cunningham, Kinematics of arm movements in elderly humans, Neurobiol Aging, 10(2), 1989, 159-165.

[11]. W. G. Darling, J. D. Cooke, and S. H. Brown, Control of simple arm movements in elderly humans, Neurobiol Aging, 10(2), 1989, 149-157.

[12]. J. A. Faulkner, L. M. Larkin, D. R. Claflin, and S. V. Brooks, Age-related changes in the structure and function of skeletal muscles, Clin Exp Pharmacol P, 34(11), 2007, 1091-1096.

[13]. R. D. Seidler, J. L. Alberts, and G. E. Stelmach, Changes in multi-joint performance with age, Motor control, 2002.

[14]. S. K. Van Den Eeden, C. M. Tanner, A. L. Bernstein, R. D.Fross, A. Leimpeter, D. A. Bloch, and L. M. Nelson, Incidence of Parkinson’s disease: variation by age, gender, and race/ethnicity, Am J Epidemiol, 157 (11), 2003, 1015-1022.

[15]. R. Mayeux, K. Marder, L. J. Cote, J. Denaro, N. Hemenegildo, H. Mejia, and A. Hauser, The frequency of idiopathic Parkinson's disease by age, ethnic group, and sex in northern Manhattan, 1988–1993. Am J Epidemiol, 142(8), 1995, 820-827.

[16]. S. L. Kowal, T. M. Dall, R. Chakrabarti, M. V. Storm, and A. Jain, The current and projected economic burden of Parkinson's disease in the United States, Movement Disord, 28(3), 2013, 311-318.

[17]. E. Rocon, J. M. Belda-Lois, J. J. Sanchez-Lacuesta, and J. L. Pons, Pathological tremor management: Modelling, compensatory technology and evaluation, Technol Disabil, 16(1), 2004, 3-18.

[18]. G. M. Hariz, and L. Forsgren, Activities of daily living and quality of life in persons with newly diagnosed Parkinson's disease according to subtype of disease, and in comparison to healthy controls, Acta Neurol Scand, 123(1), 2011, 20-27.
[19]. G. Grimaldi, and M. Manto, Tremor: from pathogenesis to treatment, Synthesis lectures on biomedical engineering, 3(1), 2008, 1-212.
[20]. S. M. Hashemi, M. F. Golnaraghi, and A. E. Patla, Tuned vibration absorber for suppression of rest tremor in Parkinson's disease, Med Biol Eng Comput, 42(1), 2004, 61- 70.
[21]. T. Igusa, and K. Xu, Vibration control using multiple tuned mass dampers, J Sound Vib, 175(4), 1994, 491-503.
[22]. M. J. Brennan, Characteristics of a wideband vibration neutralizer, Noise Control Eng, 45(5), 1997, 201-207.
[23]. L. Zuo, Effective and robust vibration control using series multiple tuned-mass dampers, J Vib Acoust, 131(3), 2009, 031003.
[24]. S. Gebai, M. Hammoud, A. Hallal, A. AL Shaer, and H. Khachfe, Passive Vibration Absorber for Tremor in Hand of Parkinsonian Patients, 3rd International Conference on Advances in Computational Tools For Engineering Applications, ACTEA2016, 2016.
[25]. S. Gebai, M. Hammoud, and H. Khachfe, Mechanical Vibration Absorber to Reduce the Neurodegenerative Disorder of Elderly, 4th International Conference on Advanced Training and Lifelong Learning Programs in Applied Health Sciences: Elderly Care, 2016.
[26]. S. Gebai, M. Hammoud, A. Hallal, and H. Khachfe, Tremor Reduction at the Palm of a Parkinson's Patient Using Dynamic Vibration Absorber, Bioengineering, 3(3), 2016, 18.
[27]. S. Gebai, M. Hammoud, A. Hallal, A. Shaito, and A. AL-Shaer, A Dual Dynamic Vibration Absorber to Attenuate Rest Tremor at the Hand of a Parkinson's Patient, 22nd International Scientific Conference of LAAS, The Social Avenues of Research, 2016.
[28]. R. Drillis, R. Contini, and M. Bluestein, Body segment parameters, Artificial Limbs, 8(1), 1966, 44-66.
[29]. D. J. Inman, Engineering Vibrations, (Virginia Tech, Prentice Hall, Upper Saddle River, 2008).
[30]. H. H. Jeffcott, The Periods of Lateral Vibration of Loaded Shafts. The Rational Derivation of Dunkerley's Empirical Rule for Determining Whirling Speeds, Proceedings of the Royal Society of London, Series A, Containing Papers of a Mathematical and Physical Character, 95(666), 1918, 106-115.
[31]. M. Rahnavard, M. Hashemi, F. Farahmand, and A. F. Dizaji, Designing a hand rest tremor dynamic vibration absorber using H 2 optimization method, J Mech Sci Technol, 28(5), 2014, 1609-1614.