Quest Journals
Journal of Research in Mechanical Engineering
Volume 8 ~ Issue 1 (2022) pp: 12-23
ISSN(Online) : 2321-8185
www.questjournals.org

Research Paper

Sensitivity Study of Indicated Mean Pressure to Working Cycle Parameters in Spark Ignition Engines

Mohamed I. Amin[1], and Waleed Neseem[2]
[1,2]*(Mechanical Power Department, Military Technical College, Cairo, Egypt.)*
Corresponding Author: Mohamed I. Amin,

ABSTRACT: *Increasing power to weight ratio is a main target for all engine designers. One of the most important parameters that assess the engine output power is the indicated mean pressure (IMP). The purpose of this work is to investigate the effect of working cycle parameters on IMP and how it is sensitive to these parameters' changes. It is shown that IMP is a function of five working cycle parameters namely pressure at the end of suction stroke(S), compression polytropic exponent (n), compression ratio (ε), Pressure rise ratio (λ), and expansion polytropic exponent (m). The analysis indicated that change of IMP is highly sensitive to expansion polytropic exponent change (m) more than other controlling parameters. Also, the sensitivity of IMP is not the same for wide ranges of other working cycle parameters change. By using regression, new correlations between IMP sensitivity and working cycle parameters have been introduced with lowest* $R^2 = 0.987$ *and error in a range of 7%. Such relations could be a useful tool for engine designers to increase IMP and power to weight ratio.*
KEYWORDS: *Sensitivity Study, Indicated Mean Pressure, Internal Combustion Engines, and Spark Ignition Engines.*

R*eceived 04 Jan, 2022;* ***R****evised 13 Jan, 2022;* ***A****ccepted 15 Jan, 2022 © The author(s) 2022.*
P*ublished with open access at* ***www.questjournals.org***

I. INTRODUCTION

Indicated mean pressure (IMP) is a parameter that measures the ability of engine to do work and it is independent on engine volume [1]. It expresses the work done by unit volume of engine and can be calculated by dividing the work done in one cycle by swept volume [2]. Previous works show that IMP is a function of many working cycle parameters, and as increasing of IMP means engine power to weight increase, it is convenient to determine which one of these working parameters has dominant effect on IMP. This encourages conducting a comprehensive sensitivity analysis of IMP to working cycle parameters.

Sensitivity analysis is a technique which findings the dependence of a system output on a specific input to evaluate the hazardousness of a certain strategy, in other words, it is the investigation of a mathematical model or system output uncertainty due to different sources of uncertainty in its inputs [3]. Sensitivity analysis approaches can be classified to mathematical, statistical, and graphical [4]. Mathematical methods assess sensitivity of a model output to a series of an input. These methods do not focus on the variation in the output due to inputs variance, but they can measure the influence of variation of input range on the output [5]. Sometimes, mathematical methods can be useful in showing the most significant inputs [6]. Statistical methods model's inputs are allocated probability distributions and considering the effect of change in inputs on the output distribution [7, 8]. Depending on the method, one or more inputs are varied at a time. Statistical methods permit to detect the effect of interactions among many inputs. Graphical methods offer illustration of sensitivity in graph forms, charts, or surfaces, and usually used to give visual sign of how an output is affected by change in inputs [9].

There are different sensitivity methods which can be followed to investigate certain model or system. *Nominal Range Sensitivity* or *local sensitivity analysis* estimates the effect of individual input variation over a reasonable range on model outputs, while keeping all other inputs at constant values [10]. The change in the model output due to the change in the input variable is concerned to the sensitivity of the model to that specific input variable [5]. Nominal range sensitivity analysis is a relatively simple method that is easily to apply but interactions among inputs are difficult to get [4].

Automatic Differentiation Technique (AD) is a technique for estimating local sensitivities for large models by calculating first-order partial derivatives of outputs with respect to small changes in the inputs [4, 11-

13]. The values of partial derivatives quantify the local sensitivity. Mathematical formulation and computer program accomplishment are the main difficulties in differentiation methods [4]. In AD, the local sensitivity is computed at one or more points in the parameter field of the model. At each point, the partial derivatives of the model output with respect to a selected number of inputs are calculated. AD is excellent to finite difference approximations of the derivatives because numerical values of the computed derivatives are more accurate, and computational effort is significantly lower, but this method can be used only if it is possible to calculate the partial derivatives locally [14].

Regression Analysis can be devoted as a probabilistic sensitivity analysis technique [15]. Regression analysis assists three main purposes: description of the relation between variables, control of predictor variables for a given value of a response variable, and prediction of a response based on predictor variables [16]. Regression analysis is mostly conducted on an independent random sample of data. The effect of inputs on the output can be investigated using regression coefficients, standard errors of regression coefficients, and the level of significance of the regression coefficients [17]. To determine statistical significance, the standard error of the regression coefficient is anticipated. If the ratio of the value of the regression coefficient divided by its standard error is greater than a critical value, then the coefficient is believed to be statistically dominant. The critical value is determined based on the desired significance level (usually 0.05) and the degrees of freedom of the regression model [17]. Regression techniques, such as those based on the use of partial correlation coefficients, can appraise the unique influence of a model input with respect to variation in a selected model output [18]. Other sensitivity analysis methods have been summarized and introduced by H. Christopher Frey, and Sumeet R. Patil [4] such as Analysis of Variance, Response Surface Method (RSM), Fourier Amplitude Sensitivity Test, Mutual Information Index, and Scatter Plots.

This work aims to investigate different parameters which affect the value of indicated mean pressure. And to determine how sensitive the indicated mean pressure is to these parameters to specify the most dominant parameter which affects IMP value. Automatic Differentiation Technique has been carried out to conduct this study. Moreover a correlation between indicated mean pressure sensitivity and working cycle parameters has been introduced, which could be a useful design tool for increasing IMP and power to weight ratio.

II. SENSITIVITY STUDY

Over past years, extensive efforts have been conducted by engine researchers for more enhancement and development. Most researches focused on either reduction of fuel consumption, or increasing power to volume ratio. One of the most important parameters which reflects the engine output per its displacement volume unit is indicated mean pressure (IMP). This pressure is defined as the conditional constant pressure which if it is acting on the piston through one stroke, will give a work equal to indicated work [1, 2]. So, the IMP is calculated by dividing the indicated work by the swept volume as shown in fig. (1), and can be expressed by equation (1), [1].

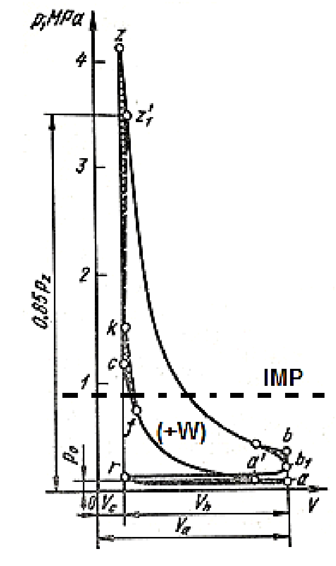


Figure (1): P-V Diagram Shows Indicated Work (W), and Indicated Mean Pressure (IMP)

$$IMP = \left(S \left(\frac{\varepsilon^{n}}{\varepsilon - 1} \right) \left(\frac{\lambda}{m-1} \left(1 - \left(\frac{1}{\varepsilon^{m-1}} \right) \right) - \frac{1}{n-1} \left(1 - \left(\frac{1}{\varepsilon^{n-1}} \right) \right) \right) \right) \quad (1)$$

Where: S Pressure at the end of admission (suction) stroke.
n Compression polytropic exponent.
ε Compression ratio.
λ Pressure rise ratio (ratio between maximum cycle pressure and compression pressure).
m Expansion polytropic exponent.

Above equation shows that IMP is a function of five working cycle parameters S, n, ε, λ, and m, and each parameter has its own effect on IMP value. To study the effect changing of each parameter individually on the IMP, it is needed to pick an average common value for each parameter. Table (1) shows typical range for each parameter for spark ignition engines as suggested by Khovakh [1], in addition to selected moderate values for each parameter. IMP has been calculated for different values of each parameter while other parameters were kept constant. Fig. (2 A-E) shows IMP trends while changing only one parameter at a time and keep others constant according to table (1). Fig. (2-A) shows increasing in IMP as suction pressure (S) increases. This trend can be explained by decreasing of negative area in P-V diagram (pumping work) as suction pressure increases, which results in increase of resulting indicated work and mean pressure. Obviously, the IMP increases as compression ratio (ε) increases due to the increase of thermal efficiency, peak pressure, and indicated work as shown in Fig. (2-B). The same behavior is experienced when increasing compression polytropic exponent (n) and compression rise ratio (λ) due to increase of positive indicated work area, Fig. (2- C, D). When increasing expansion exponent (m), it means the expansion curve becomes steeper, which means lower area under the expansion curve in P-V diagram, and lower indicated work and IMP as shown in Fig. (2-E).

Table (1): Typical Range of Working Cycle Parameters and Selected Moderate Values [1]

	S	**ε**	**n**	**λ**	**m**
Typical Range	0.8 : 0.9 bar	7 : 9	1.3 : 1.4	3 : 4	1.23 : 1.3
Selected Values	0.86 bar	8	1.4	3.5	1.25

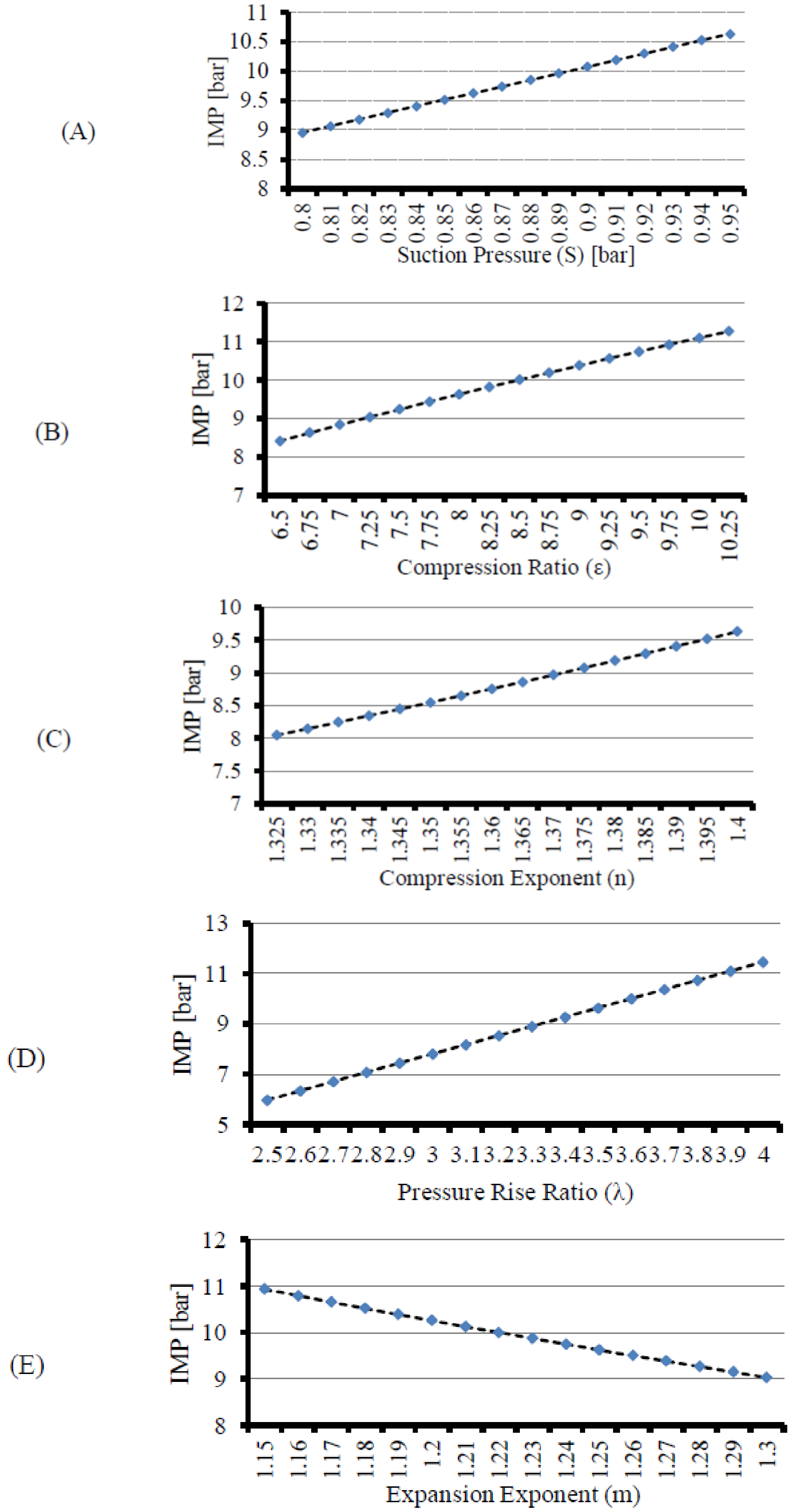


Figure (2): Effect of Change of Individual Working Cycle Parameters on IMP trends

(A) Suction Pressure, (B) compression Ratio, (C) Compression Exponent, (D) Pressure Rise Ratio, (E) Expansion Exponent

Now, the question is how sensitive is the IMP to working cycle parameters? The answer can be obtained by applying the partial derivative definition of sensitivity [4]. Automatic Differentiation Technique (AD) was conducting for calculating local sensitivities by evaluating first-order partial derivatives of outputs with respect to small changes in the inputs [4, 11-13]. Equations (2: 6) are the partial derivatives of IMP to working cycle parameters. To assess the sensitivity of IMP to working cycle parameters, data in table (1) have been substituted in equations (2: 6), fig. (3) exhibits the results of these substitutions. It is clearly noted that the compression polytropic exponent (m) is the most significant parameter on IMP changes, followed by compression exponent (n). This means that the slope of the expansion processes line in P-V diagram is the most dominate parameter that can change the area under the curve.

$$\frac{\delta IMP}{\delta S} = \frac{\varepsilon^n\left(\frac{\lambda\left(1-\varepsilon^{-m+1}\right)}{m-1} - \frac{1-\varepsilon^{-n+1}}{n-1}\right)}{\varepsilon-1} \tag{2}$$

$$\frac{\delta IMP}{\delta \varepsilon} = \frac{nS\varepsilon^{n-1}\left(\frac{\lambda\left(1-\varepsilon^{-m+1}\right)}{m-1} - \frac{1-\varepsilon^{-n+1}}{n-1}\right)}{\varepsilon-1} - \frac{S\varepsilon^n\left(\frac{\lambda\left(1-\varepsilon^{-m+1}\right)}{m-1} - \frac{1-\varepsilon^{-n+1}}{n-1}\right)}{(\varepsilon-1)^2} + \frac{S\varepsilon^n\left(\frac{\lambda}{\varepsilon^m} - \frac{1}{\varepsilon^n}\right)}{\varepsilon-1} \tag{3}$$

$$\frac{\delta IMP}{\delta n} = \frac{S\varepsilon^n\left((-n)\ln(\varepsilon)\varepsilon^{1-n} + \ln(\varepsilon)\varepsilon^{1-n} - \varepsilon^{1-n} + 1\right)}{(\varepsilon-1)\left(n^2-2n+1\right)} + \frac{S\ln(\varepsilon)\varepsilon^n\left(\frac{\lambda\left(1-\varepsilon^{-m+1}\right)}{m-1} - \frac{1-\varepsilon^{-n+1}}{n-1}\right)}{\varepsilon-1} \tag{4}$$

$$\frac{\delta IMP}{\delta \lambda} = \frac{S\varepsilon^n\left(1-\varepsilon^{-m+1}\right)}{(m-1)(\varepsilon-1)} \tag{5}$$

$$\frac{\delta IMP}{\delta m} = \frac{S\varepsilon^n\left(m\lambda\ln(\varepsilon)\varepsilon^{1-m} - \lambda\ln(\varepsilon)\varepsilon^{1-m} + \lambda\varepsilon^{1-m} - \lambda\right)}{(\varepsilon-1)\left(m^2-2m+1\right)} \tag{6}$$

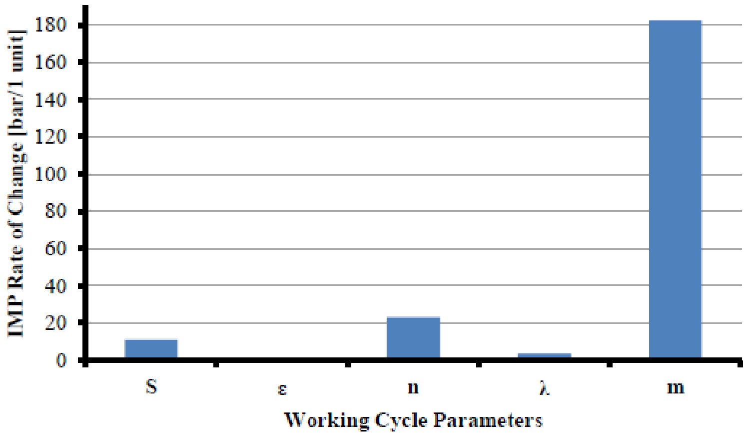
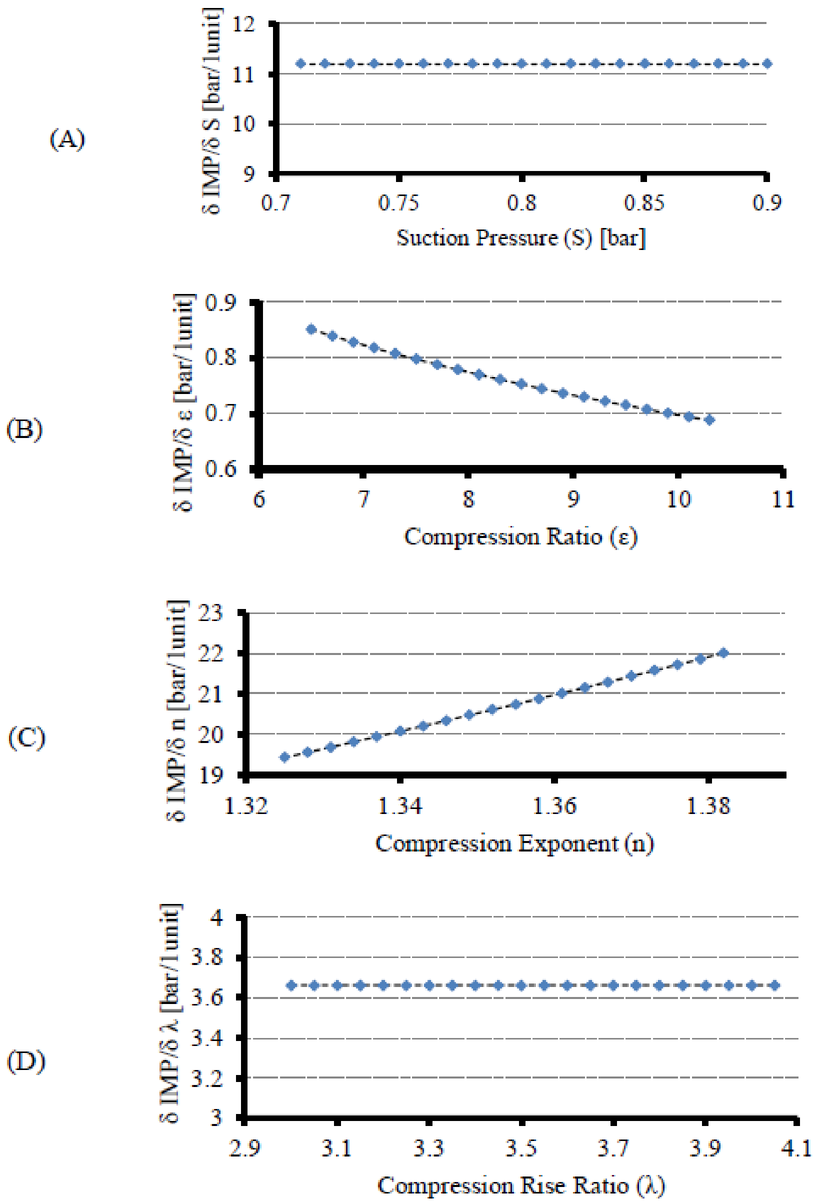


Figure (3): Rate of Change of IMP Relative to Working Cycle Parameters (Sensitivity)

These sensitivity values which are demonstrated in fig. (3) have been calculated for only selected moderate values shown in table (1). To investigate the effect of each parameter individually on IMP sensitivity, previous calculations were repeated over a wide range of each parameter while keeping other parameters constant. Fig. (4 A-E) shows the effect of individual change of working cycle parameters on IMP sensitivity using data in table (1). It is shown that the sensitivity is constant over the entire range of suction pressure (S) and pressure rise ration (λ), fig. (4 A, D). On the other hand, the sensitivity decreases by increasing compression ratio (ε) and expansion polytropic exponent (m), fig. (4 B, E). Finally, it was found that the sensitivity increases by increasing the compression polytropic exponent (n), fig. (4 C).

Figs. (3, 4) show that IMP is very sensitive to compression and expansion polytropic exponents (expansion much more). So, it is necessary to know how to control and change its values. The values of n and m are affected by the rate of heat transfer between the hot gases and walls, and affected also by the quality and rate of combustion. As the rate of heat transfer from the hot gases to walls increases, n and m decrease. And as the quality and rate of combustion increases the higher value of cycle maximum pressure is obtained which increase the value of m. Many governing parameters can be used by designers to control values of n and m. For example; they can change piston materials, cooling systems, piston clearance, compression ratio, and engine operating speed or load. Mentioned governing parameters affect somehow the heat transfer rate between gasses inside the cylinder and walls, either through time, area available for heat transfer, or thermal conductivity property.



(E)

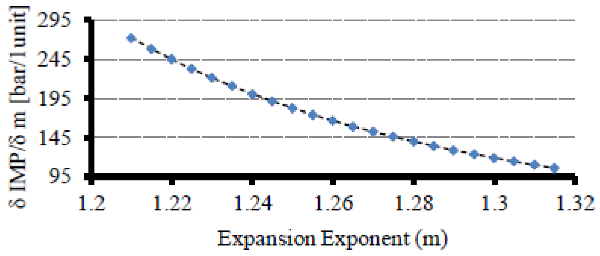


Figure (4): Effect of Individual Change of Working Cycle Parameters on IMP Sensitivity

Comprehensive investigation, it is mandatory to conduct a factorial studying for the previously determined affecting parameters, namely (ε, n, and m). For spark ignition engines, it is well known that there is a wide variety of these parameter's values. Table (2) shows selected moderate values of working cycle parameters [1]. By repeating all previous calculations for different values of ε, n, and m we can get a whole map of IMP sensitivity trends as shown in Appendix (A) and figures (5 : 9).

Table (2): Selected Moderate Values of Working Cycle Parameters for Sensitivity Study [1]

ε	n	m
6.5	1.325	1.21
7.5	1.35	1.24
8.5	1.375	1.27
9.5	1.4	1.3

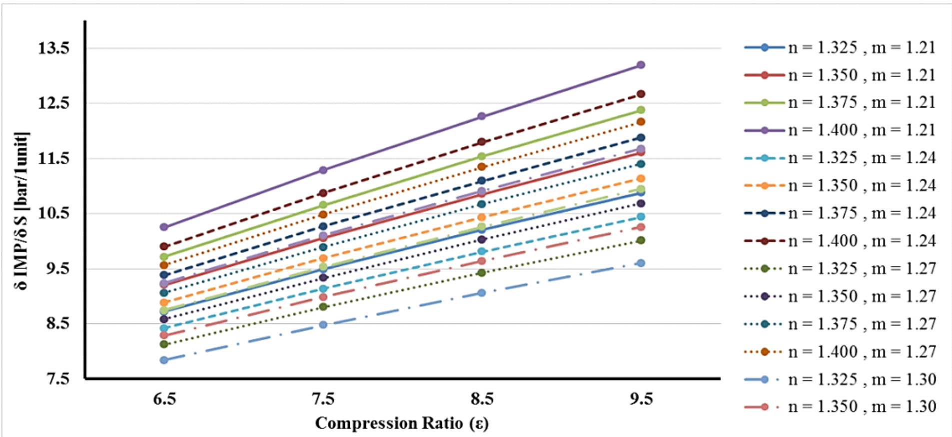


Figure (5): Sensitivity of IMP to Different Values of (S) and Different Values of Other Working Cycle Parameters

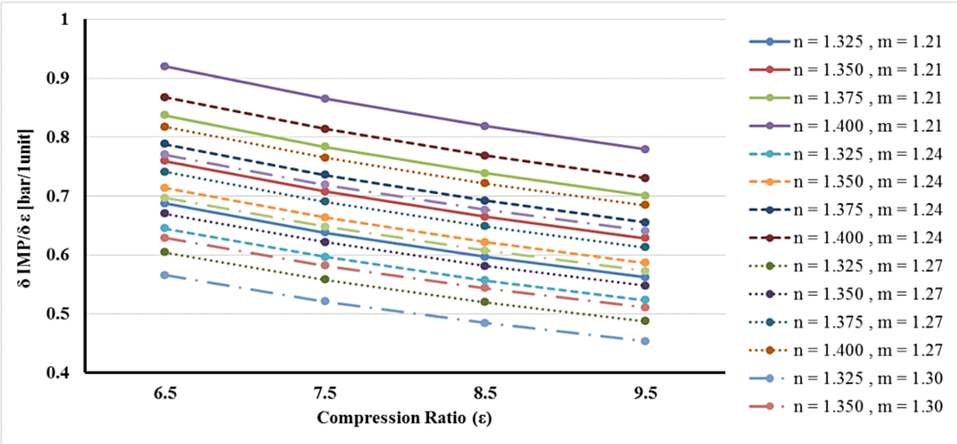


Figure (6): Sensitivity of IMP to Different Values of (ε) and Different Values of Other Working Cycle Parameters

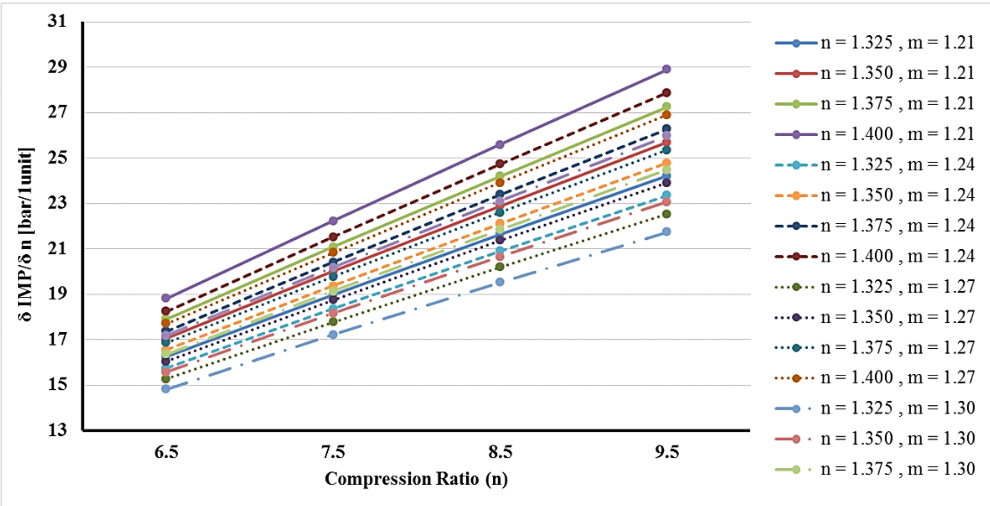


Figure (7): Sensitivity of IMP to Different Values of (n) and Different Values of Other Working Cycle Parameters

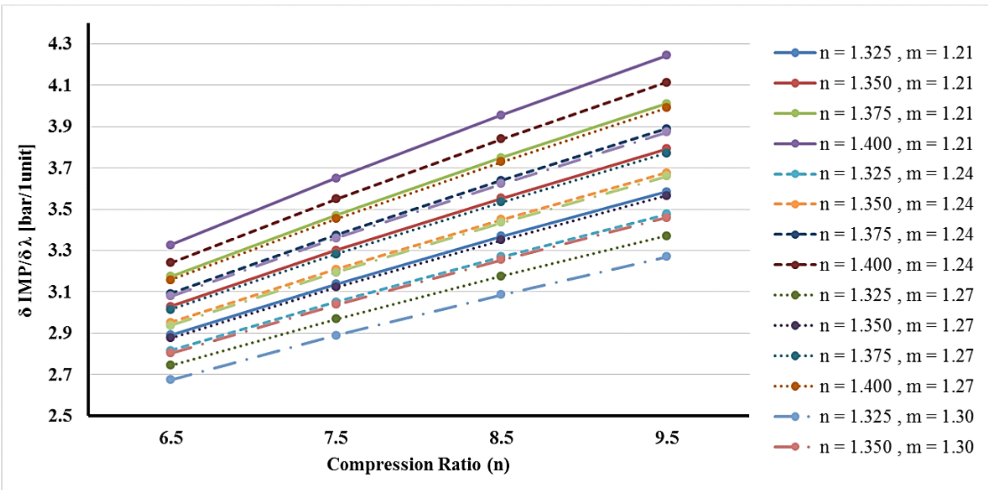


Figure (8): Sensitivity of IMP to Different Values of (λ) and Different Values of Other Working Cycle Parameters

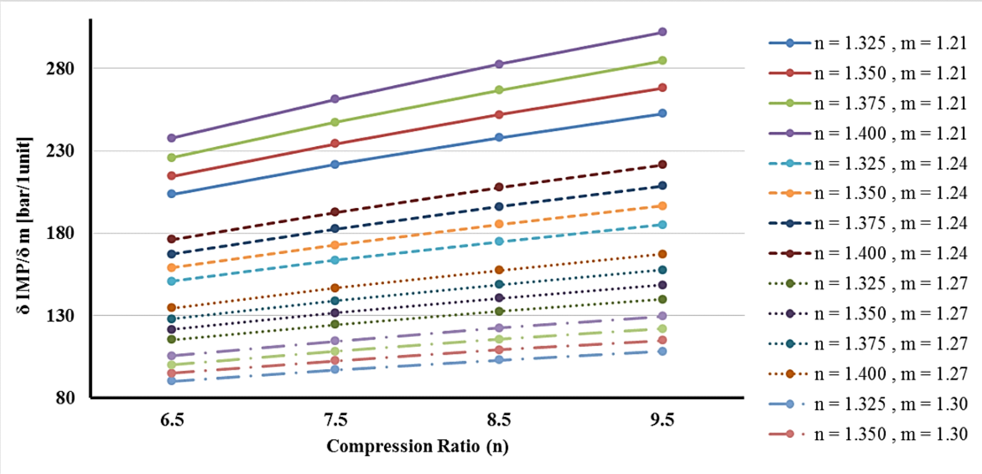


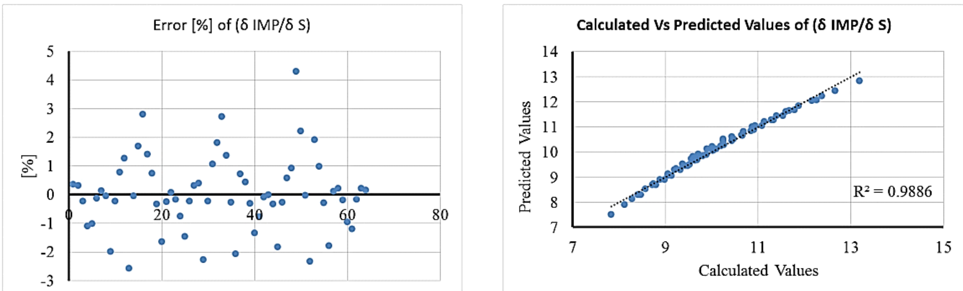
Figure (9): Sensitivity of IMP to Different Values of (m) and Different Values of Other Working Cycle Parameters

It is convenient to find a relation that can be used to predict IMP sensitivity in terms of working cycle parameters. Such relation could be a useful tool for engine designer to increase power to volume ratio. Data in appendix (A) have been utilized to correlate the values of IMP sensitivity to other dominant working cycle parameters ε, n, and m. Regression results and statistics, using least square method are illustrated in table (3). It is seen that the lowest R^2= 0.987531574 and Significance F = $2.02*10^{-55}$, appendix (B) shows sample of regression statistics results. Fig. (10) shows error % and calculated Vs. predicted values of IMP sensitivity. The obtained relations then are:

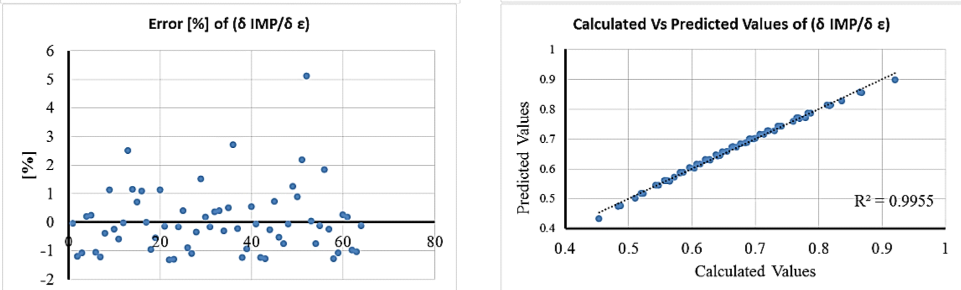
Table (3): Correlations of IMP Sensitivity to Working Cycle Parameters and Error

Correlation	R^2	Significance F
δ IMP/δ S = 0.770388 (ε) + 24.371149 (n) -13.004078 (m) - 12.877105	0.988754571	2.11E-58
δ IMP/δ ε = -0.0423894 (ε) + 2.7995753 (n) -1.4341435 (m) - 1.0105863	0.995507234	2.35E-70
δ IMP/δ n = 2.8026752 (ε) + 46.000884 (n) - 23.371002 (m) - 34.7902727	0.991165687	1.51E-61
δ IMP/δ λ = 0.2487084 (ε) + 7.0056945 (n) - 3.1952878 (m) - 4.1560376	0.990455766	1.54E-60
δ IMP/δ m = 12.1147 (ε) + 382.096 (n) + 9668.48 (m^2) - 25825.224 (m) - 16724.54	0.987531574	2.02E-55

(A)



(B)



(C)

(D)

(E)

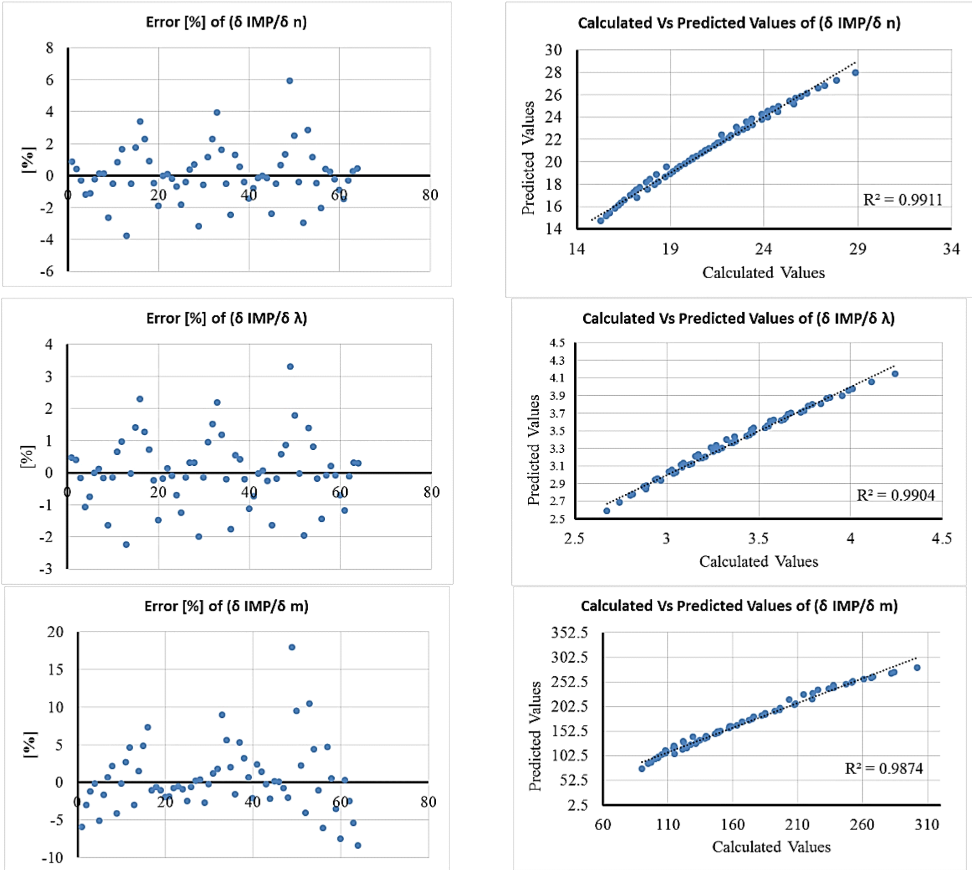


Figure (10): Error % and Calculated Vs. Predicted Values of IMP Sensitivity to Different Working Cycle Parameters

III. CONCLUSION

Indicated Mean Pressure (IMP) is an important parameter which helps in engine performance assessment. This parameter is determined from measured pressure-volume (P-V) curve by dividing the obtained work over the swept volume. So, the IMP can be expressed by a mathematical relation which is a function of five working cycle parameters namely, pressure at the end of suction stroke(S), compression polytropic exponent (n), compression ratio (ε), Pressure rise ratio (λ), and expansion polytropic exponent (m). To increase engine power to volume ratio by increasing the positive area under P-V curve and hence the IMP, it is important to study the effect of change of each parameter on IMP change.

In this work, a comprehensive sensitivity investigation has been conducted using partial derivative technique, to study the effect of changing of working cycle parameters on the value IMP. It was concluded that:

- IMP increases in as suction pressure (S), compression ratio (ε), compression polytropic exponent (n), and compression rise ratio (λ) increase. While IMP decreases by increasing expansion exponent (m).
- IMP is much more sensitive to expansion polytropic exponent change (m) over the other controlling parameters.
- IMP sensitivity is constant over the entire range of suction pressure (S) and pressure rise ration (λ), while its sensitivity decreases by increasing compression ratio (ε) and expansion polytropic exponent (m). It was also found that the IMP sensitivity increases by increasing the compression polytropic exponent (n).
- By using regression, new correlations between (IMP) sensitivity and working cycle parameters have been introduced with lowest R^2= 0.987531574. The error between predicted and calculated values of (IMP) sensitivity in a range of 7%. Such relations could be a useful tool for engine designers to increase IMP and power to weight ratio.

REFRENCES

[1]. M. Khovakh, "Motor Vehicle Engines", MIR Publisher, Mosco, (1979).
[2]. John B. Heywood, "Internal Combustion Engine Fundamentals", *McGraw-Hill Series in Mechanical Engineering*, 1988.
[3]. Saltelli, A.; Ratto, M.; Andres, T.; Campolongo, F.; Cariboni, J.; Gatelli, D.; Saisana, M.; Tarantola, S. (2008). Global Sensitivity Analysis: The Primer. John Wiley & Sons.

[4]. H. Christopher Frey, and Sumeet R. Patil, "Identification and Review of Sensitivity Analysis Methods", Risk Analysis, Vol. 22, No. 3, 2002.

[5]. Morgan, M. G., and M. Henrion (1990), Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis. Cambridge University Press: Cambridge, NY.

[6]. Brun, R., P. Reichert, and H. R. Kunsch (2001), ''Practical Identifiability Analysis of Large Environmental Simulation Models,'' Water Resources Research, 37(4):1015–1030.

[7]. Andersson, F. O., M. Aberg, and S. P. Jacobsson (2000), ''Algorithmic Approaches for Studies of Variable Influence, Contribution and Selection in Neural Networks,'' Chemo-metrics and Intelligent Laboratory Systems, 51(1):61–72.

[8]. Neter, J., M. H. Kutner, C. J. Nachtsheim, and W. Wasserman (1996), Applied Linear Statistical Models. 4th ed. McGrawHill: Chicago, IL.

[9]. Geldermann, J., and O. Rentz (2001), ''Integrated Technique Assessment with Imprecise Information as a Support for the Identification of Best Available Techniques (BAT),'' OR Spektrum, 23(1):137–157.

[10]. Cullen, A. C., and H. C. Frey (1999), Probabilistic Techniques in Exposure Assessment. Plenum Press: New York.

[11]. *Cacuci, Dan G. Sensitivity and Uncertainty Analysis: Theory. I. Chapman & Hall.*

[12]. *Cacuci, Dan G.; Ionescu-Bujor, Mihaela; Navon, Michael (2005). Sensitivity and Uncertainty Analysis: Applications to Large-Scale Systems. II. Chapman & Hall.*

[13]. *Griewank, A. (2000). Evaluating Derivatives, Principles and Techniques of Algorithmic Differentiation. SIAM.*

[14]. Bischof, C. H., A. Carle, G. Corliss, A. Griewank, and P. Hovland (1992), ''ADIFOR Generating Derivative Codes From FORTRAN programs.'' ANL-4501, Prepared by Mathematics and Computer Science Division, Aargonne National Laboratory, Argonne, Illinois.

[15]. Iman, R. L., M. J. Shortencarier, and J. D. Jhonson (1985). ''A FORTRAN 77 Program and Users Guide for the Calculation of Partial Correlation and Standardized Regression Coefficients,'' Report No. SAND85-0044, Sandia National Laboratories, Albuquerque, NM.

[16]. Sen, A., and M. Srivastava (1990), Regression Analysis: Theory, Methods, and Applications. Springer-Verlag: New York.

[17]. Devore, J. L., and R. Peck (1996), Statistics: The Exploration and Analysis of Data. 3rd ed. Brooks/Cole Publishing Company: London.

[18]. Brikes, D., and Y. Dodge (1993), Alternative Methods of Regression. John Wiley and Sons, Inc.: New York.

APPENDICES

Appendix (A) IMP Sensitivity for Different Values of n, ε, and m

		δ IMP/δ S				δ IMP/δ ε			
	ε	n				n			
		1.325	1.35	1.375	1.4	1.325	1.35	1.375	1.4
m = 1.21	6.5	8.717849	9.201651	9.70979	10.24346	0.687505	0.759622	0.837119	0.920347
	7.5	9.487354	10.05383	10.65102	11.28056	0.637716	0.707817	0.783502	0.865164
	8.5	10.20423	10.8511	11.53528	12.25886	0.596536	0.664738	0.738676	0.818773
	9.5	10.87712	11.60225	12.37143	13.18726	0.561757	0.628187	0.700462	0.779039
m = 1.24	6.5	8.414673	8.88395	9.376869	9.89459	0.644699	0.713714	0.787911	0.867628
	7.5	9.13538	9.683672	10.26174	10.87117	0.596563	0.663477	0.735756	0.813777
	8.5	9.805289	10.43024	11.09128	11.79046	0.556879	0.621837	0.692294	0.768658
	9.5	10.43287	11.13228	11.87425	12.66129	0.523457	0.586606	0.655349	0.730123
m = 1.27	6.5	8.122215	8.577482	9.055718	9.558054	0.60412	0.670178	0.741227	0.817594
	7.5	8.796742	9.327539	9.887212	10.47729	0.557681	0.621564	0.690604	0.765164
	8.5	9.422339	10.02624	10.66508	11.34084	0.519517	0.5814	0.648557	0.721379
	9.5	10.00728	10.68206	11.39796	12.15743	0.487464	0.54751	0.612912	0.684089
m = 1.3	6.5	7.840045	8.281794	8.745865	9.233356	0.56565	0.628886	0.696932	0.770103
	7.5	8.470864	8.984826	9.526792	10.09825	0.520939	0.581942	0.647902	0.719169
	8.5	9.054649	9.638343	10.25587	10.90914	0.484313	0.54328	0.607306	0.67677
	9.5	9.599469	10.25063	10.94156	11.6746	0.453634	0.510745	0.572985	0.640758

		δ IMP/δ n				δ IMP/δ λ			
	ε	n				n			
		1.325	1.35	1.375	1.4	1.325	1.35	1.375	1.4
m = 1.21	6.5	16.23742	17.05469	17.91211	18.81163	2.890296	3.028762	3.173862	3.325913
	7.5	18.97637	20.00604	21.09026	22.23188	3.137971	3.300088	3.47058	3.64988
	8.5	21.63393	22.88234	24.20109	25.59411	3.368249	3.553364	3.748652	3.954673
	9.5	24.21587	25.6876	27.24669	28.89826	3.584026	3.791528	4.011044	4.243268
m = 1.24	6.5	15.74938	16.54327	17.37619	18.25003	2.815802	2.950699	3.092059	3.240191
	7.5	18.36647	19.36463	20.41571	21.52247	3.051486	3.209135	3.374928	3.549286
	8.5	20.89969	22.10775	23.38393	24.73205	3.270223	3.44995	3.639555	3.839581
	9.5	23.35575	24.77768	26.28408	27.87993	3.474867	3.676049	3.888879	4.11403
m = 1.27	6.5	15.2786	16.04993	16.85922	17.7083	2.743941	2.875395	3.013147	3.157499
	7.5	17.77967	18.74752	19.76671	20.83995	2.968278	3.121628	3.2829	3.452504
	8.5	20.19489	21.36421	22.59953	23.90453	3.176127	3.350683	3.534832	3.729102
	9.5	22.53177	23.90699	25.36193	26.90439	3.370295	3.565422	3.771847	3.990223
m = 1.3	6.5	14.82437	15.57395	16.36043	17.18561	2.674607	2.80274	2.937012	3.077716
	7.5	17.21498	18.15366	19.14217	20.18314	2.888205	3.037418	3.19434	3.359369
	8.5	19.51817	20.6503	21.84639	23.11	3.08578	3.255371	3.434282	3.623026
	9.5	21.7422	23.07071	24.47829	25.96959	3.27009	3.459416	3.659703	3.871586

		δ IMP/δ m			
	ε	n			
		1.325	1.35	1.375	1.4
m = 1.21	6.5	203.6521	214.5066	225.8811	237.8006
	7.5	221.7469	234.2531	247.4054	261.2372
	8.5	237.9223	252.0028	266.8571	282.5279
	9.5	252.589	268.1751	284.6637	302.1068
m = 1.24	6.5	150.8696	158.9211	167.3583	176.1996
	7.5	163.5938	172.8307	182.5449	192.7608
	8.5	174.8889	185.2496	196.1796	207.7104
	9.5	185.0681	196.4985	208.5906	221.3828
m = 1.27	6.5	115.2568	121.4166	127.8714	134.6355
	7.5	124.4615	131.4979	138.8978	146.6801
	8.5	132.5724	140.4353	148.7304	157.4813
	9.5	139.8355	148.4813	157.6276	167.3035
m = 1.3	6.5	90.19977	95.02815	100.0878	105.3899
	7.5	97.00264	102.4946	108.2702	114.3442
	8.5	102.9509	109.0649	115.5149	122.3194
	9.5	108.2414	114.9417	122.03	129.5287

Appendix (B) Sample of Regression Summary Output for IMP Sensitivity

[1] SUMMARY OUTPUT OF IMP SENSITIVETY TO COMPRESSION EXPONENT (n)

Regression Statistics	
Multiple R	0.995573045
R Square	0.991165687
Adjusted R Square	0.990723971
Standard Error	0.33898163
Observations	64

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	773.5298	257.8433	2243.9	1.51E-61
Residual	60	6.894513	0.114909		
Total	63	780.4243			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-34.79027272	2.621779	-13.2697	1.68E-19	-40.0346	-29.5459	-40.0346	-29.5459
ε	2.802675222	0.037899	73.95058	1.2E-60	2.726865	2.878485	2.726865	2.878485
n	46.00088363	1.515972	30.34415	4.19E-38	42.96849	49.03328	42.96849	49.03328
m	-23.37100228	1.26331	-18.4998	1.8E-26	-25.898	-20.844	-25.898	-20.844

[2] SUMMARY OUTPUT OF IMP SENSITIVETY TO EXPANSION EXPONENT (m)

Regression Statistics	
Multiple R	0.993746232
R Square	0.987531574
Adjusted R Square	0.986686257
Standard Error	6.51769577
Observations	64

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	198508.7	49627.17	1168.238	2.02E-55
Residual	59	2506.341	42.48036		
Total	63	201015			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	16724.54052	1425.642	11.73124	4.65E-17	13871.84	19577.24	13871.84	19577.24
ε	12.11471251	0.728701	16.62509	6.19E-24	10.65659	13.57284	10.65659	13.57284
n	382.0960809	29.14802	13.10882	3.97E-19	323.771	440.4211	323.771	440.4211
m^2	9668.480867	905.2355	10.68062	2.06E-15	7857.109	11479.85	7857.109	11479.85
m	-25825.22419	2272.271	-11.3654	1.71E-16	-30372	-21278.4	-30372	-21278.4