Quest Journals
Journal of Research in Mechanical Engineering
Volume 9 ~ Issue 8 (2023) pp: 12-24
ISSN(Online):2321-8185
www.questjournals.org

Research Paper

Nonlocal Thermo-Elastic Damping of a Longitudinally Vibrating Nanotube with Internal Fluid Flow based on DPL-HCM

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ABSTRACT:*Nanotubes that exhibit fluid flow possess a wide range of potential applications in various fields. This is primarily due to their distinct properties and the capability to manipulate and control fluids at the nanoscale. The resonance frequency of nanomechanical systems are adjusted by using thermoelastic damping. The damping effect and resonance frequency are fine-tuned for different uses by manipulating the temperature and flow conditions. By inspired with the applications of fluid carrying nanotubes, a thermo-elastic damping model is derived by using nonlocal elasticity theory and dual phase lag heat conduction model. First, the nonlocal elasticity theory is used to obtain the thermo-elastic governing equation of motion for the longitudinally vibrating nanotube with internal fluid effect and then a dual phase lag-heat conduction model (DPL-HCM) is derived for one-dimensional case to consider the heat conduction phenomenon at nanoscale. The thermo-elastic and heat conduction equations are solved by using axial displacement and temperature in spectral domain to obtain a polynomial eigenvalue problem (PEP) in frequency of longitudinal vibration. The resulting PEP is solved for longitudinal vibration frequency and then the thermo-elastic damping is calculated. The effects of local and nonlocal elasticity on the thermo-elastic damping of fluid carrying nanotubes is brought out exclusively for different fluid velocities, three heat conduction models, various fluids and modenumbers. The results presented in this manuscript are very useful for the design and development of future generation nanoscale fluidic devices.*

KEYWORDS: *Thermo-elastic damping, Dual phase lag, Heat conduction, Nanotube, Nonlocal elasticity, Longitudinal Vibration, Heat Conduction, Fluid flow.*

R*eceived 22 August, 2023;* ***R****evised 31 August, 2023;* ***A****ccepted 06 Sep., 2023 © The author(s) 2023.*
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I. INTRODUCTION

Fluid-carrying nanotubes have become a subject of great interest and exploration in the field of nanotechnology in recent years. These ultra-small tube-like structures, usually made of materials like carbon or other materials at the nanoscale, provide a special and adaptable platform for manipulating fluids at the nanoscale. Fluid-carrying nanotubes offer a unique chance to utilize the exceptional properties of nanomaterials in order to control and manipulate the behavior of liquids and gases, unlike conventional macroscopic fluid conduits. This emerging field has applications in various disciplines, including nanofluidics, drug delivery, energy storage, and environmental remediation [1].

In this article, we will explore the fascinating realm of fluid-carrying nanotubes with respect to thermo-elastic damping.Thus far, numerous scholars in this particular domain have diligently explored a multitude of facets pertaining to the vibrational dynamics, employing the aforementioned theoretical frameworks. As demonstrated by the scholarly work of Yoon et al. [2], a comprehensive investigation was conducted to examine the impact of internal flowing fluid on the phenomenon of free vibration and the occurrence of flow-induced

structural instabilities, specifically divergence and flutter, in carbon nanotubes (CNTs). The authors have suggested that the presence of internal flowing fluid significantly impacts the vibrational frequencies, particularly for suspended carbon nanotubes (CNTs) with longer lengths and larger innermost radii, when subjected to higher flow velocities. Furthermore, they have observed that the rate of amplitude decay and the critical flow velocity for flutter instability may, in certain instances, have practical implications.In their study, Lee and Chang [3] employed the nonlocal elastic model to scrutinize the phenomenon of free transverse vibration in fluid-conveying single-walled carbon nanotubes (SWCNTs). The researchers discovered that augmenting the nonlocal phenomenon resulted in a reduction of the actual frequency component. Furthermore, they observed a substantial impact of the nonlocal parameter on the mode shape. Wang [4] conducted an investigation pertaining to the influence of surface characteristics on the vibration and stability of fluid-conveying nanotubes and nanopipes, taking into consideration both the inner and outer surface layers. The author effectively showcased the profound impact of surface elasticity and residual surface tension on the inherent frequency and critical flow velocity of nanotubes responsible for fluid conveyance.Narendar and Gopalakrishnan [5-15] undertake a comprehensive exploration into the intricate realm of wave propagation, torsional vibration, and spectral finite element analysis pertaining to nanorods. This scholarly pursuit is conducted through the lens of nonlocal continuum mechanics, a theoretical framework that accounts for the influence of infinitesimal elements on the overall behavior of the nanorods.

Teodor et al. [16] explores into the examination of the buckling and post-buckling phenomena exhibited by a rotating nanorod when subjected to axial compression. This investigation is grounded in the classical Euler-Bernoulli theory, which characterizes the behavior of slender structures, and Eringen's nonlocal elasticity model, which accounts for the influence of small-scale effects. The empirical findings indicate that a rise in the nonlocality parameter engenders a corresponding escalation in post-buckling deformation.Zheng et al [17] delves into the intricate realm of investigating the impact of uncertainty pertaining to material properties on the wave propagation characteristics of a nanorod that is embedded within an elastic medium. This investigation is carried out by meticulously constructing a nonlocal nanorod model that takes into account the inherent uncertainties associated with the system. The utilization of numerical outcomes not only facilitates a deeper comprehension of the intricate wave propagation characteristics exhibited by nanostructures possessing uncertain material properties, but also imparts invaluable guidance for ensuring the dependability and resilient design of forthcoming nanodevices.Uzun and Yayli [18] complete an inquiry into the intricacies of a nonlocal finite element formulation pertaining to the phenomenon of free longitudinal vibration. This formulation is specifically developed for functionally graded nano-sized rods, thereby delving into the realm of materials with varying properties at the nanoscale. The consideration of size dependency is accomplished by means of Eringen's nonlocal elasticity theory. The comparative findings indicate that the current model, which focuses on the thermal conductivity of nanorod-based nanofluids, exhibits superior predictive accuracy compared to models pertaining to nanotube-based nanofluids. Furthermore, the latter models outperform the conventional models designed for nanofluids consisting of spherical particles.

Nazemnezhad and Kamali [19] discuss the intricate realm of nonlocal free longitudinal vibration in thick nanorods. Their study centers on the profound influence of lateral motion inertia and shear stiffness effects, ultimately revealing the pivotal role interlayer shear plays in the diverse mechanical behaviors observed.Mustafa [20] investigates longitudinal dynamic analysis of carbon nanotubes which has been modelled as an axially functionally graded Rayleigh-Bishop rod by using nonlocal stress gradient elasticity theory.

In the present manuscript, a thermo-elastic damping model is constructed using nonlocal elasticity theory and a dual phase lag heat conduction model, both of which take inspiration from the uses of fluid-carrying nanotubes. To account for the heat conduction phenomenon at the nanoscale, we first derive a dual phase lag-heat conduction model for the one-dimensional case. This model is used to obtain the thermo-elastic governing equation of motion for the longitudinally vibrating nanotube with internal fluid effect. The polynomial eigenvalue problem (PEP) in frequency of longitudinal vibration is obtained by solving the thermo-elastic and heat conduction equations in the spectral domain using axial displacement and temperature. The thermo-elastic damping is determined by solving the resultant PEP for the frequency of the longitudinal vibrations. Thermo-elastic damping of fluid-carrying nanotubes is exhaustively studied over a wide range of parameters, including fluid velocity, heat conduction model, fluid type, and modenumber. The paper ends with important conclusions.

II. MATHEMATICAL FORMULATION

2.1 Nonlocal Elasticity Theory

The nonlocal elasticity theory, as postulated by Eringen [21], posits that the stress state at a given point of interest, denoted as $\boldsymbol{r}$ within the body, is not solely determined by the strain state at $\boldsymbol{r}$ itself. Rather, it is also influenced by the strain states at all other points $\boldsymbol{r'}$ within the entire domain. The constitutive relation in the

nonlocal elasticity type representation assumes its most comprehensive form when expressed as an integral spanning the entirety of the region under consideration. The integral encompasses a nonlocal kernel function, which elucidates the interplay between strains at different locations and their impact on the stress at a specific location. The constitutive equations pertaining to a linear, homogeneous, isotropic, non-local elastic solid with body forces can be written as

$$\sigma_{ij,j} = 0 \tag{1}$$

$$\sigma_{ij}(x) = \int_{\Omega} \alpha(|\boldsymbol{r} - \boldsymbol{r}'|, \xi) C_{ijkl} \varepsilon_{kl}(\boldsymbol{r}') d\Omega(\boldsymbol{r}'), \ \forall\, x \in \Omega \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

The variable C_{ijkl} represents the elastic modulus tensor in the context of classical isotropic elasticity. Meanwhile, the symbols σ_{ij} and ε_{ij} denote the stress and strain tensors, respectively andu_i signifies the displacement vector. The symbol α represents the nonlocal modulus or attenuation function, which serves to incorporate the influence of nonlocal effects into the constitutive equations. It is dependent on the distance between two points, denoted as $\boldsymbol{r}$ and $\boldsymbol{r}'$, as well as the parameter ξ. The determination of this nonlocal modulus is achieved through the process of aligning the curves of plane waves with those resulting from the dynamics of the atomic lattice. Multiple distinct variations of $\alpha(|\boldsymbol{r} - \boldsymbol{r}'|)$have been documented in the esteemed reference [21]. The expression $|\boldsymbol{r} - \boldsymbol{r}'|$ represents the Euclidean distance, a fundamental concept in mathematics. Additionally, the symbol ξ is defined as e_0 multiplied by a divided by l. Here, a represents an internal characteristic length, such as the length of a C-C bond (0.142 nm) in a carbon nanotube or a granular distance. On the other hand, l represents an external characteristic length, for instance, the wavelength (λ), crack length, or size of the sample. The symbol e_0 represents a nonlocal scaling parameter, which has been postulated as a constant that is deemed suitable for each material in the existing body of published literature. On the other hand, Ω denotes the spatial domain that is encompassed by the physical body under consideration. The selection of the parameter e_0, denoting a length dimension, holds paramount importance in guaranteeing the integrity and soundness of nonlocal models. The determination of this parameter was achieved through the process of aligning the dispersion curves, which were derived from the atomic models, as stated in reference [21]. The estimation of the nonlocal parameter associated with a particular material can be achieved by employing a fitting procedure on the outcomes of atomic lattice dynamics or experimental observations. The kernel function $\alpha(|\boldsymbol{r} - \boldsymbol{r}'|, \xi)$ that is commonly employed (as stated in Eq. (2)) can be expressed as per reference [21].

$$\alpha(|\boldsymbol{r}|, \xi) = \frac{1}{2\pi\xi^2\ell^2} H_0\left(\frac{\sqrt{\boldsymbol{r}\cdot\boldsymbol{r}}}{\xi\ell}\right) \tag{4}$$

where H_0 is the modified Bessel function.

In the realm of one-dimensional nonlocal elasticity, a differential form has been established to describe the relationship between stress and strain. This form, derived from Equation (2) and supported by references [5-15], can be expressed as follows:

$$(1 - \xi^2\ell^2\nabla^2)\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{5}$$

Here, the operator ∇^2 represents the Laplacian operator. It is worth noting that within the realm of nonlocal elasticity, the influence of minute length scales is taken into account by integrating the internal parameter length into the constitutive equation. It is worth noting that in the scenario where the internal characteristic length and is disregarded, implying that the particles within a medium are assumed to be continuously distributed; the value of ξ becomes zero. Consequently, Eq. (5) can be simplified to the constitutive equation commonly associated with classical elasticity.

2.2 Nonlocal Governing Equation for Nanotube with Internal Fluid Flow

Figure 1 provides a schematic representation of a fluid carrying nanorod that is currently being discussed. It serves the purpose of introducing several key parameters, namely the axial coordinate x, the axial displacement $u = u(x, t)$, the length L, the Young's modulus E, coss sectional area $A$and the density ρ. For fluid, the density is ρ_f and cross sectional area is A_f. The displacement field, elastic and thermal strains for this nanorod are explicitly provided within the given context.

$$u = u(x, t) \tag{6}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_T = -\alpha_T\theta \tag{7}$$

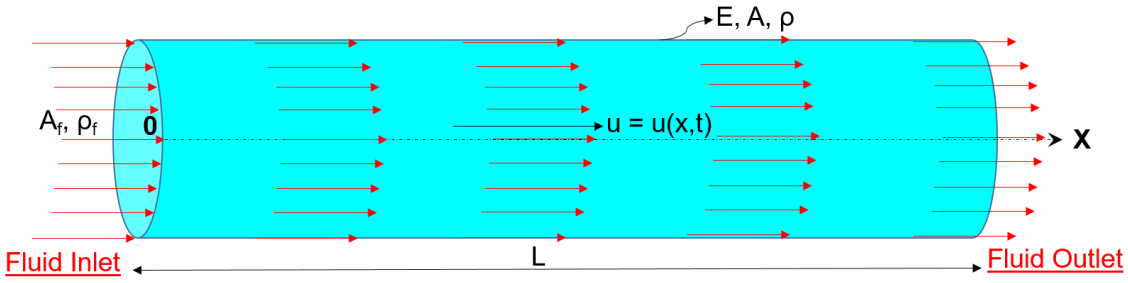

Figure 1:A Schematic of nanotube with internal fluid flow

Here α_T is coefficient of linear thermal expansion and $\theta = T - T_\infty$. For thin rods Eq. (5) can be written in the following one dimensional form

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(\varepsilon_{xx} + \varepsilon_T) = E\left(\frac{\partial u}{\partial x} - \alpha_T \theta\right) \tag{8}$$

where E is the modulus of elasticity, σ_{xx} and ε_{xx} are the local stress and strain components in the x direction, respectively. The equation of motion for a rod under longitudinal vibration with external force (F_{EXT}) can be obtained as

$$\frac{\partial N}{\partial x} + \mathrm{F_{EXT}} = \rho A \frac{\partial^2 u}{\partial t^2} \tag{9}$$

where N is the axial force per unit length and is defined by

$$N = \int_A \sigma_{xx} dA \tag{10}$$

Using Eqs. (10) and (8), we have

$$N - (e_0 a)^2 \frac{\partial^2 N}{\partial x^2} = E\left(\frac{\partial u}{\partial x} - \alpha_T \theta\right) \tag{11}$$

Substitution of the derivative of N with respect to x from Eq. (9) into Eq. (11), we obtain

$$N = EA\left(\frac{\partial u}{\partial x} - \alpha_T \theta\right) - (e_0 a)^2 \frac{\partial F_{EXT}}{\partial x} + (e_0 a)^2 \rho A \frac{\partial^3 u}{\partial x\, \partial t^2} \tag{12}$$

Substituting N from Eq. (12) into the equation of motion (9), we obtain

$$EA\left(\frac{\partial^2 u}{\partial x^2} - \alpha_T \frac{\partial \theta}{\partial x}\right) + \mathrm{F_{EXT}} - (e_0 a)^2 \frac{\partial^2 F_{EXT}}{\partial x^2} = \rho A \frac{\partial^2 u}{\partial t^2} - (e_0 a)^2 \rho A \frac{\partial^4 u}{\partial x^2\, \partial t^2} \tag{13}$$

Equation (13) represents the fundamental governing equation of motion for the nonlocal rod model with longitudinal vibration. When $e_0 a = 0$, the equation is simplified to that of the classical rod model. The force applied by the internal flowing fluid on the nanotube is given by [22]

$$F_{EXT} = -\rho_f A_f \left(\frac{\partial^2 u}{\partial t^2} + V_f^2 \frac{\partial^2 u}{\partial x^2} + 2V_f \frac{\partial^2 u}{\partial x \partial t}\right) \tag{14}$$

Substituting Eq. (14) in Eq. (13) results the following governing equation of motion

$$EA\frac{\partial^2 u}{\partial x^2} - \rho_f A_f \left(\frac{\partial^2 u}{\partial t^2} + V_f^2 \frac{\partial^2 u}{\partial x^2} + 2V_f \frac{\partial^2 u}{\partial x \partial t}\right) + \rho_f A_f (e_0 a)^2 \left(\frac{\partial^4 u}{\partial x^2 \partial t^2} + V_f^2 \frac{\partial^4 u}{\partial x^4} + 2V_f \frac{\partial^4 u}{\partial x^3 \partial t}\right) - \mathrm{EA}\alpha_T \frac{\partial \theta}{\partial x} = \rho A \frac{\partial^2 u}{\partial t^2} - \rho A (e_0 a)^2 \frac{\partial^4 u}{\partial x^2\, \partial t^2} \tag{15}$$

If we set $e_0 a = 0$, in the above equations, it results in the classical governing equation for fluid carrying nanorod.

2.3 Dual Phase-lag Heat Conduction Model for Nanotube

The Dual-Phase-Lag Heat Conduction Model (DPLHCM) serves as a mathematical construct employed to elucidate the intricate phenomenon of heat conduction in various materials, encompassing the realm of nanotubes. The proposed methodology exhibits a heightened level of sophistication and refinement in contrast to conventional heat conduction models, such as the renowned Fourier heat conduction equation. In the context of the classical Fourier heat conduction model, it is postulated that the phenomenon of heat conduction transpires with instantaneous effect, whereby a thermal perturbation imposed upon one extremity of a material promptly propagates throughout said material at an infinitely swift pace. Nevertheless, it is imperative to acknowledge that the phenomenon of heat conduction, particularly when observed at the nanoscale, is subject to temporal constraints. Furthermore, it is crucial to recognize that the velocity at which this process transpires is contingent upon the distinctive characteristics inherent to the material in question. The DPLHCM introduces two time scales to account for the finite thermal relaxation time of materials i.e., hyperbolic heat conduction and wave-like behavior. The general governing equation of DPLHCM in 1D is given as [23]

$$\boldsymbol{q}(\boldsymbol{r}, t+\tau_T) = -k\nabla T(\boldsymbol{r}, t+\tau_T), \tag{16}$$

where $\boldsymbol{q}$ is the heat flux, T is temperature,$\boldsymbol{r}$ is the position vector, k is thermal conductivity of the material,$\tau_T$and τ_q are the phase lags of the temperature gradient and the heat flux vector, respectively. The first order Taylor series expansion of Eq. (16) gives

$$\boldsymbol{q}(\boldsymbol{r}, t) + \tau_q \frac{\partial \boldsymbol{q}(\boldsymbol{r},t)}{\partial t} = -k\left\{\nabla T(\boldsymbol{r}, t) + \tau_T \frac{\partial[\nabla T(\boldsymbol{r},t)]}{\partial t}\right\}, \tag{17}$$

The heat flux, temperature and volumetric strain for a thermoelastic isotropic body have the following relation

$$-\nabla.\boldsymbol{q} = \rho C_v \frac{\partial T}{\partial t} + \frac{E\alpha_T T}{(1-2\nu)}\frac{\partial \boldsymbol{e}}{\partial t} \tag{18}$$

where $\boldsymbol{e}$ is the volumetric strain, $\alpha_T$is thermal expansion coefficient and C_v is specific heat at constant volume. The coupled heat conduction equation for a thermoelastic isotropic body in the context of DPLHCM can be derived from Eqs. (17) and (18) as

$$k\left(\frac{\partial^2\theta}{\partial x^2} + \tau_T \frac{\partial^3\theta}{\partial x^2 \partial t}\right) = \rho C_v\left(\frac{\partial\theta}{\partial t} + \tau_q \frac{\partial^2\theta}{\partial t^2}\right) - \frac{E\alpha T_0}{1-2\nu}\left(\frac{\partial e}{\partial t} + \tau_q \frac{\partial^2 e}{\partial t^2}\right) \tag{19}$$

The volumetric strain for the present case of nanotube is given in Eq. (2), so substituting $e = \partial u/\partial x$ in the Eq. (19) gives the final governing equation of the heat conduction model for the present nanotube as

$$k\left[\frac{\partial^2\theta}{\partial x^2} + \tau_T \frac{\partial^3\theta}{\partial x^2 \partial t}\right] = \rho C_v\left[\frac{\partial\theta}{\partial t} + \tau_q \frac{\partial^2\theta}{\partial t^2}\right] - \frac{E\alpha T_0}{1-2\nu}\left(\frac{\partial^2 u}{\partial x \partial t} + \tau_q \frac{\partial^3 u}{\partial x \partial t^2}\right) \tag{20}$$

If we substitute $\tau_T = 0$ in the above equation, it results in a single-phase lag heat conduction (SPLHC) equation based 1D thermoelasticity. If we substitute $\tau_q = 0$ and $\tau_T = 0$ in the above equation, it results in classical Fourier heat conduction equation based thermoelasticity in 1D.

III. SOLUTION OF THERMO-ELASTIC EQUATIONS

The solution of the thermo-elastic governing equations given in Eqs. (15) and (20) is derivedby assuming a harmonic form of displacement field and temperature as

$$u(x,t) = \hat{U}e^{-i\lambda x}e^{i\omega t},\ \theta(x,t) = \hat{\theta}e^{-i\lambda x}e^{i\omega t} \tag{21}$$

where ω is the circular frequency, λ is the axial wavenumber, $\hat{\mathrm{U}}$ and $\hat{\Theta}$ are the amplitudes of elastic and thermal deformations, respectively and $i = \sqrt{-1}$. Substituting $u(x,t)$ and $\theta(x,t)$given in above equation into Eqs. (15) and (20) leads to the following algebraic equations:

$$\begin{aligned}\Big[-EA\lambda^2 &- \rho_f A_f\left(-\omega^2 - V_f^2\lambda^2 + 2V_f\lambda\omega\right) + \rho_f A_f (e_0 a)^2\left(\omega^2\lambda^2 + V_f^2\lambda^4 - 2V_f\lambda^3\omega\right) + \rho A\omega^2 + \\ &\rho A(e_0 a)^2\omega^2\lambda^2\Big]\hat{U} + [EA\alpha_T i\lambda]\hat{\Theta} = 0\end{aligned} \tag{22}$$

$$\left[\frac{EA\alpha T_\infty}{1-2\nu}\left(\lambda\omega + \tau_q i\lambda\omega^2\right)\right]\hat{U} + \left[k(-\lambda^2 - \tau_T i\lambda^2\omega) - \rho C_v\left(i\omega - \tau_q\omega^2\right)\right]\hat{\Theta} = 0 \tag{23}$$

Rewriting Eqs. (22) and (23) as

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{Bmatrix} \hat{U} \\ \hat{\Theta} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{24}$$

where

$$\begin{aligned} Z_{11} &= R_{211}\omega^2 + R_{111}\omega + R_{011} \\ Z_{12} &= R_{012} \\ Z_{21} &= R_{221}\omega^2 + R_{121}\omega \\ Z_{22} &= R_{222}\omega^2 + R_{122}\omega + R_{022} \end{aligned} \tag{25}$$

Here the coefficients are givens as

$$\begin{aligned} R_{211} &= \rho_f A_f + \rho_f A_f (e_0 a)^2 \lambda^2 + \rho A + \rho A (e_0 a)^2 \lambda^2 \\ R_{111} &= -2\rho_f A_f V_f \lambda - 2\rho_f A_f (e_0 a)^2 V_f \lambda^3 \\ R_{011} &= -EA\lambda^2 + \rho_f A_f V_f^2 \lambda^2 + \rho_f A_f (e_0 a)^2 V_f^2 \lambda^4 \\ R_{012} &= iEA\alpha_T \lambda \\ R_{221} &= \frac{EA\alpha T_\infty}{(1-2\nu)} i\tau_q \lambda \\ R_{121} &= \frac{EA\alpha T_\infty}{(1-2\nu)} \lambda \\ R_{222} &= -\rho C_v \tau_q \\ R_{122} &= i\rho C_v - ik\tau_T \lambda^2 \\ R_{022} &= k\lambda^2 \end{aligned} \tag{26}$$

Equation (24) can be written in Polynomial Eigenvalue Problem (PEP) form as

$$\begin{bmatrix} R_{211} & 0 \\ R_{221} & R_{222} \end{bmatrix} \omega^2 + \begin{bmatrix} R_{111} & 0 \\ R_{121} & R_{122} \end{bmatrix} \omega + \begin{bmatrix} R_{011} & R_{012} \\ 0 & R_{022} \end{bmatrix} = 0 \tag{27}$$

The above equation is solved numerically using “polyeig” command of MATLAB [24]. The part of the code is given below:

```
    R211=rhof*Af+rhof*Af*S^2*lam^2+rho*A+rho*A*S^2*lam^2;
    R111=-2*rhof*Af*V*lam-2*rhof*Af*S^2*V*lam^3;
    R011=-E*A*lam^2+rhof*Af*V^2*lam^2+rhof*Af*S^2*V*lam^4;
    R012=j*E*A*at*lam;
    R221=E*A*at*T0*j*tq*lam/(1-2*nu);
    R121=E*A*at*T0*lam/(1-2*nu);
    R222=-rho*Cv*tq;
    R122=j*rho*Cv-j*k*tt*lam^2;
    R022=k*lam^2;
      G2 = [R211 0;R221 R222];
      G1 = [R111 0; R121 R122];
      G0 = [R011 R012; 0 R022];
            w_j=polyeig(G0,G1,G2);
            [wj,I]=sort(w_j);
      m=1;
for n=1:4
        Q(m,i)=2*abs(imag(wj(n))/real(wj(n)));
        m=m+1;
end
```

(28)

The PEP is solved to obtain the frequency of the fluid carrying nanotube in the form as $\omega = \omega_R + i\omega_I$. The thermo-elastic damping is defined as

$$Q^{-1} = 2\left|\frac{\omega_I}{\omega_R}\right| \tag{29}$$

In the next section, the results are analyzed for various cases.

IV. NUMERICAL RESULTS AND DISCUSSION

For the present analysis, a nanorod composed of silicon materialis considered with the following material properties:
Young's modulus E= 169 GPa,
density ρ = 2330 kg/m^3,
thermal conductivity k = 160 W/m-K,
specific heat at constant volume C_v= 715 J/kg-K,
coefficient of thermal expansion α_T= 2.60×10^{-6}K^{-1} and
initial temperature T_∞= 290 K.
Fluid is considered as water with density of ρ_f = 1000 kg/m^3.

The phenomenon of thermoelastic damping in nanotubes and its dependence on the length of the nanorod is an intriguing aspect of nanoscale heat transfer and mechanical vibrations. Thermoelastic damping arises due to the interaction between mechanical deformation and the associated temperature changes in a material. When considering nanotubes with fluid flow, the effects of length become particularly relevant. Let's explore the relationship between thermoelastic damping, nanotube length, and fluid flow: Thermo-elastic damping variation of a nanotube with respect to its length is shown in Fig. 2 for both the local and nonlocal elasticity cases. The effect of the fluid on the thermo-elastic damping is also shown in the graph.As the nanotube undergoes stress and strain, it generates thermal energy owing to the thermoelastic characteristics inherent in the material. The thermal energy, in its essence, causes temperature gradients within the nanotube.It can be seen that the thermo-elastic damping under predicted by the classical elasticity ($\epsilon = e_0 a = 0\ nm$) as compare to nonlocal elasticity ($\epsilon = 1\ nm$). This is clearly understood that why nonlocal elastic model is needed for ultra-small scale structures like nanotubes. The magnitude of a nanotube or nanorod's length is a pivotal determinant that exerts a profound impact on its thermoelastic characteristics. As the longitudinal dimension of the nanotube is extended, the thermal energy arising from thermoelastic phenomena is afforded a greater expanse for dissipation along the tubular axis.

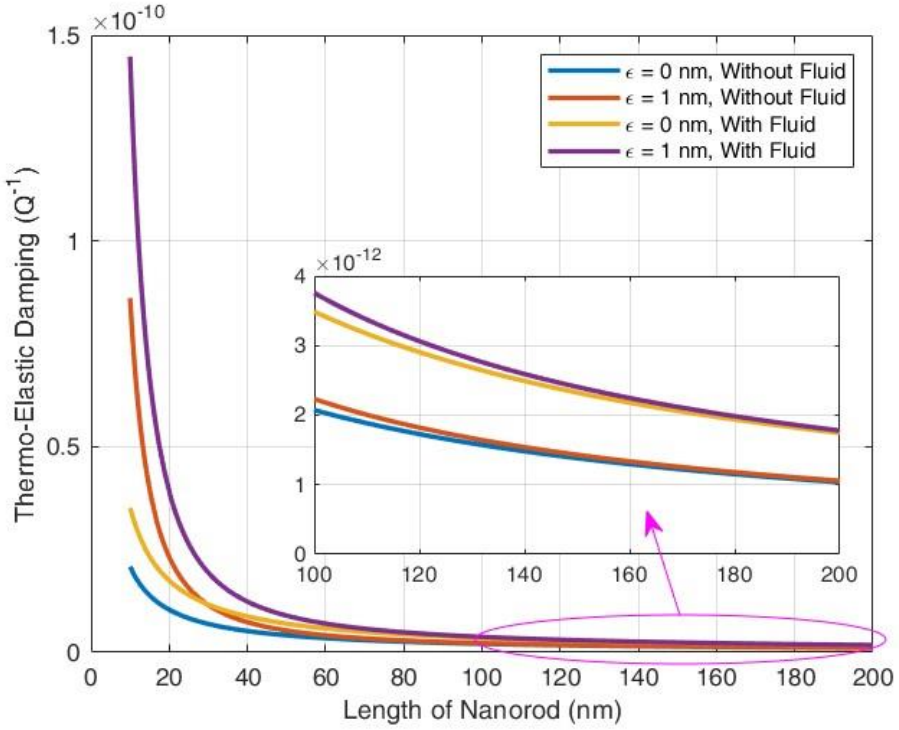

Figure2: Effect of internal fluid on thermo-elastic damping of nanotube with length of nanorod based on local ($\epsilon = 0\ nm$) and nonlocal ($\epsilon = 1\ nm$) elasticity theories and classical heat conduction model

In the case of nanotubes with reduced length, it is observed that the resonance frequencies pertaining to mechanical vibrations exhibit an elevation, thereby rendering them higher in magnitude. Furthermore, it is noteworthy that these vibrations are subject to intensified thermoelastic damping phenomena. The resonance frequencies of elongated nanotubes may exhibit a reduction, potentially influencing the magnitude and attributes of thermoelastic damping.

When fluid is present either within or around the nanotube, the temperature gradients have the potential to engage in interactions with the fluid. The fluid present in nanotube can also influence on thermoelastic damping as seen in Fig. 2.When fluid is filled in nanotube ($V_f = 0$), the thermo-elastic damping is enhanced for lengths less than 40 nm. For longer lengths, the phenomenon is negligible as clearly seen in Fig. 2.

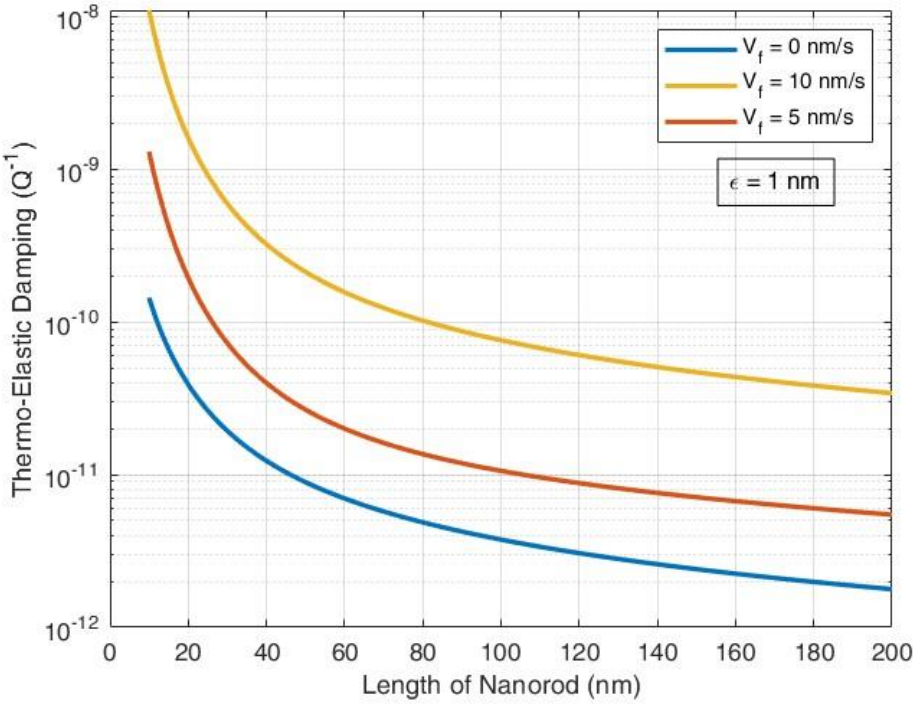

Figure3: Effect of velocity of internal fluid on thermo-elastic damping of nanotube with length of nanorod nonlocal ($\epsilon = 1\ nm$) elasticity theory and classical heat conduction model

The phenomenon of fluid flow (V_f) occurring within or in the vicinity of the nanotube has the potential to augment thermoelastic damping by facilitating the more effective convection of heat away from the surface of said nanotube. The dissipation of heat can be contingent upon the velocity of fluid flow and the thermal characteristics of said fluid.The influence of fluid velocity in nanotube on thermo-elastic damping is shown in Fig. 3. The results are presented for nonlocal elasticity case. It can be clearly observed that as the fluid velocity increases, the thermo-elastic damping also increase.Boundary conditions also play a pivotal role in determining the behavior of a system, particularly when considering the interplay between fluid flow patterns and thermoelastic damping. Factors such as whether a nanotube is clamped at its ends or allowed to move freely can significantly affect these dynamics.So, a clamped-free nanotube is considered in the complete analysis withwavenumber $\lambda = m\pi/L$, where m is the modenumber and L is the length of the nanotube.

Classical, single-phaselag, and dual-phase lag heat conduction models are different mathematical approaches used to describe heat transfer or heat conduction in materials. The effects of these three heat conduction models on thermo-elastic damping of fluid carrying nanotube are shown in Fig. 4 for classical and nonlocal elasticity models too.The classical heat conduction model (CHCM) assumes that heat conduction occurs instantaneously and at an infinite speed throughout the material. This means that any change in temperature at one point in the material immediately affects all other points.While the classical model can adequately represent heat transmission on a macroscopic scale, it may fail to do so on a nanoscale or in materials subject to fast temperature variations. Due to its fundamental assumption, the CHCM along with classical elasticity model predicts lower thermo-elastic damping for the fluid carrying nanotubes as shown in Fig. 4. The incorporation of the single-phase lag heat conduction model (SPL-HCM) introduces a finite thermal relaxation time for materials. It is duly recognized that the process of heat conduction necessitates a finite duration to transpire, rather than manifesting instantaneously. The utilization of the single-phase lag model is employed as a means to elucidate the intricate phenomenon of heat conduction in substances that possess finite thermal relaxation durations. Notably, this model finds relevance in the analysis of various materials, including but not limited to polymers, nanomaterials, and thin films.The SPL-HCM predicts the correct thermo-elastic damping for the fluid filled nanotubes as it is directly applicable for nanoscale structures. The predictions of SPL-HCM are higher than the classical ones as shown in Fig. 4. The dual-phase lag heat conduction model

(DPL-HCM) expands upon the notion of thermal relaxation by incorporating the temporal delays in both temperature gradient and heat flux. It elucidates the presence of distinct temporal scales of relaxation, one pertaining to the thermal gradient and the other to the thermal flux. The utilization of the dual-phase lag model is imperative in scenarios wherein the process of heat conduction is substantially influenced by the finite relaxation times of both temperature and heat flux. The examination of heat transfer at the nanoscale and in materials exhibiting expeditious heat transfer dynamics holds significant pertinence.The predictions of DPL-HCM are higher than that of the SPL-HCM and classical ones as shown in Fig. 4.Due to the additional thermal relaxation parameter, the DPL-HCM predicts the accurate thermo-elastic damping of the fluid carrying nanotubes.

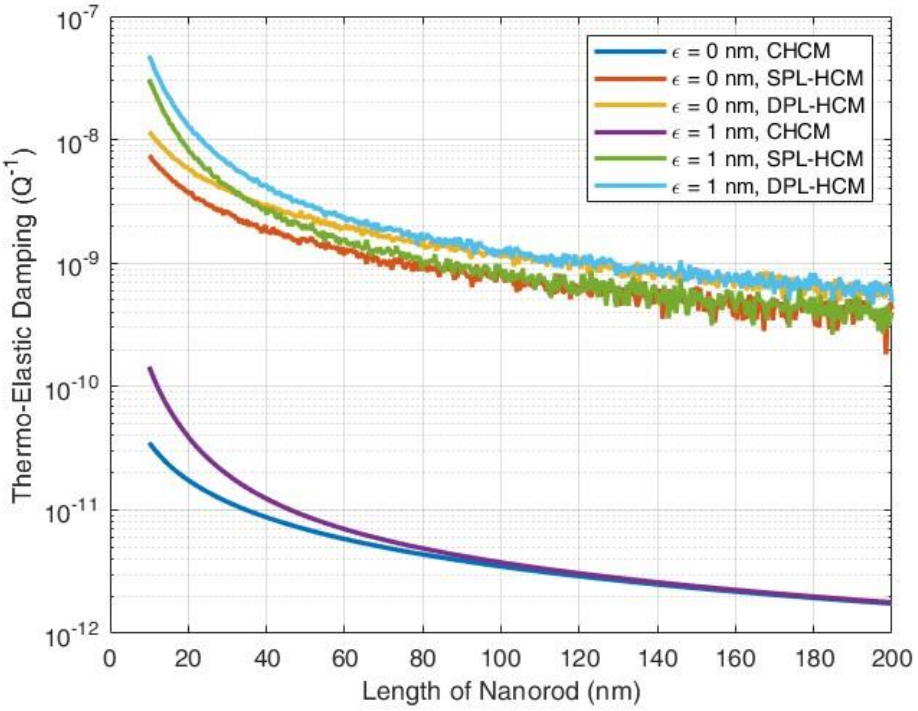

Figure 4: Variation of thermo-elastic damping of nanotube with internal fluid versus length of nanorod based on classical, single-phase-lag and dual-phase-lag heat conduction models and local ($\epsilon = 0\ nm$) and nonlocal ($\epsilon = 1\ nm$) elasticity theories

The thermo-elastic damping phenomenon exhibited by a fluid-filled nanotube is subject to the influence of various factors, one of which pertains to the inherent characteristics of the fluid involved. The property in question, known as fluid density, holds considerable influence over the thermo-elastic damping behavior. Let us embark upon an investigation into the intricate interplay between fluid density and thermo-elastic damping within the confines of a nanotube that is replete with fluid as shown in Fig. 5.Application-specific thermo-elastic damping may be achieved by selecting a fluid density in addition to other fluid parameters like thermal conductivity and specific heat capacity. Designers may choose fluids for resonant sensors or energy harvesting systems based on their damping properties. For the present graph, the fluid velocity is assumed as $V_f = 0$.

The heat capacity of the fluid within the nanotube is often greater when the fluid has a higher density (such as Trichlor Ethylene). This implies that there will be less temperature differences between different parts of the nanotube due to the increased ability to absorb and release heat.However, fluids with a lower density (such as Propane R-290) might undergo larger temperature shifts for the same mechanical deformation because their heat capacities are smaller. The nanotube's internal temperature gradient may increase as a consequence. These effects are clearly observed in Fig. 5. The amount of thermo-elastic damping is often reduced in fluids with higher densities. This is because they are better at dampening vibrations by absorbing and spreading the heat produced by mechanical deformation.Thermo-elastic damping may be improved with fluids of lower density due to the lower heat capacity and perhaps more noticeable temperature gradients inside the nanotube that these fluids may display. The dampening effects may intensify as a result.

Fluid movement ($V_f = 10\ nm/s$) inside the nanotube is also modify the impact of fluid density on thermo-elastic damping. The temperature distribution and the subsequent damping behavior of nanotubes is

affected by the convection of heat away from the surface by the flow of fluid. Such effects are shown in Fig. 6 from nonlocal elasticity ($\epsilon = 1\ nm$).

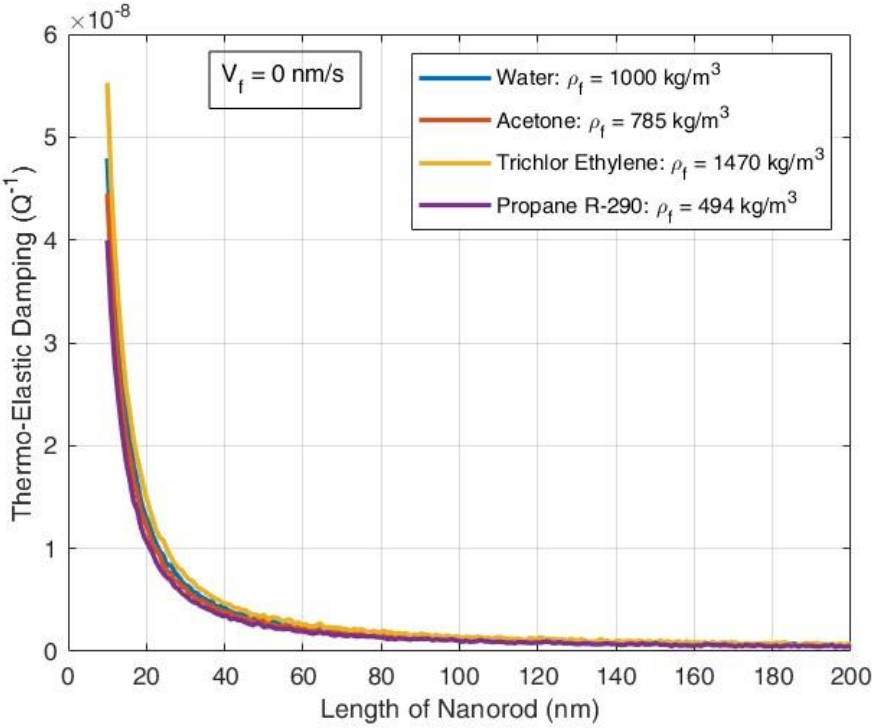

Figure 5: Variation of thermo-elastic damping of nanotube with different internal fluids having zero velocity versus length of nanorod based on dual-phase-lag heat conduction model and nonlocal ($\epsilon = 1\ nm$) elasticity theory

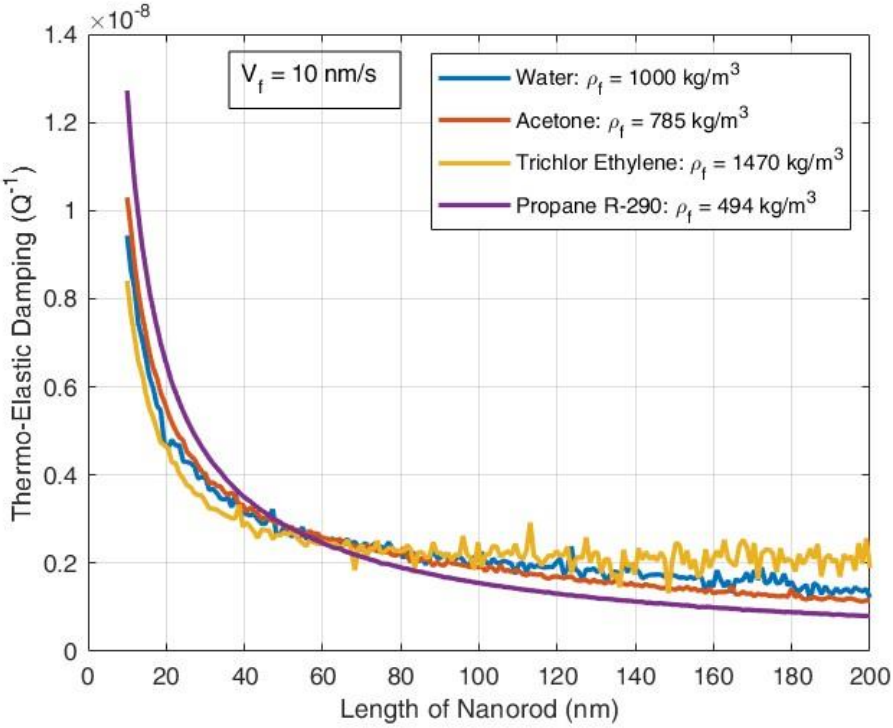

Figure 6: Variation of thermo-elastic damping of nanotube with different internal fluids having velocity of 10 nm/s versus length of nanorod based on dual-phase-lag heat conduction model and nonlocal ($\epsilon = 1\ nm$) elasticity theory

With fluid velocity and higher the density of the fluid a vibrational disturbance is seen in the thermo-elastic damping response of the nanotube in Fig. 6. As the density of the fluid reduces, these disturbances are vanished for the longer lengths of the nanotube. For shorter lengths i.e., less than 20 nm, such disturbance is not observed even for higher density fluid flow.

In a nanotube filled with fluid, the thermo-elastic damping is significantly affected by the number of vibration modes (m). The effect of vibrational modenumber on thermo-elastic damping of nanotube with fluid flow is shown in Fig. 7. The amplitude and frequency of mechanical vibrations, which in turn influence the amount of thermo-elastic damping, are both affected by the mode number of the vibrations. Every distinct vibration mode possesses a singular frequency and mode shape, thereby delineating the specific manner in which the nanotube undergoes deformation during the process of vibration. When the nanotube is undergoing vibrational oscillations in low mode numbers or low-frequency modes, the mechanical vibrations are generally characterized by reduced velocity and increased temporal duration. Vibrations at lower frequencies cause the nanotube to retain heat from thermo-elastic effects for longer. In the realm of high modes or modes characterized by high frequencies, mechanical vibrations manifest themselves through oscillations occurring at elevated frequencies and correspondingly reduced periods. The occurrence of vibrations with higher frequencies induces expeditious and repetitive deformations as well as alterations in temperature within the nanotube. Such effects are clearly observed in Fig. 7.

When building fluid-filled nanotube-based devices like resonators or sensors, engineers and researchers may take the vibration mode number into account. Thermo-elastic damping may be optimized or controlled by selecting the appropriate vibration mode.Other aspects, such as fluid properties, fluid flow conditions, and nanotube shape, must be taken into account in addition to thermo-elastic damping in order to produce the necessary damping characteristics and device performance.

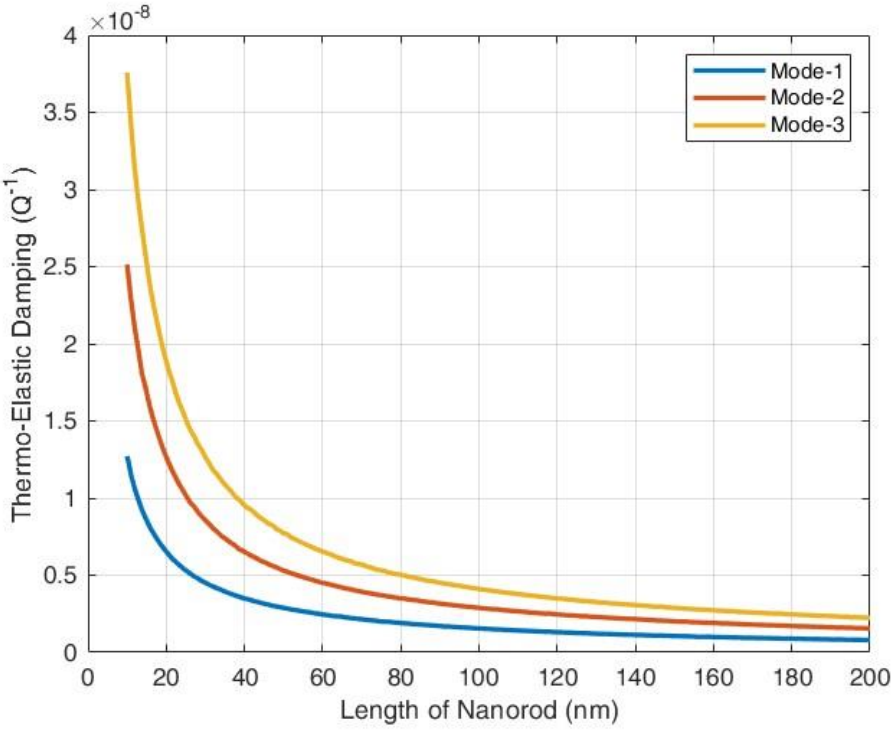

Figure 6: Variation of thermo-elastic damping of nanotube with propane R-290 with a velocity of 10 nm/s versus length of nanorod based on dual-phase-lag heat conduction model and nonlocal ($\epsilon = 1\ nm$) elasticity theory for first three modes of vibration in longitudinal direction

V. CONCLUSIONS

The design of nanomechanical sensors and resonators relies heavily on an appreciation of the interaction between nanotube length, fluid flow, and thermoelastic damping. To attain the appropriate resonance characteristics, engineers may try to optimize the length and fluid environment. To understand these phenomenon with respect to thermo-elastic damping of fluid filled nanotubes, a nonlocal elasticity theory and nonclassical heat conduction models are utilized. These nonlocal thermo-elastic governing equations are solved

for the thermo-elastic damping of nanotubes with fluid flow and the following major outcomes are observed in this manuscript:

- The thermo-elastic damping under predicted by the classical elasticity as compare to nonlocal elasticity.
- As the longitudinal dimension of the nanotube is extended, the thermal energy arising from thermoelastic phenomena is
- When fluid is filled in nanotube with zero velocity, the thermo-elastic damping is enhanced for lengths less than 40 nm. For longer lengths, the phenomenon is negligible afforded a greater expanse for dissipation along the tubular axis.
- As the fluid velocity increases, the thermo-elastic damping also increase.
- The predictions of thermo-elastic damping of fluid filled nanotubes by DPL-HCM are higher than that of the SPL-HCM and C-HCM.
- Due to the additional thermal relaxation parameter over SPL-HCM, the DPL-HCM predicts the accurate thermo-elastic damping of the fluid carrying nanotubes.
- The amount of thermo-elastic damping is often reduced in fluids with higher densities. This is because they are better at dampening vibrations by absorbing and spreading the heat produced by mechanical deformation.
- Vibrations at lower frequencies cause the nanotube to retain heat from thermo-elastic effects for longer.
- In the realm of high modes or modes characterized by high frequencies, mechanical vibrations manifest themselves through oscillations occurring at elevated frequencies and correspondingly reduced periods.
- Thermoelastic damping may be used to increase the efficiency of energy harvesting systems by increasing the rate at which energy is converted.
- The balance between damping and energy conversion may be modified by adjusting the length of the nanotubes and the fluid conditions.

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