Quest Journals
Journal of Software Engineering and Simulation
Volume 9 ~ Issue 3 (2023) pp: 50-56
ISSN(Online) :2321-3795 ISSN (Print):2321-3809
www.questjournals.org

Research Paper

Fixed-time Consensus of Nonlinear Multi-agent Systems with External Disturbance under Antagonistic Network

Jing Han[1], Tao Li[1,*], Kaiyin Huang[1], Shihao Li[2]
[1]*School of Electrical Engineering and Automation, Hubei Normal University, Huangshi 435002, RP China*
[2]*School of Electronics and Information, Yangtze University, Jingzhou 434023, RP China*
Corresponding Author: Tao Li

ABSTRACT:*Under a signed graph, the fixed-time bipartite consensus problem for the non-linear multi-agent systems with external disturbance is studied in this paper. Considering antagonistic and cooperative relationship among the agents, we design the fixed-time distributed control law, which assures that system states converge to two values of the same modulus but in contrary directions within fixed time, the upper bound of convergence time is independent of original state. Then, the sufficient conditions are proposed to reach fixed-time bipartite consensus by applying Lyapunov stability theory and algebraic graph theory, the upper bound of the convergence time is proposed and the stability about control algorithm is discussed in detail. Finally, simulation examples verify the theoretical analytical results.*

KEYWORDS:*nonlinear multi-agent system, antagonistic network, external disturbance, fixed-time control, bipartite consensus*

Received 10Mar., 2023; Revised 20Mar., 2023; Accepted 23 Mar., 2023

I. INTRODUCTION

Recently, a growing number of people have paid attention to multi-agent systems which are widely applied in various fields, for example, drone formation, biology and intelligent transportation and so on [1-4].

Cooperative control of multi-agent systems is that agents work together and accomplish complicated tasks, and the consensus problem is the foundation of cooperative control [5-11]. Consensus problem is to contrive a protocol to reach consensus after the agents communicate with each other, which promotes the reliability and robustness of the system effectively. In reference [12], the consensus questions about dynamic agents with fixed and switched topologies were discussed. Aiming at second-order non-linear multi-agent systems, Qian et al. discussed the consensus question owning time delay [13]. For linear and non-linear multi-agent systems, Li et al. provided relative status distributed consensus laws with adaptive protocol which can modify coupling weights between neighboring agents to achieve consensus [14]. For second-order strict feedback non-linear leader-follower multi-agent systems, Wang et al. investigated the distributed consensus problem [15]. However, in real application, there is not only cooperation but also antagonism among agents. This phenomenon generally exists in nature, industry and other fields, and the concept of bipartite consensus emerges. Altafini was the first one to introduce the conception of bipartite consensus [16]. It was to say that agents were classified as two different sets and converged to states of the same value but in opposite directions. And the work of bipartite consensus has achieved great development. Under symbolic digraphs, Zhang et al. discussed the bipartite consensus for the linear multi-agent systems [17]. Guo et al. looked into the consensus for multi-agent systems with finite communication delay based on signed digraphs [18]. Qi et al. provided a totally distributed regular event-triggered law in response to the single-integral multi-agents bipartite consensus problem [19]. Aiming at linear multi-agent systems of one leader and uncharted leader input, Nan et al. researched the bipartite consensus tracking problems [20].

In the discussion of multi-agent system consensus, not only are final convergence states of agents considered, but also the speed of achieving consensus is considered. To solve this problem, scholars come up with the concept of asymptotic consensus that states of agent converge to same value at infinite time. Obviously, due to the limitation of time and cost, how to improve the speed of convergence is an important research direction. Finite-time control can make agents achieve consensus in finite time which saves resources compared with asymptotic consensus. Considering second-order systems, Wang et al. designed distributed coordination finite-time controllers to reach consensus [22]. In accordance with distributed finite-time state observer, Wang et al. put a control protocol to solve leader-following output consensus for multi-agent systems in the directed topology with disturbances [23]. For non-linear second-order systems with external disturbance, Lü et al. designed two containment control protocols terminal and nonsingular terminal sliding mode to solve finite-time containment control problems [24]. Anyway, the convergence time required to get to finite-time consensus is influenced by the original value of system. In real application, it is hard to get the original value of system, thus, the conception of fixed-time consensus emerges. Polyakov was the first to come up with the conception of fixed-time control, then gave sufficient condition of fixed-time stability about non-linear systems [25]. Dfoort et al. researched fixed-time consensus for the first-order unknown inherent nonlinear multi-agent systems [26]. In reference [27], aiming at the linear multi-agent systems with disturbance, bipartite fixed-time consensus was studied. Ning et al. researched fixed-time consensus and finite-time consensus for multi-agent systems with the leader [28].

Researchers have made great contributions on research of multi-agent systems consensus, however, these work only take into account one or two aspects of nonlinearity, external disturbances and competition among agents, but not comprehensively considering all these factors. So, here we comprehensively consider these factors to study fixed-time consensus of non-linear multi-agent systems with disturbance under antagonistic network. It is more significant in practical application.

Structure of this paper is described as following. Section 2 presents preparatory work and statement of problem. Section 3 provides the fixed-time bipartite consensus control protocol, the upper bound of the convergence time and analyses its stability in detail. Section 4 provides simulations. Finally, section 5 contains the conclusions.

II. PRELIMINARIES AND STATEMENT OF PROBLEM

2.1 GRAPH THEORY

The graph $G=(V,E,A)$ represents a graph which includes N nodes. It is made up of three parts: $V=\{v_1,\ldots,v_N\}$ 、 $A=\left[a_{ij}\right]\in R^{N\times N}$ and $E\subseteq V\times V$. $V=\{v_1,\ldots,v_N\}$ denotes the vertices of graph G , the i th agent is represented by v_i . They are connected when a path is in nodes v_i and v_j , when two nodes are connected, the undirected graph is connected. $E\subseteq V\times V$ is a set called edge set. Two agents are said to be neighbors in case agent i and agent j are capable of delivering message. $N_i=\left\{v_j\in V \mid \left(v_i,v_j\right)\in E\right\}$ is a set denoting all neighbour nodes of node v_i . $A=\left[a_{ij}\right]\in R^{N\times N}$ is adjacency matrix of graph G . When value of a_{ij} in the weight matrix can be greater than or less than 0, graph G is a signed graph. The G can be repartitioned into $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$, this two graphs are nonempty and disjoint graphs, then signed graph G is considered to be structural balance. Details are in definition 1. If agents j and i are cooperatives, then $a_{ij}>0$. If agents j and i are competitive, then $a_{ij}<0$. Moreover, The graph G contains no self-loops, $a_{ii}=0$. $d_i=\sum_{i=1,j=i}^{N}\left|a_{ij}\right|$, and $D=\mathrm{diag}\left[d_1,\ldots,d_N\right]$ is termed as the degree matrix. For the conception of adjacency matrix and degree matrix, Laplace matrix is $L=D-A$. $l_{ij}=\begin{cases}\sum_{j=1,j\neq i}^{N}\left|a_{ij}\right|, i=j\\ -a_{ij}, i\neq j\end{cases}$.

2.2 SOME LEMMAS AND DEFINITIONS

Definition 1[16]**.** If there exists two subsets V_1 and V_2 in a node set V that meets the following two factors, signed graph is considered to be structural balance.

(1) $V_1\cup V_2=V\, and\, V_1\cap V_2=\varnothing$.

(2) $\begin{cases} a_{ij}\geq 0,\forall i,j\in V_l\ (l=1,2)\\ a_{ij}\leq 0,\forall i\in V_l, j\in V_h, j\neq h\ (j,h=1,2)\end{cases}$.

Definition 2[25]**.** For system that: $\dot{x}(t)=f\left(x\left(t\right),t\right)$, $x\left(0\right)=x_0$, where $\mathrm{X}\subseteq R^n$ and $f:R^{+}\times R^n\rightarrow R^n$ is a non-linear function. The solutions are known in the sense of Filippov. If it is globally finite-time stable and the convergence time function $T\left(x_0\right)$ is bounded, then equilibrium point of the system is called to be fixed-time stable, i.e., $\exists T_{max}>0$, s.t. $T\left(x_0\right)\le T_{max}$, $\forall x_0\in R^n$.

Lemma 1[27]**.** Assuming that $\lambda_k\left(L\right)$ represents the k th eigenvalue of Laplace matrix L ,if the graph G is a connected graph, $0=\lambda_1\left(L\right)<\lambda_2\left(L\right)\le\ldots\le\lambda_N\left(L\right)$.

Lemma 2[12]**.** When graph G is connected and undirected, L as Laplace matrix of graph G is positive semi-definite, and its simple eigenvalue is 0. The L meets $x^TLx=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\left(x_i-x_j\right)^2$. Representing $\lambda_2\left(L\right)$ as the second smallest eigenvalue of L, $\lambda_2\left(L\right)=\min_{x\neq0,1^Tx=0}\frac{x^TLx}{x^Tx}$, then $x^TLx\ge\lambda_2\left(L\right)x^Tx$.

Lemma 3[25]**.** $V\left(x\right)$ is a positive definite and continuous radially unbounded function and it meets $\dot{V}(x)\le-\eta V^m\left(x\right)-\gamma V^n\left(x\right)$ for some $\eta>0$, $\gamma>0$, $0<n<1$, $m>1$, then the system is fixed-time stable globally, the convergence time T is valued as $T\le T_{max}=\frac{1}{\eta\left(m-1\right)}+\frac{1}{\gamma\left(1-n\right)}$, T_{max} is the upper boundary.

Lemma 4[27]**.**Let $\xi_1,\xi_2,\ldots,\xi_M\ge0$. When $0\le q\le1$,

$$\sum_{i=1}^{M}\xi_i^q\ge(\sum_{i=1}^{M}\xi_i)^q.$$

When $q>1$,

$$M^{1-q}(\sum_{i=1}^{M}\xi_i)^q\le\sum_{i=1}^{M}\xi_i^q\le(\sum_{i=1}^{M}\xi_i)^q.$$

2.3 PROBLEM STATEMENT

To settle fixed-time bipartite consensus of non-linear multi-agent systems with disturbance, we consider the whole N agents making up the multi-agent systems, Following is the dynamic model of the i thagent $\left(i=1,2,\ldots\mathrm{N}\right)$.

$$\dot{x}_i\left(t\right)=f\left(x_i\left(t\right),t\right)+u_i\left(t\right)+\omega_i\left(t\right)\tag{1}$$

$x_i\left(t\right)\in R$ is position state, $f\left(x_i\left(t\right),t\right)$ is a non-linear function, the control input is $u_i\left(t\right)\in R$. the external disturbance $\omega_i\left(t\right)$ meets $\left|\omega_i\left(t\right)\right|\le d,d\ge0$. This paper supposes that the system state space is in the real space R, but the study conclusion can be applied into R^n by using Kronecker production operator.

The local error about the i th agent is confirmed by following

$$e_i(t)=\sum_{j=1}^{N}a_{ij}(x_i(t)-sign(a_{ij})x_j(t))\tag{2}$$

Definition 3. For system (1), when there exists T that makes

$$\lim_{t\to T}\left|x_i(t)-sign\left(a_{ij}\right)x_j(t)\right|=0,i,j=1,2,\ldots,N\tag{3}$$

and when $t\ge T$, $\left|x_i(t)\right|=\left|x_j(t)\right|$.This denotes that the multi-agent systems reach fixed-time bipartite consensus. The convergence time T is bounded, and exists T_{max} that satisfies $T\le T_{max}$.

Assumption 1. The non-linear function $f\left(x_i\left(t\right),t\right)$ is bounded. It is to say that a positive constant c exists and $\left|f\left(x_i\left(t\right),t\right)\right|\le c$.

III. CONTROL PROTOCOL DESIGN AND STABILITY ANALYSIS

For the nonlinear multi-agent systems with disturbances, a control protocol is provided. The communication topology for multi-agent system is structural balance and undirected, the fixed-time control protocol is represented by

$$\begin{aligned} u_i(t) = &-\alpha\Big(\sum_{j=1}^{N} a_{ij}\left(x_i - sign\left(a_{ij}\right)x_j\right)\Big)^{\frac{b}{a}} - \beta\Big(\sum_{j=1}^{N} a_{ij}\left(x_i - sign\left(a_{ij}\right)x_j\right)\Big)^{\frac{q}{p}} \\ &-\rho sign\sum_{j=1}^{N} a_{ij}\left(x_i - sign\left(a_{ij}\right)x_j\right) \end{aligned} \tag{4}$$

Where $\alpha > 0, \rho > 0, \beta > 0, q > p > 0, a > b > 0$. a, b, p and q are positive odd numbers.

Theorem 1. Under the circumstance that communication topology is undirected connected signed graph and structural balance, supposing that multi-agent system (1) meets assumption 1. If $\rho \geq d + c$, with the above control protocol (4), the system (1) is able to get to the fixed-time bipartite consensus. Namely,

$$\lim_{t \to T(x)} \left| x_i - sign\left(a_{ij}\right)x_j \right| = 0, i, j = 1, 2, \ldots, N .$$

And the convergence time meets

$$T(x) \leq \frac{1}{\alpha\left(2\lambda_2(L)\right)^{\frac{b+a}{2a}}}\frac{2a}{a-b} + \frac{1}{\beta N^{\frac{p-q}{2p}}\left(2\lambda_2(L)\right)^{\frac{p+q}{2p}}}\frac{2p}{p-q} .$$

Proof. Construct the Lyapunov function

$$V(\mathrm{t}) = \frac{1}{2}x^T(t)Lx(\mathrm{t}) \tag{5}$$

$$\begin{aligned} \dot{V}(t) &= x^T(t)L\dot{x}(t) \\ &= \sum_{i=1}^{N}\Big(\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t))\big(f(x_i(t),t) + u_i(t) + \omega_i(t)\big) \\ &= -\alpha\sum_{i=1}^{N}\Big(\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t))\Big)^{\frac{b}{a}+1} - \beta\sum_{i=1}^{N}\Big(\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t))\Big)^{\frac{q}{p}+1} \\ &\quad -\rho\sum_{i=1}^{N}\left|\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t)\right| + \sum_{i=1}^{N}\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t))f(x_i(t),t) \\ &\quad +\sum_{i=1}^{N}\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t))\omega_i(t) \\ &\leq -\alpha\sum_{i=1}^{N}\Big(\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t))\Big)^{\frac{b+a}{a}} - \beta\sum_{i=1}^{N}\Big(\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t))\Big)^{\frac{q+p}{p}} \\ &\quad -(\rho - d - c)\sum_{i=1}^{N}\left|\sum_{j=1}^{N}\left|a_{ij}\right|(x_i(t) - sign(a_{ij})x_j(t)\right| \end{aligned} \tag{6}$$

For $\rho \geq d + c$, we can get

$$\left(\rho - d - c\right)\sum_{i=1}^{N}\left|\sum_{j=1}^{N}\left|a_{ij}\right|\left(x_i(t) - sign\left(a_{ij}\right)x_j(t)\right|\right. \geq 0 \tag{7}$$

Based on lemma 2 and lemma 4, combining (6) and (7), we can gain

$$\dot{V}(t) \le -\alpha \sum_{i=1}^{N} (\sum_{j=1}^{N} \left|a_{ij}\right| (x_i(t) - sign(a_{ij}) x_j(t)))^{\frac{b+a}{a}} - \beta \sum_{i=1}^{N} (\sum_{j=1}^{N} \left|a_{ij}\right| (x_i(t) - sign(a_{ij}) x_j(t)))^{\frac{q+p}{p}}$$
$$= -\alpha \sum_{i=1}^{N} ((\sum_{j=1}^{N} \left|a_{ij}\right| (x_i(t) - sign(a_{ij}) x_j(t)))^2)^{\frac{b+a}{2a}}$$
$$-\beta \sum_{i=1}^{N} ((\sum_{j=1}^{N} \left|a_{ij}\right| (x_i(t) - sign(a_{ij}) x_j(t)))^2)^{\frac{q+p}{2p}}$$
$$\le -\alpha \sum_{i=1}^{N} ((\sum_{j=1}^{N} \left|a_{ij}\right| (x_i(t) - sign(a_{ij}) x_j(t)))^2)^{\frac{b+a}{2a}} \tag{8}$$
$$-\beta N^{\frac{p-q}{2p}} \sum_{i=1}^{N} ((\sum_{j=1}^{N} \left|a_{ij}\right| (x_i(t) - sign(a_{ij}) x_j(t)))^2)^{\frac{q+p}{2p}}$$
$$\le -\alpha (2\lambda_2(L) V(x))^{\frac{b+a}{2a}} - \beta N^{\frac{p-q}{2p}} (2\lambda_2(L) V(x))^{\frac{q+p}{2p}}$$
$$= -\alpha (2\lambda_2(L))^{\frac{b+a}{2a}} V^{\frac{b+a}{2a}}(x) - \beta N^{\frac{p-q}{2p}} (2\lambda_2(L))^{\frac{q+p}{2p}} V^{\frac{q+p}{2p}}(x)$$

By lemma 3, $\lim\limits_{t \to T(x)} V(t) = 0$ can be gained, so $\lim\limits_{t \to T(x)} \left| x_i - sign\left(a_{ij}\right) x_j \right| = 0$, and $\left|x_i(t)\right| \equiv \left|x_j(t)\right|$ if $t \ge T(x)$. The upper bound of $T(x)$ is

$$T_{\max}(x) = \frac{1}{\alpha \left(2\lambda_2(L)\right)^{\frac{b+a}{2a}}} \frac{2a}{a-b} + \frac{1}{\beta N^{\frac{p-q}{2p}} \left(2\lambda_2(L)\right)^{\frac{p+q}{2p}}} \frac{2p}{p-q} \tag{9}$$

This means that system (1) is able to get to the bipartite consensus within a fixed time and the convergence time satisfies

$$T \le \frac{1}{\alpha \left(2\lambda_2(L)\right)^{\frac{b+a}{2a}}} \frac{2a}{a-b} + \frac{1}{\beta N^{\frac{p-q}{2p}} \left(2\lambda_2(L)\right)^{\frac{p+q}{2p}}} \frac{2p}{p-q} \tag{10}$$

The proof is completed.

IV. SIMULATION

The effectiveness of the theory is tested by numerical simulations in the section. Fig.1 is structurally balanced topological graph among the six agents. We choose a multi-agent system containing six agents. The nodes v_1, v_2, v_4 and v_3, v_5, v_6 are different groups. The original state of the agents is [5.5 -4 7 4 -9.6 -7.3].

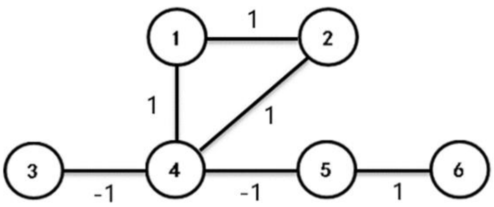

Fig.1 structural balance topological graph

According to Fig.1, adjacency matrix and Laplace matrix have following forms.

$$A=\begin{bmatrix}0&1&0&1&0&0\\1&0&0&1&0&0\\0&0&0&-1&0&0\\1&1&-1&0&-1&0\\0&0&0&-1&0&1\\0&0&0&0&1&0\end{bmatrix}\quad L=\begin{bmatrix}2&-1&0&-1&0&0\\-1&2&0&-1&0&0\\0&0&1&1&0&0\\-1&-1&1&4&1&0\\0&0&0&1&2&-1\\0&0&0&0&-1&1\end{bmatrix}$$

According to the Laplace matrix L, the second smallest eigenvalue $\lambda_2(L)=0.49$. $a=5$, $b=3$, $p=5$, $q=7$, $\alpha=0.2$, $\beta=10$, $\rho=0.4$ are the coefficients of the controller (4). $f\left(x_i\left(t\right),t\right)=0.2\sin(x_i)$, the external disturbance is $\omega_i\left(t\right)=0.2\cos(W_i t)$. The results are presented below. Fig.2 is state curves about agents. We can see that the six agents reach bipartite consensus within 1.2 seconds from Fig.2. The $T_{\max}=25.07s$ can be calculated according to expression (9).Obviously, the convergence time $T=1.2$ is less than $T_{\max}=25.07$ .So it meets $T\le T_{\max}$ .The local error curves about six agents are demonstrated in Fig.3.

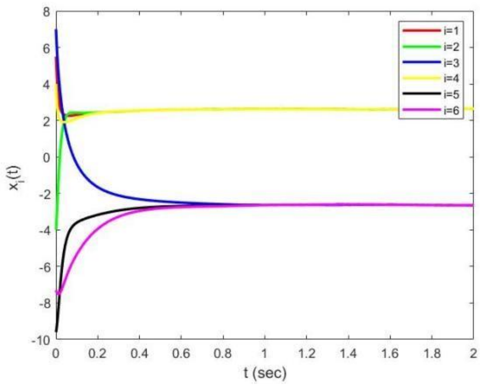

Fig.2 states curves

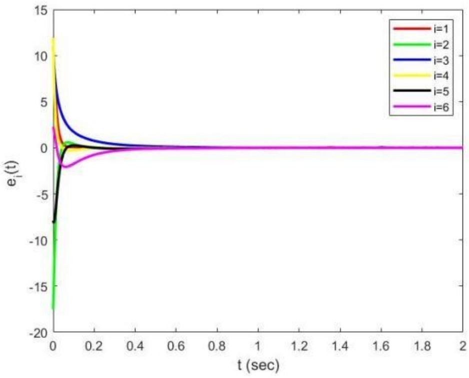

Fig.3 the local error curves

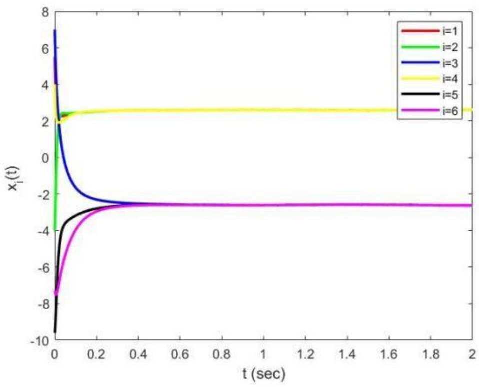

Fig.4 states after adjusting gain

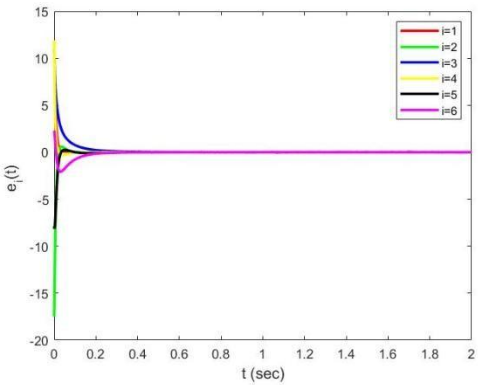

Fig.5 the local errors after adjusting gain

Then, we choose $\alpha=0.5$, $\beta=20$ as the parametersfor the controller (4).Fig.4 is state curves about agentsafter adjusting the controller gain. We can see that the six agents reach bipartite consensus within 0.6 seconds from Fig.4. The $T_{\max}=9.95s$ can be calculated according to expression (9).Obviously, the convergence time $T=0.6$ is less than $T_{\max}=9.95$ .So it meets $T\le T_{\max}$. We can find that α and β are the factors that affect the upper bound $T_{\max}$, andthe controller gain is larger, the convergence time is shorter. The local error curves about six agents after adjusting the gain are demonstrated in Fig.5.

V. CONCLUSION

Combining the cooperative and antagonistic relationshipsamongthe agents, this paper has researched bipartite fixed-time consensus problem for the nonlinear multi-agent systems with disturbances. A distributed control law has been proposed for this system. By applying graph theory and Lyapunov stability theory, the sufficient condition about achieving bipartite consensus and an upper bound ofconvergence time have been proposed. The influence of control gain on the convergence time has been analyzed in detail. The simulation has shown the validity of the analysis results.

REFERENCES

[1] X Dong, Y Zhou, Z Ren, et al. Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application to quadrotor formation flying. IEEE Transactions on Industrial Electronics, 2016, 64(6): 5014-5024.

[2] V Rodin, A Benzinou, A Guillaud, et al. An immune oriented multi-agent system for biological image processing. Pattern Recognition, 2004, 37(4): 631-645.

[3] M Li, S Ma, Z Yan. Robust admissible consensus of singular multi- agent systems using sliding mode approach. Asian Journal of Control, 2021, 23(4): 2029-2036.

[4] S Satunin, E Babkin. A multi-agent approach to intelligent transportation systems modeling with combinatorial auctions. Expert Systems with Applications, 2014, 41(15): 6622-6633.

[5] S H Hosseini, M S Tavazoei, N V Kuznetsov. Agent-based time delay margin in consensus of multi-agent systems by an event-triggered control method: Concept and computation. Asian Journal of Control, 2022, https://doi.org/10.1002/asjc.2814.

[6] G Antonelli, F Arrichiello, F Caccavale, et al. A decentralized controller-observer scheme for multi-agent weighted centroid tracking. IEEE Transactions on Automatic Control, 2012, 58(5): 1310-1316.

[7] G Notarstefano, M Egerstedt, M Haque. Containment in leader–follower networks with switching communication topologies. Automatica, 2011, 47(5): 1035-1040.

[8] M Donganont, X Liu. Scaled consensus problems of multi agent systems via impulsive protocols. Applied Mathematical Modelling, 2023, 116: 532-546.

[9] M H Trinh, S Zhao, Z Sun, et al. Bearing-based formation control of a group of agents with leader-first follower structure. IEEE Transactions on Automatic Control, 2018, 64(2): 598-613.

[10] S M Erlien, S Fujita, J C Gerdes. Shared steering control using safe envelopes for obstacle avoidance and vehicle stability. IEEE Transactions on Intelligent Transportation Systems, 2015, 17(2): 441-451.

[11] L Yao, D Xie, H Wang. Complex group consensus of multi-agent systems with cooperative-competitive mechanisms: acyclic partition method. Applied Mathematics and Computation, 2023, 443: 127770.

[12] R Olfati-Saber, R M Murray. Consensus problems in networks of agents with switching topology and time-delays. IEEE Transactions on automatic control, 2004, 49(9): 1520-1533.

[13] Y Qian, X Wu, J Lü, et al. Consensus of second-order multi-agent systems with nonlinear dynamics and time delay. Nonlinear Dynamics, 2014, 78(1): 495-503.

[14] Z Li, W Ren, X Liu, et al. Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols. IEEE Transactions on Automatic Control, 2012, 58(7): 1786-1791.

[15] C Wang, L Zhang, F Cao. Leader-following consensus control of second-order nonlinear multi-agent systems with applications to slew rate control. Asian Journal of Control, 2022, https://doi.org/10.1002/asjc.2925.

[16] C Altafini. Consensus problems on networks with antagonistic interactions. IEEE transactions on automatic control, 2012, 58(4): 935-946.

[17] H Zhang, J Chen. Bipartite consensus of general linear multi-agent systems. 2014 American Control Conference. IEEE, 2014: 808-812.

[18] X Guo, J Lu, A Alsaedi, et al. Bipartite consensus for multi-agent systems with antagonistic interactions and communication delays. Physica A: Statistical Mechanics and its Applications, 2018, 495: 488-497.

[19] Y Qi, R Mu, X Zhang. Periodic event-triggered bipartite consensus for multi-agent systems with partial information transmission. Asian Journal of Control, 2022, 24(5): 2778-2786.

[20] X Nan, Y Lv, Z Duan. Bipartite consensus tracking for antagonistic topologies with leader's unknown input. Asian Journal of Control, 2022, 24(2): 547-561.

[21] W Meng, Q Yang , J Sarangapani, et al. Distributed control of nonlinear multi-agent systems with asymptotic consensus. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2017, 47(5): 749-757.

[22] X Wang, Y Hong. Finite-time consensus for multi-agent networks with second-order agent dynamics. IFAC Proceedings volumes, 2008, 41(2): 15185-15190.

[23] Y Wang, Y Yuan, J Liu. Finite-time leader-following output consensus for multi-agent systems via extended state observer. Automatica, 2021, 124: 109133.

[24] H Lü, W He, Q L Han, et al. Finite-time containment control for nonlinear multi-agent systems with external disturbances. Information Sciences, 2020, 512: 338-351.

[25] A Polyakov. Nonlinear feedback design for fixed-time stabilization of linear control systems. IEEE Transactions on Automatic Control, 2011, 57(8): 2106-2110.

[26] M Defoort, A Polyakov, G Demesure, et al. Leader- follower fixed- time consensus for multi- agent systems with unknown non- linear inherent dynamics. IET Control Theory & Applications, 2015, 9(14): 2165-2170.

[27] Q Deng, J Wu, T Han, et al. Fixed-time bipartite consensus of multi-agent systems with disturbances. Physica A: Statistical Mechanics and its Applications, 2019, 516: 37-49.

[28] B Ning, J Jin, J Zheng, et al. Finite-time and fixed-time leader-following consensus for multi-agent systems with discontinuous inherent dynamics. International Journal of Control, 2018, 91(6): 1259-1270.

[29] Y Luo, J Zhu, J Cao, et al. Aperiodically Intermittent Output Feedback Consensus Control for Second-order Leader-following Multi-agent Systems Based on Observer. Journal of the Franklin Institute, 2023, https://doi.org/10.1016/j.jfranklin.2023.01.043

[30] Z Zuo, J Ji, Z Zhang, et al. Consensus of double-integrator multi-agent systems with asymmetric input saturation. Systems & Control Letters, 2023, 172: 105440.