



ForLoveDCs: A Set of MATLAB Function for Calculating the Forward Modeling of Multimode Love Wave Dispersion Curves

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ABSTRACT :Detection of an earth's subsurface velocity profile is very essential for geotechnical site investigation. Basically, multimode Love wave dispersion curves (DCs) can be used for detecting the subsurface velocity structure. Here, the ForLoveDCs software is introduced to calculate the forward modeling of multimode Love wave dispersion curves by employing a set of MATLAB function. In general, ForLoveDCs has running well and successfully computing the multimode Love Wave DCs. The computation procedure tested on the synthetic model representing the regularly and irregularly shear wave velocity (SWV) structure profiles. Manifestly, all computation results show the reasonable and reliability of DCs.

KEYWORDS:MATLAB function; Forward modeling; Multimode Love wave; Shear wave velocity profile; Dispersion curves (DCs)

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I. INTRODUCTION

Multimode Love wave dispersion curves (DCs) can be used for geotechnical site investigation, such as to detect a subsurface structure having an irregular velocity profile. However, an irregular velocity comprising a soft layer below a stiff one is common and very important in geophysical, environmental, and civil engineering [1]. Computations of multimode Love wave DCs as an initial stage of forward modeling play an important role for engine of inversion processing. In the previous effort the multimode Love wave and the effective Rayleigh wave DCs have developed in the joint inversion approach [2].

In the perspective of inversion modeling, model modification is carried out systematically so that solutions or models are obtained automatically from the data. In this case, the model modification is based on the difference between the observed data and the theoretical data for the model so that the model modification is expected to minimize the difference. It can be seen that inverse modeling can be done if we can predict the data for a certain model (forward modeling is available or known). In general, the term inverse modeling is used in contrast to forward modeling, which is defined as the process of predicting the results of measurements (predicting data) on the basis of some general principle or model and a set of specific conditions relevant to the problem at hand. Inverse modeling, roughly speaking, addresses the reverse problem: starting with data and a general principle, theory, or quantitative model, it determines estimates of the model parameters [3].

As discussed in the previous study, the multimode Love wave dispersion curves were synthesized from the profile representing shear-wave velocity reversal using a full shear horizontal (SH) waveform. Furthermore, a dispersion curve from the full SH waveform is extracted by using a frequency-slowness transforms. The dispersion curves overlain in dispersion images were picked manually. In this study, all inversion codes involving in the genetic algorithm (GA) and forward modeling engine have been developed under a full FORTRAN computer program using Visual Compact Fortran Version 6.6 [1].

In this paper, a set MATLAB function is developed to perform the calculation of forward modeling multimode Love wave dispersion curves called ForLoveDCs software. It aims to propose a sustainable open-source software that based MATLAB language environment. The use of a MATLAB package has also presented for calculating partial derivatives of surface-wave dispersion curves by a reduced delta matrix method [4]. Overall, the ForLoveDCs software can be used to promote a collaboration among researchers in analyzing the multimode Love wave dispersion curves. Here, as the beginning innovation, the implementation of this software is focused

to analyze several velocity profiles including regular and/or irregular velocity structures using a synthetic model. In the coming days, this software can be developed into a program package of MATLAB integrated simultaneously with inversion modeling.

$$\frac{dy_1}{dz} = \frac{1}{\mu} y_2, \quad \frac{dy_2}{dz} = \mu \left(k^2 - \frac{\omega^2}{\beta^2} \right) y_1 \quad (11)$$

Equations (11) are simultaneous equations for y_1 and y_2 , where β is shear wave (S-wave) velocity (V_s) defined as [7]

II. SOFTWARE DESCRIPTION

2.1 Software Architecture

The ForLoveDCs is a software which is fully written in MATLAB language environment and contains three functions that is ForLoveDCs.m, layer_iteration.m, and plot_profile.m. The main function named ForLoveDCs.m contains a set of program codes that involve all equations of the Love wave propagation and calculation of the Love wave dispersion curves. In general, the calculation forward modeling of multimode Love wave DCs involve the matrix method called Haskell layer matrix. However, the calculation of the effective Love wave dispersion curve employing full SH-wavefield reflectivity method has been investigated in our previous study [5]. The details of the Love waves propagation, characteristic equation of Love wave and algorithm of the calculation the multimode Love wave dispersion curves utilized in this software are introduced as follows.

2.2 Love waves propagating in the layered medium

The description of Love waves propagating in the layered medium is based on the matrix method described by Haskell (1953) [6]. Equation of motion in the elastic medium can be generally rewritten as follows.

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \quad (1)$$

This equation shows x component of displacement (u) and other components can be written as similarly. The σ_{xx} , σ_{xy} and σ_{xz} are stresses and can be written as follow using Lamé constants λ and μ .

$$\sigma_{xx} = \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + 2\mu \frac{\partial u}{\partial x} \quad (2)$$

$$\sigma_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (3)$$

$$\sigma_{xz} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \quad (4)$$

where v and w are y component and z component of displacement respectively.

Type of solution for SH waves that satisfies equations (1)-(4) is generally assumed as follows.

$$u = y_1(z) \frac{\partial Y}{k \partial y}, \quad v = -y_1(z) \frac{\partial Y}{k \partial x}, \quad \text{and } w = 0 \quad (5)$$

where

$$Y(x, y, t) = e^{i(\omega t - k_x x - k_y y)} \quad (6)$$

Stresses in x - y plane can be written as follows in terms of separation of variables.

$$\sigma_{xz} = y_2(z) \frac{\partial Y}{k \partial y}, \quad \sigma_{yz} = -y_2(z) \frac{\partial Y}{k \partial x}, \quad \sigma_{zz} = 0 \quad (7)$$

Here, $y_1(z)$ and $y_2(z)$ are simultaneous solution for layered medium. Substituting set solutions from equation (5) into equation (4) yields,

$$\sigma_{xz} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = \mu \frac{dy_1}{dz} \frac{\partial Y}{k \partial y} \quad (8)$$

Comparison between (8) and (7) indicates in explicit that,

$$y_2 = \mu \frac{dy_1}{dz} \quad (9)$$

Writing σ_{xx} , and σ_{xy} using y_1 and substituting them into the equation of motion (1) gives

$$-\rho\omega^2 y_1 = -k^2 \mu y_1 + \frac{dy_2}{dz} \quad (10)$$

The equations (9) and (10) are basic equation for SH waves. These equations can be rewritten as

$$\beta = \sqrt{\frac{\mu}{\rho}} \text{ or } \mu = \rho \beta^2 = \rho V_S^2 \quad (12)$$

2.3 Calculation of Love wave dispersion curve

Solution for equation (9) can easily be obtained in the layered medium, in which each layer has constant density ($\rho = \rho_m$) and S-wave velocity ($\beta = \beta_m$) in the layer of index m, as follow

$$y_1(z) = A_m e^{v_{\beta m}(z-z_{m+1})} + B_m e^{-v_{\beta m}(z-z_{m+1})} \quad (13)$$

$$y_2(z) = \mu_m v_{\beta m} (A_m e^{v_{\beta m}(z-z_{m+1})} + B_m e^{-v_{\beta m}(z-z_{m+1})}) \quad (14)$$

or

$$y_1(z) = A'_m \cosh[v_{\beta m}(z-z_{m+1})] + B'_m \sinh[v_{\beta m}(z-z_{m+1})] \quad (15)$$

$$y_2(z) = \mu_m v_{\beta m} \{A'_m \sinh[v_{\beta m}(z-z_{m+1})] + B'_m \cosh[v_{\beta m}(z-z_{m+1})]\} \quad (16)$$

where

$$v_{\beta m}^2 = k^2 - (\omega/\beta_m)^2 \quad (17)$$

Substituting $z = z_{m+1}$ into the equations (15) and (16) gives the solution on the boundary of (m+1) as follows

$$y_1^{m+1} = A'_m, \quad y_2^{m+1} = \mu_m v_{\beta m} B'_m \quad (18)$$

Substituting these A'_m and B'_m into the equations (15) and (16) yields

$$y_1(z) = y_1^{m+1} \cosh[v_{\beta m}(z-z_{m+1})] + y_2^{m+1} \sinh[v_{\beta m}(z-z_{m+1})] \quad (19)$$

$$y_2(z) = (y_1^{m+1} \mu_m v_{\beta m}) \sinh[v_{\beta m}(z-z_{m+1})] + (y_2^{m+1} / \mu_m v_{\beta m}) \cosh[v_{\beta m}(z-z_{m+1})] \quad (20)$$

These equations can be written as a matrix form

$$\begin{bmatrix} y_1^m \\ y_2^m \end{bmatrix} = \begin{bmatrix} \cosh(v_{\beta m} d_m) & (1/\mu_m v_{\beta m}) \sinh(v_{\beta m} d_m) \\ \mu_m v_{\beta m} \sinh(v_{\beta m} d_m) & \cosh(v_{\beta m} d_m) \end{bmatrix} \begin{bmatrix} y_1^{m+1} \\ y_2^{m+1} \end{bmatrix} \quad (21)$$

where $d_m = z_m - z_{m+1}$ is the thickness of layer mth. Using matrix notation, equation (21) can be rewritten as follow

$$\mathbf{y}^m = \mathbf{B}_m \mathbf{d}_m \mathbf{y}^{m+1} \quad (22)$$

The matrix $\mathbf{B}_m(\mathbf{d}_m)$ is so called a layer matrix for layer m. The matrix is essentially equal to the matrix so called Haskell layer matrix. Advantage of the matrix (21) is that the elements of the matrix are always real. Solution on the layer boundary can then be calculated using the layer matrix iteratively from the initial condition \mathbf{y}^n . Initial condition in the layer boundary n is

$$\mathbf{y}^n = \begin{pmatrix} 1 \\ \mu_n v_{\beta n} \end{pmatrix} \quad (23)$$

where $\mu_n v_{\beta n}$ must be larger than 0 so that y_1 and y_2 go to 0 where z goes to infinite. This condition implies that phase velocity or dispersion curves of Love wave must be slower than S-wave velocity of bottom layer (β_n). At

the free surface, y_2 must be 0 so that stress goes to 0. Therefore, characteristic equations for Love wave can be written as follows [7].

$$F_L(c,\omega)=y_2^0=0 \quad (24)$$

or

$$F_L(c,\omega)=y_2^0/y_1^0=0 \quad (25)$$

where c is dispersion curve. The dispersion curve of Love waves is based on the integrating the equation (9) and its actual calculation procedure uses the equation (24) or (25).

2.4 Algorithm of the calculation the multimode Love wave dispersion curves

Actual calculation procedure of Love wave dispersion curves using compound matrix method is succinctly outlined as follow:

- Define the model parameters of surface wave including number of layer (n), S-wave velocity values for each layer (β), the value of density constants for each layer (ρ), thickness values of each layer (H).
- Read input of global parameters of surface waves including minimum, maximum and shift/increments of frequency desired, as well as minimum, maximum and shift/increments of S-wave velocity.
- Setup a Lamé constant (μ) using equation (12).
- Process the looping for the frequency.
 - Setup the angular frequency ($\omega=2\pi f$).
 - Process the looping for the S-wave velocity.
 - Setup the wave number using equation (17).
 - Calculate the initial condition based on the equation (23).
 - Process the looping for the layer iteration.
 - Calculate Q-matrix (layer matrix) using equation (22).
 - Calculate variable $y_1(z)$ and $y_2(z)$ from bottom to top layer using equation (21).
 - End the looping for the layer iteration.
 - Check if $FL(c,\omega)$ is zero (equation 25).
 - End the looping for the S-wave velocity.
- End the looping for the frequency.
- Display the dispersion curves data (pairs of frequency and velocity).

2.5 Description of Code Metadata

The software of ForLoveDCs is first created and published as a capsule using the Code Ocean platform on Feb 21, 2023. The Code Ocean integrative computational research experience is one place where great computational research is created, organized, and shared. This platform is deployed on the Amazon Web Services (AWS) cloud with a dedicated Virtual Private Cloud (VPC). Code Ocean delivers a unique experience with a standard, secure, and executable research package called a Compute Capsule. Product datasheet of Code Ocean platform can be in detail accessed through the link: <https://codeocean.com/wp-content/uploads/2020/12/Product-Datasheet.pdf>.

To reproduce the run of the ForLoveDCs, the reader can join in the Code Ocean platform with the permanent link to reproducible capsule is <https://codeocean.com/capsule/7821371/tree/v2> and its DOI is <https://doi.org/10.24433/CO.9681215.v2>. Currently, the ForLoveDCs software is one of the capsules in the Code Ocean platform that has been republished on Apr 20, 2023 with current code version is v2.0. Legal code license is under MIT license. A screenshot of the metadata of the ForLoveDCs software in the capsule of Ocean Code is shown in Fig. 1.

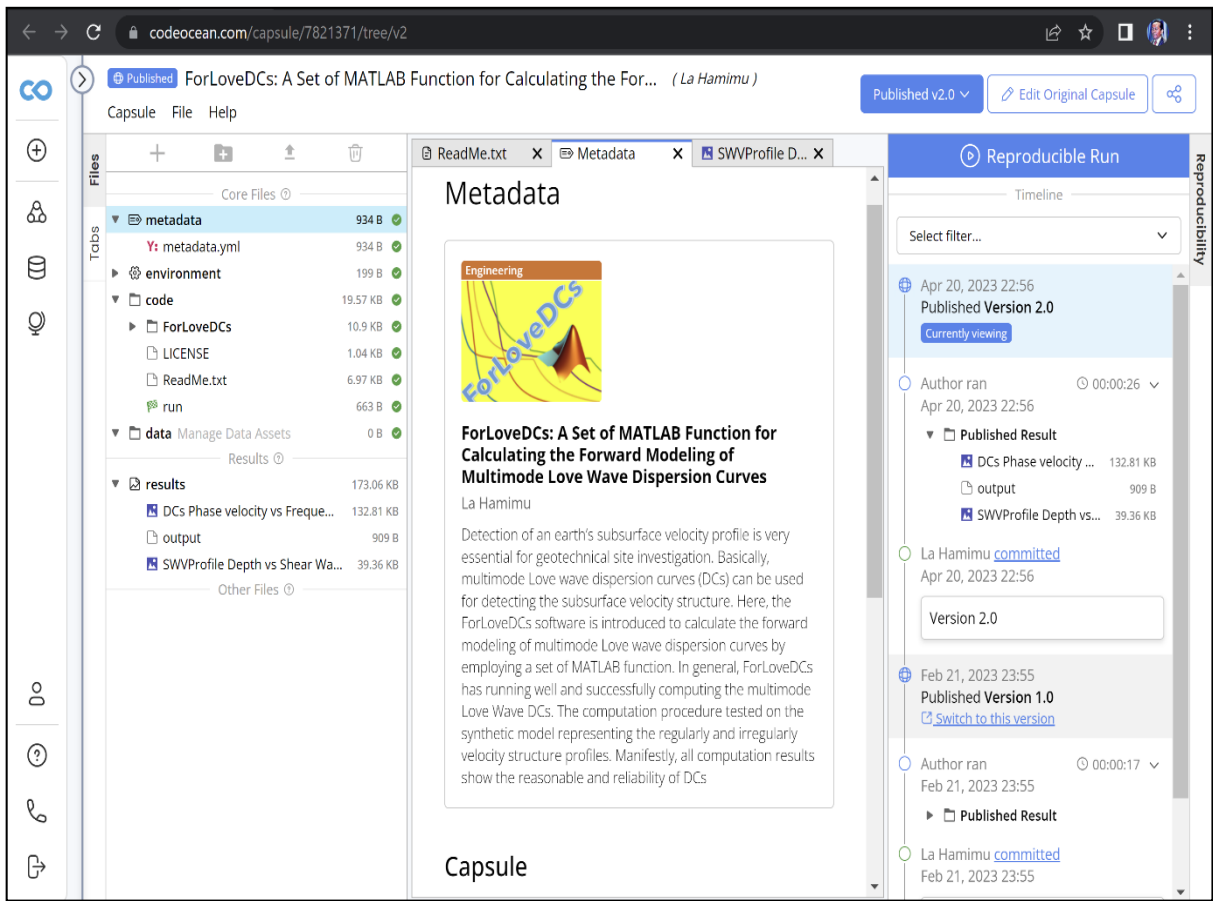


Fig.1 A screenshot of the metadata for the capsule of ForLoveDCs software

III. ILLUSTRATIVE EXAMPLE

To test the reasonable and reliability of the ForLoveDCs software, this section will be reported the calculation of multimode Love wave dispersion curves resulted from four profiles different. These profiles are synthetic site models called Profile 1, Profile 2, Profile 3, and Profile 4.

3.1 Profile 1

The Profile 1 is an example of synthetic model representing the regularly velocity structure where the value of the S-wave velocity (V_s) in each layer increases monotonically with increasing layers ($V_s^{i+1} > V_s^i$). This profile consists of four layers over a half-space. The model parameters of this profile and the calculation results of the cut-off frequency (CF) are listed in Table 1. The visualization of this profile is shown in Fig. 2. Computation results of multimode Love wave DCs obtained from the Profile 1 can be seen in Fig. 3.

Table 1 Model Parameters of the Profile 1

Nr	Parameters	V_s (m/s)	$\rho \times 10^3$ (kg/m ³)	Thickness (m)	CF (Hz)
1	Top layer	200	1.7	2.5	3
2	Second layer	300	1.7	3.5	13
3	Third layer	450	1.8	5	21
4	Fourth layer	600	1.8	7.5	32
5	Half space	700	2.0	∞	41

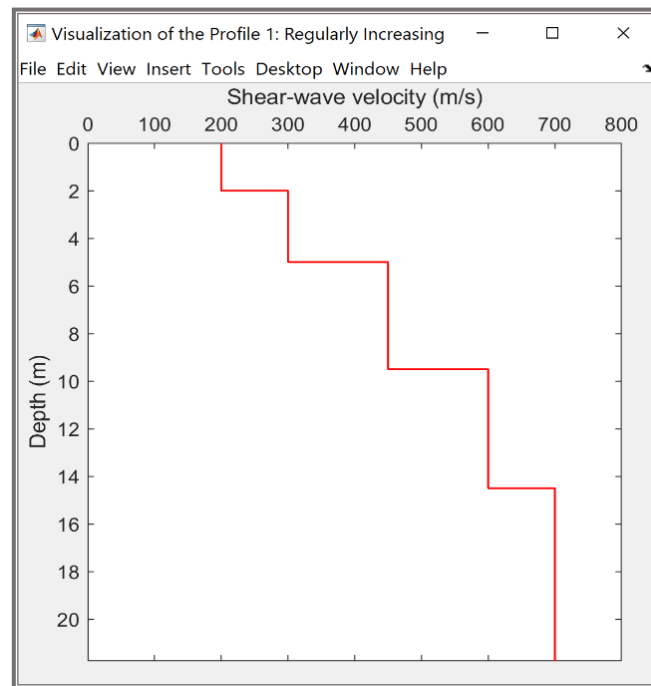


Fig. 2. S-wave velocity structure of the Profile 1

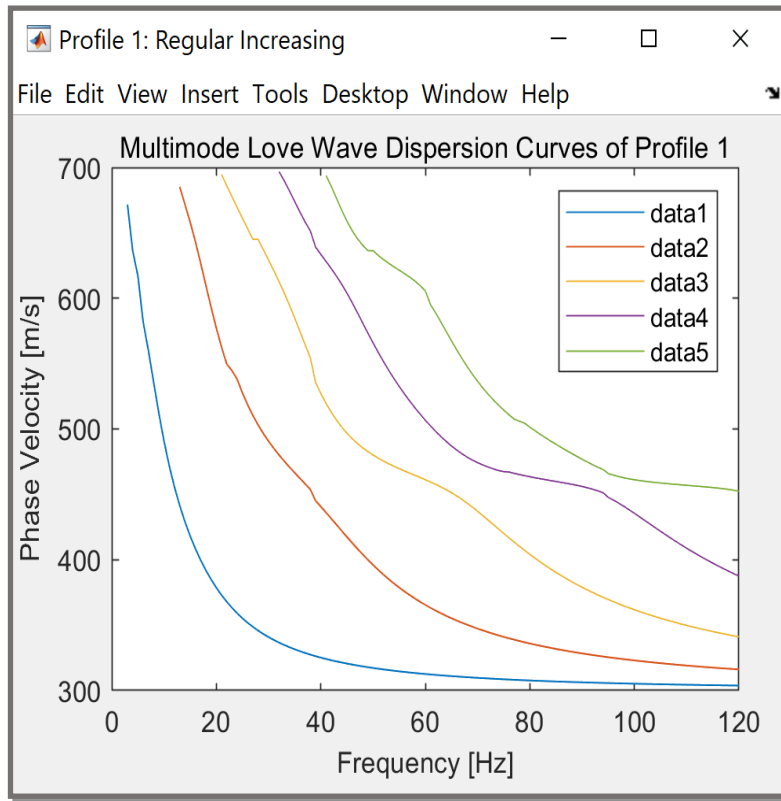


Fig. 3 DCs trend resulted from the Profile 1

In general, the legends displayed in Fig. 3 use text of data1, data2, data3, etc. Here, data 1 is correlated by the first mode or fundamental mode, data2 for the second mode, data3 for the third mode, etc. It is an essential assumption that the first curve closest to the vertical axis will always be associated with the first mode, followed by the second mode, the third mode and so on. Trend of the multimode Love wave DCs produced from a velocity structure profile can be analyzed employing the cut-off frequency (CF) approach. The quantity of the cut-off frequency is defined as a value at which the S-wave velocity in a certain mode reaches its peak velocity value (maximum velocity). The structure of the S-wave velocity illustrated by Profile 1 has the cut-off frequency as presented in Table 1 with name of header column is CF (Hz). The calculation of an average cut-off frequency obtained from Table 1 is defined as:

$$\overline{CF} = \frac{1}{N-1} \left(\sum_{h=1}^N CF^{h+1} - CF^h \right) \quad (26)$$

where \overline{CF} is the average cut-off frequency, N is number of modes, $h=1,2, \dots, N$. For the Profile 1 the average cut-off frequency calculated from the CF (Hz) column of Table 1 is 9.5 Hz.

3.2 Profile 2

The Profile 2 is the synthetic model representing the regularly velocity structure where the value of the S-wave velocity (V_s) in each layer decreases monotonically with increasing layers ($V_s^{i+1} < V_s^i$). This profile consists also of four layers over a half-space. The model parameters of the second profile and the calculation results of the cut-

off frequency (CF) are listed in Table 2. The visualization of the Profile 2 is shown in Fig. 4. Results of the calculation of forward modeling the multimode Love wave DCs for the Profile 2 are shown in Fig. 5. The cut-off frequency resulted from the Profile 2 is presented in the CF (Hz)'s column of Table 2 with the average cut-off frequency (\overline{CF}) is 3.25 Hz.

Table 3 Model Parameters of the Profile 2

Nr	Parameters	V_s (m/s)	$\rho \times 10^3$ (kg/m ³)	Thickness (m)	CF (Hz)
1	Top layer	700	2.0	2.5	1
2	Second layer	600	1.8	3.5	4
3	Third layer	450	1.8	5	7
4	Fourth layer	300	1.7	7.5	10
5	Half space	200	1.7	∞	14

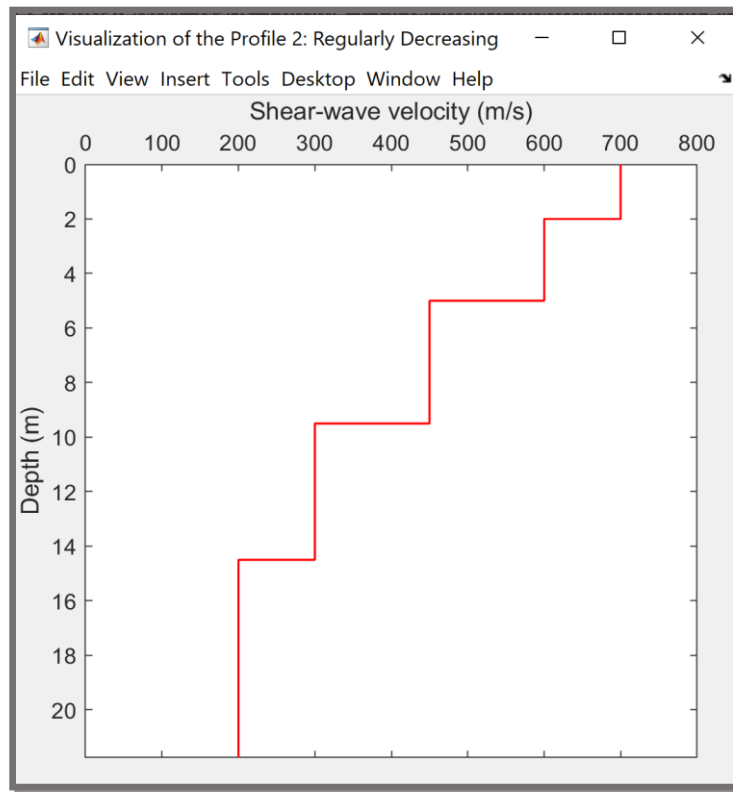


Fig. 4. S-wave velocity structure of the Profile 2

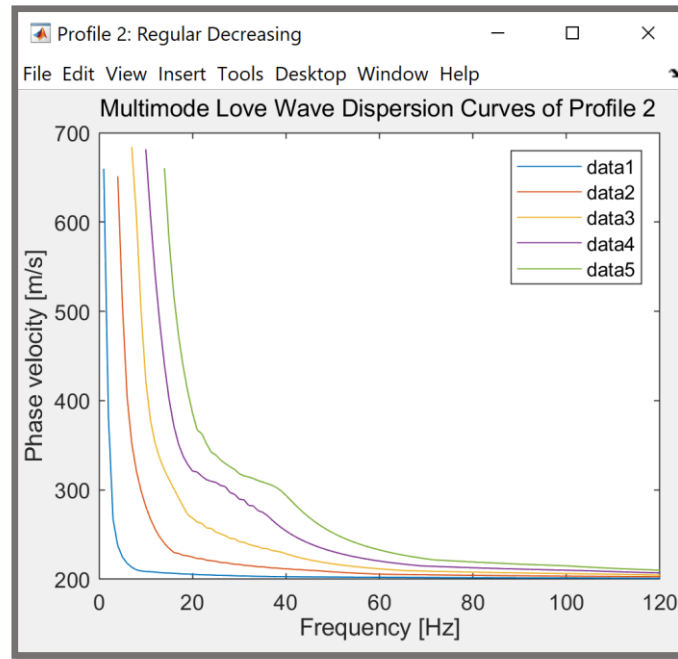


Fig. 5 DCs trend resulted from the Profile 2

3.3 Profile 3

The Profile 3 is performing an irregularly dispersive profiles comprising the high velocity layer (HVL) implanted by different stiff layers. This profile consists of four layers over a half-space. The model parameters of this profile and the calculation results of the cut-off frequency (CF) are listed in Table 3. In addition, the visualization of the Profile 3 is shown in Fig. 6. The implementation of the ForLoveDCs software for computing the multimode Love wave DCs for the Profile 3 is illustrated in Fig. 7. The cut-off frequency resulted from Profile 3 is shown in Table 3 with the name of column header is also CF (Hz). For this profile, the average cut-off frequency (\overline{CF}) is 6 Hz.

Table 3 Model Parameters of the Profile 3

Nr	Parameters	V_s (m/s)	$\rho \times 10^3$ (kg/m ³)	Thickness (m)	CF (Hz)
1	Top layer	200	1.7	2.5	1 Hz
2	Second layer	300	1.7	3.5	7 Hz
3	Third layer	700	2.0	5	12 Hz
4	Fourth layer	450	1.7	7.5	19 Hz
5	Half space	600	1.7	∞	25 Hz

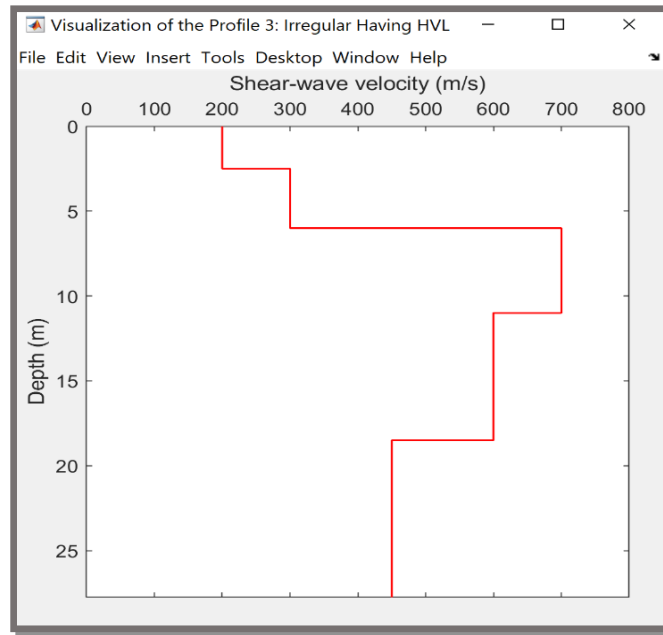


Fig. 6. S-wave velocity structure of the profile 3

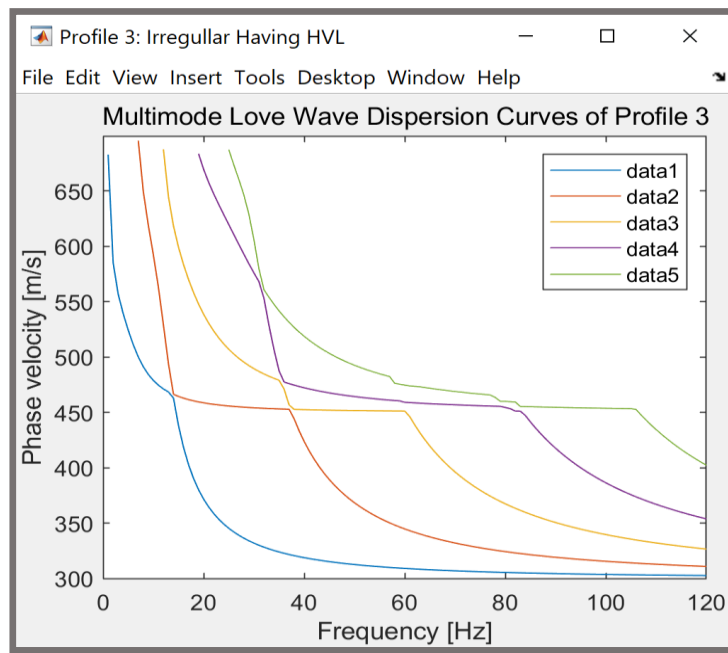


Fig. 7. DCs trend resulted from the Profile 3

3.4 Profile 4

The Profile 4 is an irregularly dispersive layer structure representing the shear-wave velocity reversal profile having low velocity layer (LVL) bounded by different stiff layers. This profile consists also of four layers over a half-space. The model parameters of the Profile 4 and the calculation results of the cut-off frequency (CF) are listed in Table 4. The parameter visualization of this profile is shown in Fig. 8. The computation of forward modeling the multimode Love wave DCs for the Profile 4 employing the ForLoveDCs software is performed in Fig. 9. The cut-off frequency resulted from Profile 4 is presented in the CF (Hz)'s column of Table 4. It can also be verified that the average cut-off frequency (\overline{CF}) of the Profile 4 is 3 Hz.

Table 4 Model Parameters of the Profile 4

Nr	Parameters	V_s (m/s)	$\rho \times 10^3$ (kg/m ³)	Thickness (m)	CF (Hz)
1	Top layer	600	1.8	2.5	1 Hz
2	Second layer	700	2.0	3.5	4 Hz
3	Third layer	200	1.7	5	6 Hz
4	Fourth layer	450	1.8	7.5	9 Hz
5	Half space	300	1.7	∞	13 Hz

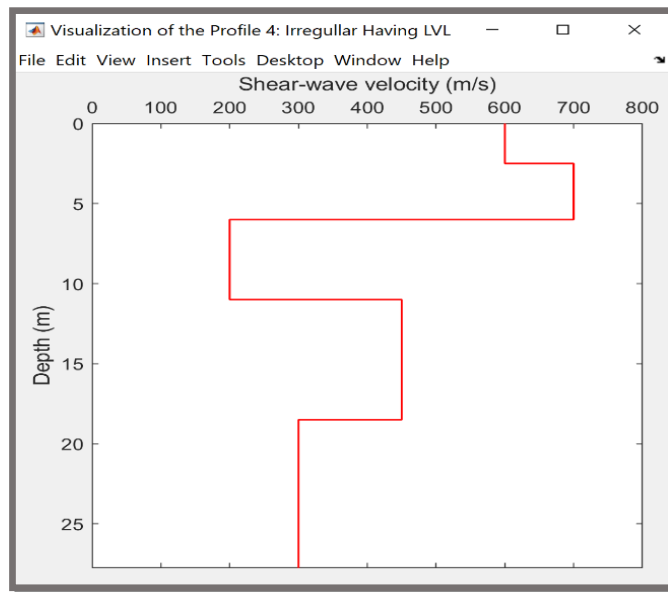


Fig. 8. S-wave velocity structure of the Profile 4

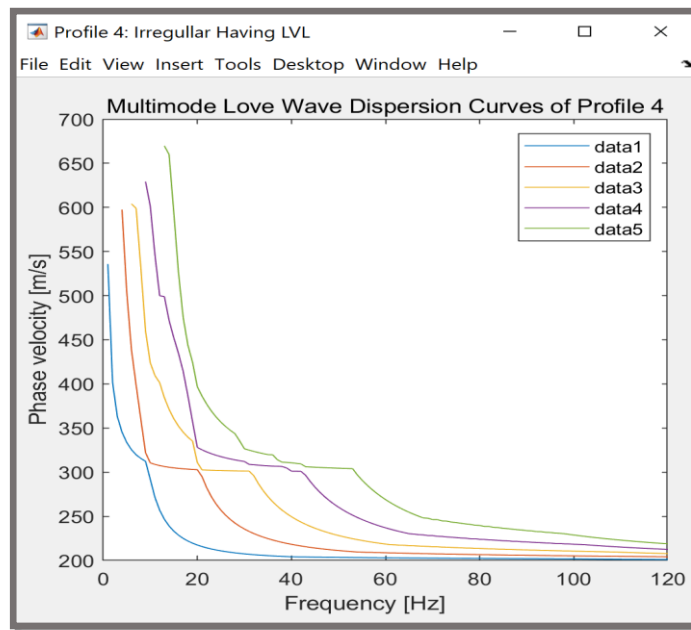


Fig. 9. DCs trend resulted from the Profile 4

IV. IMPACT OF THE SOFTWARE

In the perspective of inversion modeling, model modification is carried out systematically so that solutions or models are obtained automatically from the data. In this case, the model modification is based on the difference between the observed data and the theoretical data for the model so that the model modification is expected to minimize the difference [3]. Clearly, the inverse modeling of multimode Love wave DCS can be properly done if the calculation of the forward modeling for a certain model is available or known.

In the previous study, the multimode Love wave dispersion curves were synthesized from the profile using a full shear horizontal (SH) waveform. A dispersion curve from the full SH waveform is extracted by using a frequency-slowness transforms. The dispersion curves overlain in dispersion images were picked manually. All codes involving in the computation of the forward modeling engine have been developed under a full FORTRAN computer program using Visual Compact Fortran Version 6.6 [1]. Here, the ForLoveDCs software is presented to calculate the forward modeling of multimode Love wave dispersion curves by employing a set of MATLAB function. In this software, there is no treatments to extract DCs and there is no any action to pick manually of DCs.

Computation of the multimode Love wave DCs using the ForLoveDCs software can be used to verify important presumption that is the difference of the shear wave velocity (SWV) structure profile should be affecting the difference of DCs trend. Based on the computational results of forward modeling multimode Love DCs on the different profiles, it is obtained that ForLoveDCs software was able to demonstrate the significant differences of DCs trend between the profile of the SWV structures regularly increasing or decreasing and irregularly structures having velocity reversal such as high velocity layer (HVL) or low velocity layer (LVL). The cut-off frequency expressing a different value for each mode in a specific profile can also be utilized to analyze the difference between the SWV profile increasing regularly and decreasing regularly.

V. CONCLUSION

As a new set function of MATLAB, the ForLoveDCs can be implemented to compute the forward modeling multimode Love wave dispersion curves (DCs). In general, the ForLoveDCs software has running well, and it has been successful in proving the trend of the DCs curve which corresponds to a given profile of shear wave velocity structure. Based on the trend of DCs generated from 4 different profiles can be conclude that the distribution of DCs for the SWV structure profile increasing regularly (Profile 1) is more dispersed than those profile decreasing regularly (Profile 2). A consistently result, it is also obtained that the distribution of DCs for the SWV structure profile having high velocity layer (HVL) reversal (Profile 3) is also more dispersed than those having low velocity layer (LVL) reversal (Profile 4). Manifestly, the average value of the cut-off frequency (\bar{CF}) from the SWV profile regularly increasing is in any case greater than those profile regularly decreasing.

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