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Research Paper

Analytical and Numerical Calculation of the Lift Force Due to Roughness

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ABSTRACT**:** *We consider a rigid spherical particle fixed in a viscous incompressible fluid, driven by a flow at low Reynolds number in parallel to a plane and rough wall. The model used is that of Stokes equations quasi-stationary; respectively associated with the condition of the adhesion surface of the sphere and the surface of the rough wall [1, 2]. With using the Lorentz reciprocity theorem, we calculated the lift force, perpendicular to the flow, due to the roughness. We have verified that this calculation is valid in the critical area of lubrication. The calculation of this force has been performed for different values of, the between particle and wall, the period of the roughness, and its amplitude.*

Keywords:- *Lift force, rough wall, viscous fluid, Stokes equations, shear flow.*

I. INTRODUCTION

The hydrodynamic interactions between particles and walls are important in flows with small scale (microhydrodynamics), involved in various applications, such as separation techniques of field- flow fractionation (FFF) [3], the movement of cells in biological fluids, the detachment of the grains of sand under the effect of wind, etc. We consider here a Newtonian, incompressible fluid such that the Reynolds number relative to a particle is small.

The problem of a sphere, maintained fixed in a shear flow at vicinity of a wall plane was examined by Goldman and al [4] as well as by Goren and 0'Neill [5]. This problem was solved later by Tözeren and Sakalak [6] using the technique of bispherical coordinates. The details of this solution can also be found in the thesis of Chaoui [7] and in Chaoui and Feuillebois [2] in which the last two have developed a technique for precise calculation of the coefficients of the bispherical coordinates solution, which gave the coordinates method bispherical better precision.

The problem of the flow of a viscous fluid on a rough surface has been addressed in a number of articles. Indeed B. Cichocki and P. Szymczak [8] have managed to determine the boundary conditions for this problem by using a rough and periodic area. Furthermore, N Lecoq and R Anthore [9] were established the drag force on a sphere moving under the action of gravity to a plane rough wall. They considered that the roughness, are periodic waves whose wavelength is very small compared with the radius of the sphere. Moreover, N.Lecoq [10] treated the problem of the vertical displacement of the particle sedimentation to rough wall, he took different type of roughness and observed that when the surface presents valleys there is a decrease in the friction factor. On the contrary if the surface presents bumps there is a increase of friction factor.

In this paper, we used an analytical treatment to calculate the lift force due to the roughness, then we calculated numerically this force, with a high precision for any distance between the sphere and the wall. For this, we use a formulation in spherical coordinates which had been developed for asymmetric problems [11, 12, 13]. In addition, our calculations are based on very precise results about the Stokes flow, that allows exploring the area of lubrication.

In the first part, we present a succinct theoretical approach that allows to get the roughness's force. In the second part, we explicit integral giving the roughness's force, using bispherical coordinates. And finally, we present the results and conclusion.

II. ANALYTICAL CALCULATION OF THE LIFT FORCE ROUGHNESS

The lift force experienced by the sphere under the influence of the roughness of the wall is defined by:

$$\boldsymbol{F}^{(1)}_{z\ f\to s} = \int_S \sigma\left(\boldsymbol{v}^{(1)}\right).\ \boldsymbol{n}_s\ ds \tag{1}$$

Our goal is to calculate the contribution of roughness in the forces applied in the sphere. The Lorentz reciprocity theorem [14, 15] allows to calculate the efforts without having to calculate the constraint tensor in all the fluid's field but only on the edges. This theorem express that if a fluid bounded by a border $\partial\boldsymbol{D}$ there are two different flows $(\boldsymbol{v}',\ \sigma'')$ and $(\boldsymbol{v}'',\ \sigma')$ then These flows are related by the following relationship:

$$\int_{\partial D} \boldsymbol{v}'.\sigma''.d\boldsymbol{S} = \int_{\partial D} \boldsymbol{v}''.\sigma'.d\boldsymbol{S} \tag{2}$$

with $d\boldsymbol{S}$ is the vector surface element on the edge $\partial\boldsymbol{D}$.

In practice, one of the two flows interests physically to us, and the other is called reciprocal flow, it is chosen according to the problem studied. We are interested in the lateral disturbance due to the roughness of the range $O(\varepsilon)$.

The reciprocal flow that calculates the disturbance flow is generated by the movement of sedimentation of a sphere whose radius a and whose constant speed $\boldsymbol{U}_z = U_z\,\boldsymbol{i}_z$, in a fluid at rest.

By applying the reciprocity theorem of Lorentz, the analytical expression of the lift force experienced by the sphere due to the roughness of the wall can be written:

$$F^{(1)}_z = \frac{a}{U_z}\int_p \boldsymbol{\mathcal{R}}(x,y)\,\frac{\partial \boldsymbol{v}^{(0)}}{\partial z}.\sigma(\boldsymbol{u}^s).\boldsymbol{i}_z ds \tag{3}$$

CHOICE OF THE ROUGHNESS FUNCTION $\boldsymbol{\mathcal{R}}(x,y)$

The roughness profile is chosen according to the studied case. In this case we consider a profile of periodic roughness, whose period L and whose stiffness $(a\ \varepsilon)$. And we take fluctuations of this profile, small, for avoiding generate dead zone (recirculation zones of the fluid).

In cylindrical coordinates

$$\boldsymbol{\mathcal{R}}(x,y) = \boldsymbol{\mathcal{R}}(\rho,\varphi) = \boldsymbol{\mathcal{R}}(\rho)\ cos\ \varphi \tag{4}$$

Let $\boldsymbol{\mathcal{R}}_n(\rho)$ be the Fourier series corresponding to the roughness function $\boldsymbol{\mathcal{R}}(\rho)$.

$$\boldsymbol{\mathcal{R}}_n(\rho) = c_0 + \sum_{n=0}^{+\infty}(c_n\ cos\ n\omega\rho + s_n\ sin\ n\omega\rho) \tag{5}$$

where

$$c_0 = \frac{1}{2} \tag{6}$$

$$c_n = \frac{1}{(2\pi n)^2}\frac{2}{\delta(\delta-1)}\ (1 - cos\ 2\pi n\delta) \tag{7}$$

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$$s_n = -\frac{1}{(2\pi n)^2}\frac{2}{\delta(\delta-1)} \sin 2\pi n\delta \quad (8)$$

For having slender profiles of the roughness, the value of δ should be small such as $(1/2 < \delta < 1, \delta \sim 3/4)$. That will allow to avoid the disturbances having large amplitude. Finally in order to have the adhesion to the wall, satisfied, it must avoid angular peaks. For this reason $\boldsymbol{\mathcal{R}}_n$ series must be calculated with a reduced number of terms (n is in the order of 3). With the angular peaks, we may have an adhesion to the wall, but against we may perhaps have recirculation.

III. NUMERICAL METHOD OF RESOLUTION OF THE LIFT FORCE DUE TO ROUGHNESS

After a first integration on φ in the interval of revolution [0,2π], we get, in cylindrical coordinates:

$$F_z^{(1)} = -\frac{\pi\mu_f a}{U_z}\int_0^{+\infty} \boldsymbol{\mathcal{R}}(\rho)\frac{\partial h_p^{(k)}}{\partial z}\cdot\frac{\partial u_\rho^s}{\partial z}\rho d\rho \quad (9)$$

The quantity $h_\rho^{(k)}$ is independent of φ in speed expression v_ρ^k

where

$$v_\rho^k = \frac{1}{2}(\rho Q_1^K + c(U_0^k + U_2^k)) \cos\varphi \quad (10)$$

and

$$u_\rho^s = \rho\frac{Q_0^s}{2} + U_1^s \quad (11)$$

With U_0^k, Q_1^K, U_2^k the spherical harmonics in the shear case and U_1^s, Q_0^s the spherical harmonics in the sedimentation case.

Through the use of bispherical coordinates the integral in unbounded field is reduced to an integral over a finite field and determining this integral Maple language, the expression of the lift force in the database becomes:

$$F_p = \varepsilon F_z^{(1)} \quad (12)$$

$$F_z^{(1)} = \frac{-\tan h(\alpha)}{24}(F_1 + F_2 + F_3 + F_4 + F_5 + F_6) \quad (13)$$

$$F_1 = \int_{-1}^{1} \boldsymbol{\mathcal{R}}(\mu)\ \frac{\sin^3(\eta)}{1-\mu}\sum_{n=1}^{+\infty} C_n\,\gamma_n\,P_n'(\mu)\sum_{m=0}^{+\infty} B_m\,\gamma_m\,P_m(\mu)\,d\mu \quad (14)$$

$$F_2 = 2\int_{-1}^{1} \boldsymbol{\mathcal{R}}(\mu)\ \sin^3(\eta)\sum_{n=1}^{+\infty} C_n\,\gamma_n\,P_n'(\mu)\sum_{m=1}^{+\infty} D_m\,\gamma_m\,P_m'(\mu)\,d\mu \quad (15)$$

$$F_3 = \int_{-1}^{1} \boldsymbol{\mathcal{R}}(\mu)\ \sin(\eta)\sum_{n=0}^{+\infty} E_n\,\gamma_n\,P_n(\mu)\sum_{m=0}^{+\infty} B_m\,\gamma_m\,P_m(\mu)\,d\mu \quad (16)$$

$$F_4 = 2\int_{-1}^{1} \boldsymbol{\mathcal{R}}(\mu)\ (1-\mu)\sin(\eta)\sum_{n=0}^{+\infty} E_n\,\gamma_n\,P_n(\mu)\sum_{m=1}^{+\infty} D_m\,\gamma_m\,P_m'(\mu)\,d\mu \quad (17)$$

$$F_5 = \int_{-1}^{1} \mathcal{R}(\mu)\ sin^3(\eta) \sum_{n=2}^{+\infty} G_n\ \gamma_n\ P_n''(\mu) \sum_{m=0}^{+\infty} B_m\ \gamma_m\ P_m(\mu)\ d\mu \tag{18}$$

$$F_6 = 2\int_{-1}^{1} \mathcal{R}(\mu)\ (1-\mu)\, sin^3(\eta) \sum_{n=2}^{+\infty} G_n\ \gamma_n\ P_n''(\mu) \sum_{m=1}^{+\infty} D_m\ \gamma_m\ P_m'(\mu)\ d\mu \tag{19}$$

with $\mu = cos(\eta)$, $\gamma_n = n + 1/2$, $P_n(\mu)$ is the Legendre polynomial whose degree n, and $P_n'(\mu)$ its derivative, C_n, E_n, G_n the coefficients of spherical harmonics in the shear case, and B_m, D_m the coefficients of spherical harmonics in the sedimentation case.

IV. RESULTS

We have got new results based on the results done by M. Chaoui [1,7]. We calculated the lift force roughness when certain parameters are varied (the gap sphere-rough wall (*gap*), the period (*L*), the amplitude of the roughness (ε), the factor (δ), and the translation factor (*T*)). Although, we get more precise results, even for a very small particle- rough wall.

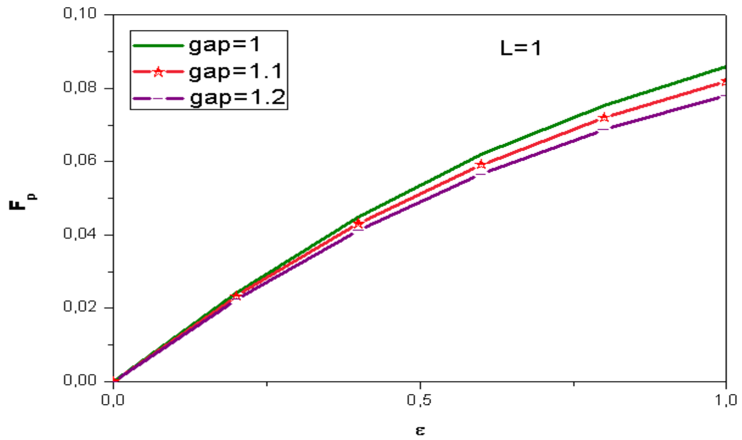

Fig.1: The lift force based on the roughness amplitude, for different values of the gap sphere-wall.

Figure 1 presents the variation of the lift force based on the amplitude of the roughness, for different gap sphere rough wall (*gap*) values, with the period of the roughness *L=1*. We observe that the force increases when the amplitude of the roughness increases. And when the sphere gap-rough wall increases the force decreases.

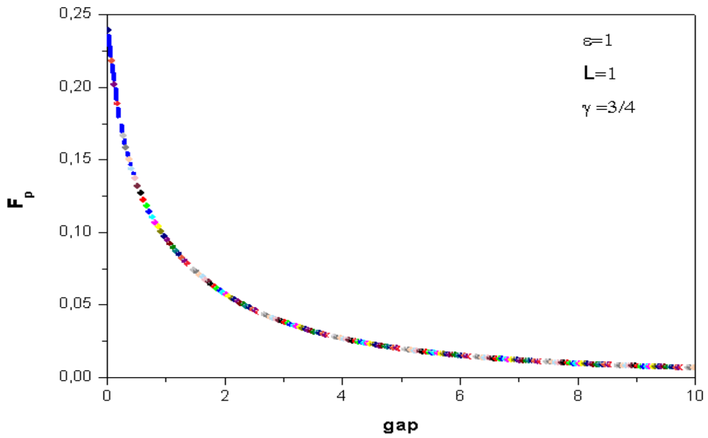

Fig.2: The lift force based on the gap sphere-wall.

Figure 2 presents the variation of the lift force based on the gap sphere rough wall (*gap*), with the value of the amplitude of the roughness $\varepsilon=1$, and with the period of the roughness $L=1$. We observe that the lift force reach zero when the gap sphere rough wall increases, so we can say that at the infinity, the disturbances due to the rough wall is canceled out and we only observe the undisturbed shear flow.

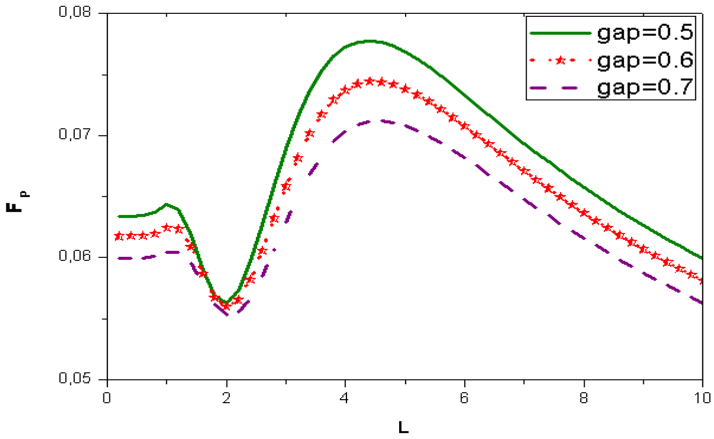

Fig.3: the lift force based on the roughness period, for different values of the gap sphere-wall.

Figure 3 presents the variation of the lift force based on the period of the roughness, for different values of the gap sphere-rough wall, with the amplitude of the roughness $\varepsilon=0.5$. We observe that the lift force increases when the gap sphere rough wall decreases. Furthermore we remark that the force takes a constant value in the area of small values of L, When $L=2$ (the value of the period equal to the diameter of the sphere) the lift force presents a minimum. Increasing the value of L, the force increases and reaching a maximum value ($L=4.5$), after it decreases slowly. We remark that the lift force depends on the structure of the wall roughness and to the particle size. The influence of the structure of the roughness on the lift force will be explained in Figures (4,5,6,7).

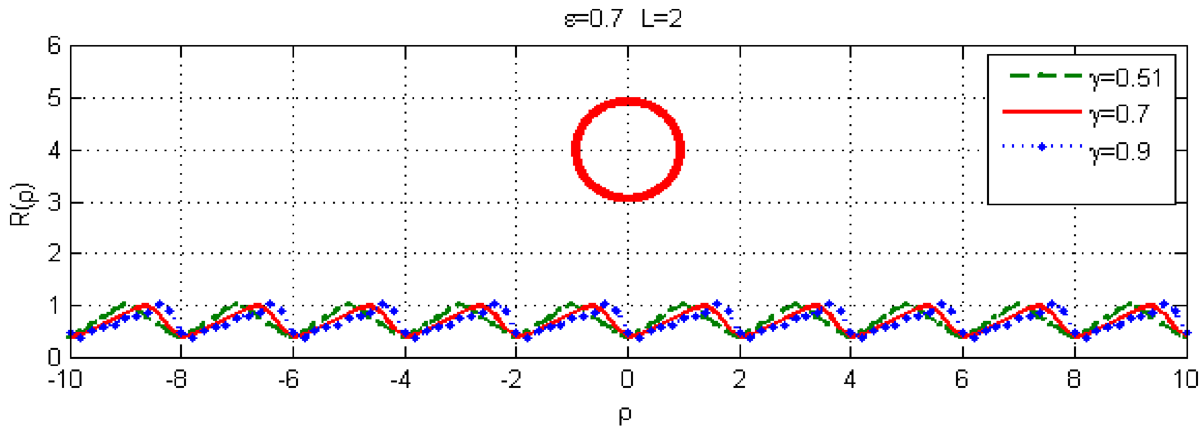

Fig.4: The function of the roughness $\mathcal{R}(\rho)$ based on bispherical coordinates ρ for different values of factor δ.

Figure 4 presents the variation of the function of the roughness based on bispherical coordinates ρ, for different values of the factor δ. We find that if we change the value of the factor δ, the local structure of the roughness changes.

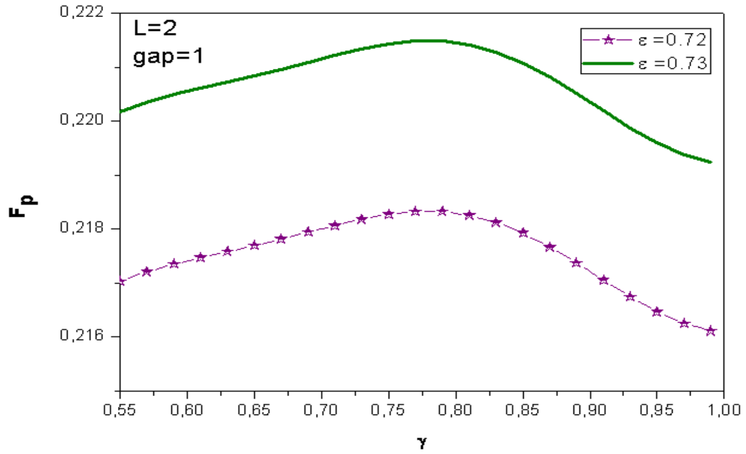

Fig.5: the lift force based on values of factor *δ*, for different values of the roughness amplitude

Figure 5 presents the variation of the lift force based on values of the factor *δ*, for two amplitude values of the roughness (*ε= 0.72* and *ε= 0.73*), we observe that the force increases when the amplitude of the roughness increases. Furthermore, we note that when the values of the factor δ increases the force increases until a maximum value of the factor (*δ = 0.8*), after it decreases. So, it is clear that the local shape of the roughness affects the lift force.

Figure 6 presents the variation of the function of the roughness $\mathcal{R}(\rho)$ based on bispherical coordinates ρ for different values of the translation factor *T*, this figure illustrates the position of a particle when the translation is on the rough surface.

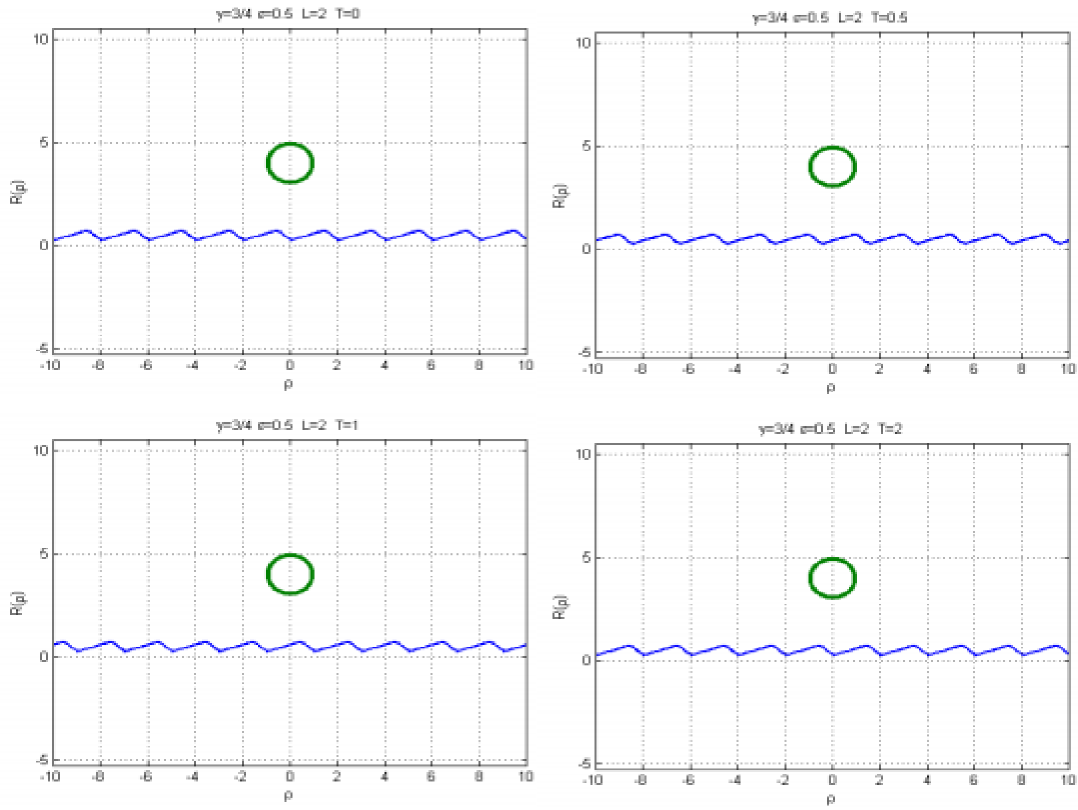

Fig.6: The function of the roughness $\mathcal{R}(\rho)$ based on bispherical coordinates ρ for different Translation factor *T* values *T*(*T=0, T=0.5, T=1, T=2*).

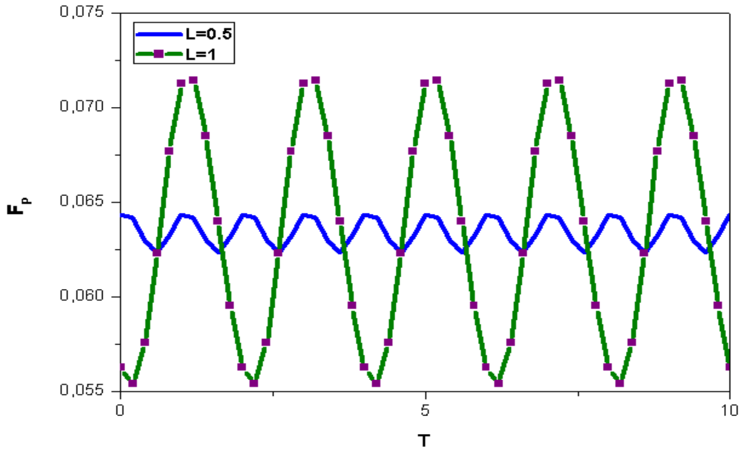

Fig.7: the lift force based on translation factor T for different roughness period values.

To explain the results obtained in Figure 3, we plot in Figure 7 the variation of the lift force based on translation factor T for different roughness period values with *gap=0.5* and *ε=0.5*, when we make a translation on a rough surface, we remark that the curve of the lift force is periodic for all values of *L* and is related to the position of the particle with respect to the rough wall. It is clear that the local shape of the roughness affects the lift force, these results are similar to those obtained in [10].

V. CONCLUSION

We have established a numerical technique allows to get very important results for the roughness force. These precise results, achieved in bispherical coordinates, are valid until the lubrication area and they are similar to those obtained in [10].

We concluded that the lift force increases with the amplitude of the roughness but the gap rough wall decreases with increasing the lift force.

We can also conclude that when the period of the roughness increases, the force reach zero, in this case the rough wall behaves like a smooth wall. In conclusion the local shape of the rough surface, has a great influence on the lift force.

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